

HC VERMA Solutions for Class 11 Physics Chapter 8 Work and Energy

Question 1

The mass of cyclist together with the bike is 90 kg. Calculate the increase in kinetic energy if the speed increases from 6'0 km/h to 12 km/h.

Solution 1

$$K.E. = \frac{1}{2}mv^2$$

$$\text{Change in K.E.} = \frac{1}{2}m[v^2 - u^2]$$

$$= \frac{1}{2} \times 90 \times [12^2 - 6^2]$$
$$= 375\text{J}$$

Question 2

A block of mass 2.00 kg moving at a speed of 10.0 m/s accelerates at 3.00 m/s² for 5.00 s. Compute its final kinetic energy.

Solution 2

$$\text{Final speed } v = u + at$$

$$= 10 + 3 \times 5$$

$$v = 25\text{m/s}$$

Now,

$$K.E. = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 2 \times 25^2$$

$$K.E. = 625\text{J}$$

Question 3

A box is pushed through 4.0 m across a floor offering 100 N resistance. How much work is done by the resisting force?

Solution 3

Resisting force

$$F_R = F \cdot d$$

$$F_R = F d \cos\theta = 100 \times 4$$

$$F_R = 400\text{ J}$$

Question 4

A block of mass 5.0 kg slides down an incline of inclination 30° and length 10 m. Find the work done by the force of gravity.

Solution 4

Work done,

$$W = mgh$$

$$= 5 \times 9.8 \times (\sin 30^\circ \times 10)$$

$$W = 245\text{ J}$$

Question 5

A constant force of 2.50 N accelerates a stationary particle of mass 15 g through a displacement of 2.50 m. Find the work done and the average power delivered?

Solution 5

$$W = F.d$$

$$W = Fd \cos\theta$$

$$W = 2.5 \times 2.5 \times \cos\theta$$

$$W = 6.25\text{J}$$

Now,

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 - 0 = 6.25$$

$$v = 28.8 \text{ m/s}$$

$$\text{And } F = ma$$

$$a = \frac{F}{m} = 2.5 / 0.015 = 166.66 \text{ m/s}^2$$

$$\text{So, } v = u + at$$

$$t = \frac{(v-u)}{a} = 28.8 / 166.66 = 0.1728 \text{ sec}$$

$$\text{And average power, } P_w = \frac{W}{t} = 6.25 / 0.1728 = 36.1 \text{ W}$$

Question 6

A particle moves from a point $r = (2 \text{ m})\hat{i} + (3 \text{ m})\hat{j}$ to another point $r_2 = (3 \text{ m})\hat{i} + (2 \text{ m})\hat{j}$ during which a certain force $F = (5 \text{ N})\hat{i} + (5 \text{ N})\hat{j}$ acts on it. Find the work done by the force on the particle during the displacement.

Solution 6

$$r = r_2 - r_1$$

$$= (3\hat{i} + 2\hat{j}) - (2\hat{i} + 3\hat{j})$$

$$r = \hat{i} - \hat{j}$$

$$\text{Work done } W = F \times r$$

$$W = (5\hat{i} + 5\hat{j}) \times (\hat{i} - \hat{j})$$

$$W = 0$$

Question 7

A man moves on a straight horizontal road with a block of mass 2 kg in his hand. If he covers a distance of 40 m with an acceleration of 0.5 m/s^2 , find the work done by the man on the block during the motion.

Solution 7

$$F = ma$$

$$W = F.d$$

$$W = 1 \times 40$$

$$W = 40 \text{ J}$$

Question 8

A force $F = a + bx$ acts on a particle in the x-direction, where a and b are constants. Find the work done by this force during a displacement from $x = 0$ to $x = d$.

Solution 8

Work done, $W = \int_0^d F \, dx$ ($F = a + bx$)

$$W = \left(a + \frac{1}{2}bd \right) d$$

Question 9

A block of mass 250 g slides down an incline of inclination 37° with a uniform speed. Find the work done against the friction as the block slides through 1 m.

Solution 9

Force = μR and $F = mg \sin \theta$

$mg \sin \theta = \mu R$

Work done $W = \mu R \cos \theta$

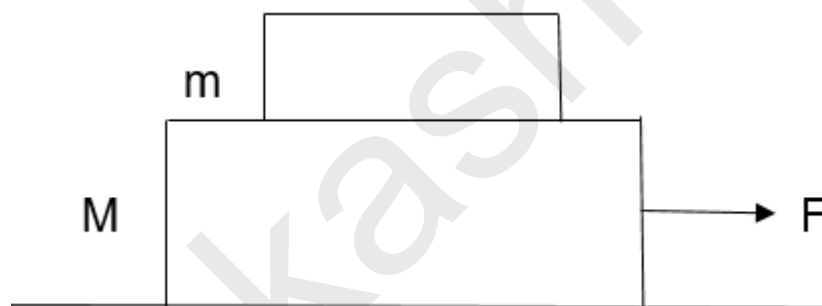
$W = mg \sin \theta \times \cos \theta \times S$

$W = 0.25 \times 9.8 \times 0.60 \times 1 \times 1$

$W = 1.5 \text{ J}$

Question 10

A block of mass m is kept over another block of mass M and the system rests on a horizontal surface (figure 8-E1). A constant horizontal force F acting on the lower block produces an acceleration $F/2$ ($m + M$) in the system, the two blocks always move together. (a) Find the coefficient of kinetic friction between the bigger block and the horizontal surface. (b) Find the frictional force acting on the smaller block. (c) Find the work done by the force of friction on the smaller block by the bigger block during a displacement d of the system.



Solution 10

$$(a) \mu_k = \frac{ma}{R} = \frac{ma}{mg} = \frac{a}{g}$$

$$\mu_k = \frac{F}{2(m+M)g}$$

$$(b) F_s = \mu_k R = \frac{F}{2}(m+M)g \times mg$$

$$F_s = \frac{mF}{2}(m+M)$$

$$(c) W = F_s d$$

$$W = \frac{mFd}{2} (m + M)$$

Question 11

A box weighing 2000 N is to be slowly slid through 20 m on a straight track having friction coefficient 0.2 with the box. (a) Find the work done by the person pulling the box with a chain at an angle θ with the horizontal. (b) Find the work when the person has chosen a value of θ which ensures him the minimum magnitude of the force.

Solution 11

$$(a) R + P \sin \theta = 2000 \dots\dots 1$$

$$P \cos \theta - 0.2 R = 0 \dots\dots (2)$$

Solving 1 and 2 equations we get,

$$P = \frac{400}{\cos \theta + 0.2 \sin \theta}$$

$$\text{and work done } W = PS \cos \theta$$

$$W = \frac{40000}{5 + \tan \theta}$$

(b) For minimum force,

$$\frac{dW}{d\theta} [\cos \theta + 0.2 \sin \theta] = 0 \text{ (from equation 1)}$$

$$\text{Also, } W = 40000 / 5 + \tan \theta = 40000 / 5 + 0.2$$

$$W = 7690 \text{ J}$$

Question 12

A block of weight 100 N is slowly slid up on a smooth incline of inclination 37° by a person. Calculate the work done by the person in moving the block through a distance of 2.0 m, if the driving force is (a) parallel to the incline and (b) in the horizontal direction.

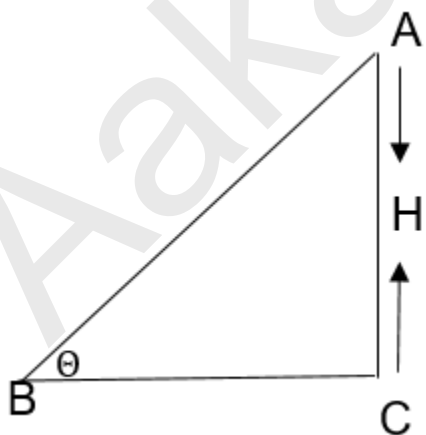
Solution 12

$$\text{Force } F = mg \sin \theta$$

$$F = 100 \times \sin 37^\circ = 60 \text{ N}$$

$$(a) \text{ Work done } W = Fd \cos \theta$$

$$W = 60 \times 2 \times \cos 0 = 120 \text{ J}$$



(b) In triangle ABC, AB = 2m, $\theta = 37^\circ$

$$AC = h = 1.3$$

$$\text{work done } W = mgh$$

$$W = 100 \times 1.2$$

$$W = 120 \text{ J}$$

Question 13

Find the average frictional force needed to stop a car weighing 500 kg in a distance of 25 m if the initial speed is 72 km/h.

Solution 13

$$v^2 - u^2 = -2as$$

$$a = \frac{(v^2 - u^2)}{-2s}$$

$$a = \frac{(0 - 400)}{-2} \times 25 = 8 \text{ m/s}^2$$

$$\text{frictional force } F = ma = 500 \times 8$$

$$F = 4000 \text{ N}$$

Question 14

Find the average force needed to accelerate a car weighing 500 kg from rest to 72 km/h in a distance of 25 m.

Solution 14

$$v^2 - u^2 = 2as$$

$$a = \frac{(v^2 - u^2)}{2s}$$

$$a = \frac{(400 - 0)}{2} \times 25$$

$$a = 8 \text{ m/s}^2$$

$$\text{Force, } F = ma = 500 \times 8$$

$$F = 4000 \text{ N}$$

Question 15

A particle of mass m moves on a straight line with its velocity varying with the distance travelled according to the equation $v = a\sqrt{x}$, where a is a constant. Find the total work done by all the forces during a displacement from $x = 0$ to $x = d$.

Solution 15

$$v = a\sqrt{x}$$

$$v_1 = 0 \text{ and } v_2 = a\sqrt{d}$$

$$v_2^2 - v_1^2 = 2a'd$$

$$a' = \frac{(v_2^2 - v_1^2)}{2d} = \frac{a^2}{2}$$

$$\text{Force } F = ma' = (ma^2)/2$$

$$\text{Work done } W = Fd \cos \theta$$

$$W = \frac{ma^2}{2} \times d$$

$$W = \frac{ma^2 d}{2}$$

Question 16

A block of mass 2.0 kg kept at rest on an inclined plane of inclination 37° is pulled up the plane by applying a constant force of 20 N parallel to the incline. The force acts for one second. (a) Show that

the work done by the applied force does not exceed 40 J. (b) Find the work done by the force of gravity in that one second if the work done by the applied force is 40 J. (c) Find the kinetic energy of the block at the instant the force ceases to act. Take $g = 10 \text{ m/s}^2$.

Solution 16

(a) For a free body,

$$\text{Force } F = mg \sin \theta + ma$$

$$a = \frac{m}{(F - mg \sin \theta)}$$

$$a = \frac{2}{2}$$

$$a = 4 \text{ m/s}^2$$

$$s = ut + \frac{1}{2} at^2$$

$$s = 0 \times 1 + \frac{1}{2} \times 4 \times 1^2$$

$$s = 2 \text{ m}$$

$$W = Fs = 20 \times 2$$

$$W = 40 \text{ J}$$

(b) $W = Fs$

$$s = \frac{W}{F} = \frac{40}{20}$$

$$s = 2$$

$$W = -mgh = -mg [s \sin 37^\circ]$$

$$W = -20 \times 1.2$$

$$W = -24 \text{ J}$$

(c) $v = u + at = 4 \times 10$

$$v = 40 \text{ m/s}$$

$$K.E. = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times 2 \times 16$$

$$K.E. = 16 \text{ J}$$

Question 17

A block of mass 2.0 kg is pushed down an inclined plane of inclination 37° with a force of 20 N acting parallel to the incline. It is found that the block moves on the incline with an acceleration of 10 m/s^2 . If the block started from rest, find the work done (a) by the applied force in the first second, (b) by the weight of the block in the first second and (c) by the frictional force acting on the block in the first second. Take $g = 10 \text{ m/s}^2$.

Solution 17

$$(a) S = ut + \frac{1}{2} at^2$$

$$S = 0 + \frac{1}{2} at^2$$

$$S = 5 \text{ m}$$

$$\text{Work done } W = Fs \cos \theta = 20 \times 5 \times 1$$

$$W = 100 \text{ J}$$

$$(b) h = S \sin \theta = 5 \sin 37^\circ = 3 \text{ m}$$

work done, $W = mgh = 2 \times 10 \times 3$
 $W = 60 \text{ J}$

(c) Frictional force $F_f = mg \sin\theta$
and work done $W = F_f s \cos\theta$
 $W = mg \sin\theta s \sin\theta$
 $W = 20 \times 0.60 \times 5$
 $W = 60 \text{ J}$

Question 18

A 250 g block slides on a rough horizontal table. Find the work done by the frictional force in bringing the block to rest if it is initially moving at a speed of 40 cm/s. If the friction coefficient between the table and the block is 0.1, how far does the block move before coming to rest?

Solution 18

We know,

$$ma = \mu R$$

$$(\mu mg)$$

$$a = \frac{\mu mg}{m} = \mu g$$

$$a = 0.1 \times 9.8 = 0.98 \text{ m/s}^2$$

and

$$v^2 - u^2 = 2as$$

$$s = \frac{v^2 - u^2}{2a} = (0 - 0.4^2) / 2 \times 0.98$$

$$s = 0.082 \text{ m}$$

Work done

$$W = -\mu R s \sin\theta = -0.1 \times 2.5 \times 0.082 \times 1$$

$$W = -0.02 \text{ J}$$

Question 19

Water falling from a 50 m high fall is to be used for generating electric energy. If $1.8 \times 10^5 \text{ kg}$ of water falls per hour and half the gravitational potential energy can be converted into electric energy, how many 100 W lamps can be lit.

Solution 19

Potential Energy

$$\text{P.E.} = mgh = 1.8 \times 10^5 \times 9.8 \times 50$$

$$\text{P.E.} = 882 \times 10^5 \text{ J/hr}$$

$$\text{Electrical energy} = \frac{1}{2} \text{P.E.} = 441 \times 10^5 \text{ J/hr}$$

$$\text{Power } P = (441 \times 10^5) / 3600$$

$$P = 12.25 \times 10^3 \text{ W}$$

$$\text{No. of 100W lamps that can be lit} = (12.25 \times 10^3) / 100$$

$$\text{Number of lamps} = 122.5 \approx 123$$

Question 20

A person is painting his house walls. He stands on a ladder with a bucket containing paint in one hand and a brush in other. Suddenly the bucket slips from his hand and falls down on the floor. If the bucket with the paint had a mass of 6.0 kg and was at a height of 2.0 m at the time it slipped, how much gravitational potential energy is lost together with the paint?

Solution 20

Initial P.E. at height h ,

$$\text{P.E.}_i = mgh = 6 \times 9.8 \times 2 = 117.6 \text{ J}$$

Final P.E. at the floor,

$$P.E_f = mgh = 0$$

$$P.E. = 117.6 - 0$$

$$P.E. = 117.6 \text{ J}$$

Question 21

A projectile is fired from the top of a 40 m high cliff with an initial speed of 50 m/s at an unknown angle. Find its speed when it hits the ground.

Solution 21

By law of conservation of energy

$$mgh + \frac{1}{2} m v_i^2 = \frac{1}{2} m v_f^2$$

$$10 \times 40 + \frac{1}{2} \times 50^2 = \frac{1}{2} \times v_f^2$$

$$v = v_f = 57.4 \text{ m/s}$$

$$v \approx 58 \text{ m/s}$$

Question 22

The 200 m free style women's swimming gold medal at Seol Olympic 1988 went to Heike Friedrich of East Germany when she set a new Olympic record of 1 minute and 57.56 seconds. Assume that she covered most of the distance with a uniform speed and had to exert 460 W to maintain her speed. Calculate the average force of resistance offered by the water during the swim.

Solution 22

$$\text{Power, } P = \frac{1}{2} W/t$$

$$W = P \times t = 460 \times 117.56 \dots\dots(1)$$

Also

$$W = F \times d = F \times 200 \dots\dots(2)$$

From 1 and 2

$$F = (460 \times 117.56)/200 = 270.3$$

$$F \approx 270 \text{ N}$$

Question 23

The US athlete Florence Griffith-Joyner won the 100 m sprint gold medal at Seol Olympic 1988 setting a new Olympic record of 10.54 s. Assume that she achieved her maximum speed in a very short-time and then ran the race with that speed till she crossed the line. Take her mass to be 50 kg. (a) Calculate the kinetic energy of Griffith-Joyner at her full speed. (b) Assuming that the track, the wind etc. offered an average resistance of one tenth of her weight, calculate the work done by the resistance during the run. (c) What power Griffith-Joyner had to exert to maintain uniform speed?

Solution 23

$$(a) v = \frac{d}{t} = 100/10.54 \text{ m/s}$$

$$K.E. = \frac{1}{2} m v^2 = \frac{1}{2} \times 50 \times 9.487^2$$

$$K.E. = 2250 \text{ J}$$

$$(b) \text{Weight} = mg = 50 \times 9.8 = -490 \text{ J}$$

and work done

$$W = -R d = \frac{-mg}{10} \times d = \frac{-490 \times 100}{10}$$

$$W = -4900 \text{ J}$$

$$(c) \text{ Power } P = \frac{W}{t} = \frac{4900}{10.54}$$

$$P = 465 \text{ W}$$

Question 24

A water pump lifts water from a level 10 m below the ground. Water is pumped at a rate of 30 kg/minute with negligible velocity. Calculate the minimum horsepower the engine should have to do this.

Solution 24

$$\frac{m}{t} = 30 \text{ kg/min} = 0.5 \text{ kg/s}$$

and

$$\text{Power} = \frac{m}{t} \times g \times h = 0.5 \times 9.81 \times 10$$

$$P = 49 \text{ W}$$

$$\text{Horse power} = \frac{P}{746} = 6.6 \times 10^{-2} \text{ hp}$$

Question 25

An unruly demonstrator lifts a stone of mass 200 g from the ground and throws it at his opponent. At the time of projection, the stone is 150 cm above the ground and has a speed of 3.00 m/s. Calculate the work done by the demonstrator during the process. If it takes one second for the demonstrator to lift the stone and throw, what horsepower does he use?

Solution 25

$$\text{Work done } W = mgh + \frac{1}{2}mv^2$$

$$W = 0.2 \times 9.8 \times 1.5 + \left(\frac{1}{2} \times 0.2 \times 3^2\right)$$

$$W = 3.84 \text{ J}$$

Question 26

In a factory it is desired to lift 2000 kg of metal through a distance of 12 m in 1 minute. Find the minimum horsepower of the engine to be used.

Solution 26

$$W = Fd \cos \theta$$

$$W = mg \cos \theta = 2000 \times 10 \times 12 \times 1$$

$$W = 24 \times 10^4 \text{ J}$$

$$P = \frac{W}{t} = (24 \times 10^4) / 60 = 4000 \text{ W}$$

$$\text{Horse power} = P / 746 = 4000 / 746 = 5.3 \text{ hp}$$

Question 27

A scooter company gives the following specifications about its product.

Weight of the scooter - 95 kg

Maximum speed - 60 km/h

Maximum engine power - 3.5 hp

Pick up time to get the maximum speed - 5 s

Check the validity of these specifications.

Solution 27

Max. acceleration,

$$a = \frac{v - u}{t}$$

$$a = (50/3 - 0) / 5 = 3.33 \text{ m/s}^2$$

Force

$$F = ma = 95 \times 3.33$$

$$F = 316.6 \text{ N}$$

Velocity

$$v = \frac{F}{m} = (3.5 \times 746)/316.6$$

$$v = 8.2 \text{ m/s}$$

Question 28

A block of mass 30.0 kg is being brought down by a chain. If the block acquires a speed of 40.0 cm/s in dropping down 2.00 m, find the work done by the chain during the process.

Solution 28

$$v^2 - u^2 = 2as$$

$$a = (0.4^2 - 0)/2 \times 2$$

$$a = 0.04 \text{ m/s}^2$$

Force

$$F = ma - mg$$

and

Work done

$$W = Fs \cos\theta = m(a-g) s \cos\theta$$

$$W = 30 (0.04 - 9.8) \times 2 \times 1$$

$$W = -585.5 \text{ J} \approx -586 \text{ J}$$

Question 29

The heavier block in an Atwood machine has a mass twice that of the lighter one. The tension in the string is 16.0 N when the system is set into motion. Find the decrease in the gravitational potential energy during the first second after the system is released from rest.

Solution 29

Net force is

$$T - 2mg + 2ma = 0 \dots\dots 1$$

$$T - mg - ma = 0 \dots\dots 2$$

from equation 1 and 2

$$a = \frac{T}{4m} = \frac{16}{4m}$$

$$a = \frac{4}{m} \text{ m/s}^2$$

Now,

$$s = ut + \frac{1}{2} at^2$$

$$s = 0 + \frac{1}{2} \times \frac{4}{m} \times 1^2$$

$$s = \frac{2}{m}$$

Decrease in P.E. = mgh

$$\text{P.E.} = (2m - m) \times 9.8 \times (2/m)$$

$$\text{P.E.} = 19.6 \text{ J}$$

Question 30

The two blocks in an Atwood machine have masses 2.0 kg and 3.0 kg. Find the work done by gravity during the fourth second after the system is released from rest.

Solution 30

$$T - 3g + 3a = 0 \quad 1$$

$$T - 2g - 2a = 0 \quad 2$$

Solving equation 1 and 2

$$a = \frac{g}{5} \text{ m/s}^2$$

Now,

$$S = \frac{a}{2} (2m - 1)$$

$$S = \frac{g}{5} [2 \times 4 - 1]$$

$$h = S = 6.86 \text{ m}$$

Mass

$$m = m_1 - m_2$$

$$m = 3 - 2 = 1 \text{ kg}$$

Decrease in P.E. = mgh

$$\text{P.E.} = 1 \times 9.8 \times 6.86$$

$$\text{P.E.} = 67 \text{ J}$$

Question 31

Consider the situation shown in figure (8-E2). The system is released from rest and the block of mass 1.0 kg is found to have a speed 0.3 m/s after it has descended through a distance of 1 m. Find the coefficient of kinetic friction between the block and the table.

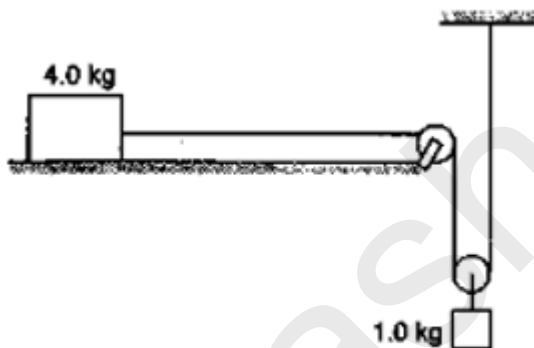


Figure 8-E2

Solution 31

Work done = change in K.E.

$$-\mu R S + mg = \left[\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right] - 0$$

$$-\mu \times 40 \times 2 + 1 \times 10 = \frac{1}{2} (4 \times 0.3^2 + 1 \times 0.6^2)$$

$$\mu = 0.12$$

Question 32

A block of mass 100 g is moved with a speed of 5.0 m/s at the highest point in a closed circular tube of radius 10 cm kept in a vertical plane. The cross-section of the tube is such that the block just fits in it. The block makes several oscillations inside the tube and finally stops at the lowest point. Find the work done by the tube on the block during the process.

Solution 32

Work done by block

$$= \text{T.E. at A} - \text{T.E. at B}$$

$$= \frac{1}{2}mv^2 + mgh - 0$$

$$= \frac{1}{2} \times 0.1 \times 5^2 + 0.1 \times 10 \times 0.2$$

$$W = 1.45 \text{ J}$$

Work done by tube = - Work done by block

$$W' = -1.45 \text{ J}$$

Question 33

A car weighing 1400 kg is moving at a speed of 54 km/h up a hill when the motor stops. If it is just able to reach the destination which is at a height of 10 m above the point, calculate the work done against friction (negative of the work done by the friction).

Solution 33

Work done,

$$W = K.E_T - P.E_T$$

$$W = (0 + \frac{1}{2}mv^2) - mgh$$

$$W = [\frac{1}{2} \times 1400 \times 15^2] - [1400 \times 9.8 \times 10]$$

$$W = 20300 \text{ J}$$

Question 34

A small block of mass 200 g is kept at the top of a frictionless incline which is 10 m long and 3.2 m high. How much work was required (a) to lift the block from the ground and put it at the top, (b) to slide the block up the incline? What will be the speed of the block when it reaches the ground, if (c) it falls off the incline and drops vertically on the ground (d) it slides down the incline? Take $g = 10 \text{ m/s}^2$.

Solution 34

(a) Work done $W = mgh = 0.2 \times 10 \times 3.2$ (to lift the block)

$$W = 6.4 \text{ J}$$

(b) Work done $W = mg \sin \theta$ (to slide the block)

$$W = 6.4 \text{ J}$$

(c) When block falls on ground work done is

$$W = \frac{1}{2}mv^2 - 0$$

$$6.4 = \frac{1}{2} \times 0.2 \times v^2$$

$$v = 8 \text{ m/s}$$

$$(d) W = \frac{1}{2}mv^2 - 0$$

$$6.4 = \frac{1}{2} \times 0.2 \times v^2$$

$$v = 8 \text{ m/s}$$

Question 35

In a children's park, there is a slide which has a total length of 10 m and a height of 8.0 m (figure 8-E3). Vertical ladder are provided to reach the top. A boy weighing 200 N climbs up the ladder to the

top of the slide and slides down to the ground. The average friction offered by the slide is three tenth of his weight. Find (a) the work done by the ladder on the boy as he goes up, (b) the work done by the slide on the boy as he comes down. Neglect any work done by forces inside the body of the boy.



Figure 8-E3

Solution 35

(a) Work done, $W = 0$ because no work is done by ladder.

(b) Work done $W = \mu RS = fd$
 $= - (200 \times 3/10) \times 10 = -600 \text{ J}$

(c) Work done, $W = mg \sin\theta \times d = 200 \times 8/10 \times 10$
 $W = 1600 \text{ J}$

Question 36

Figure (8-E4) shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground?



Figure 8-E4

Solution 36

By conservation of energy of point A and B

$$mgH = \frac{1}{2}mv^2 + mgh$$

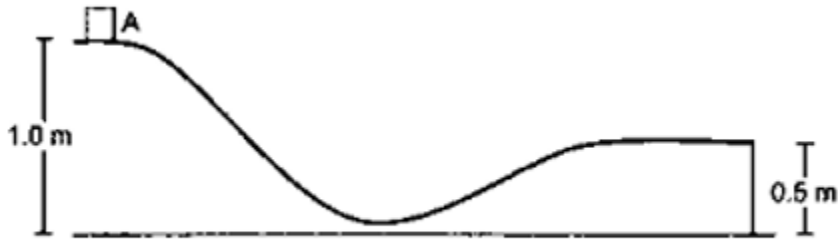


Figure 8-E4

$$g \times 1 = \frac{1}{2}v^2 + g \times 0.5$$

$$v = \sqrt{9.8} = 3.13 \text{ m/s}$$

After B, body obeys projectile motion, so

$$S = ut + \frac{1}{2}at^2$$

$$-0.5 = u \sin\theta t + \frac{1}{2}(-9.8)t^2$$

$$t = 0.31 \text{ s}$$

$$\text{and } X = u \cos\theta \times t$$

$$X = 1$$

Question 37

A block weighing 10 N travels down a smooth curved track AB joined to a rough horizontal surface (figure 8-E5). The rough surface has a friction coefficient of 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface, how far will it move on the rough surface?

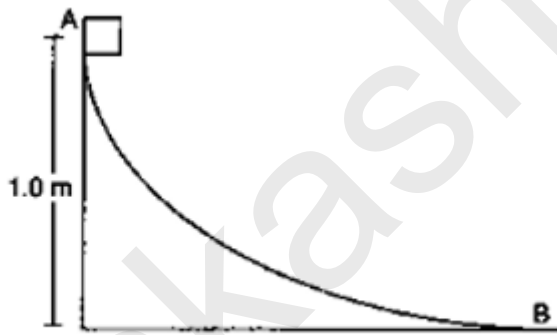


Figure 8-E5

Solution 37

$$\text{Work done, } W = mgh = 10 \times 1$$

$$W = 10 \text{ J}$$

$$\text{Frictional force, } F = \mu R = \mu mg = 0.2 \times 10$$

$$F = 2 \text{ N}$$

$$\text{and } W = Fs$$

$$s = \frac{W}{F} = \frac{10}{2} = 5 \text{ m}$$

Question 38

A uniform chain of mass m and length l overhangs a table with its two third part on the table. Find the work to be done by a person to put the hanging part back on the table.

Solution 38

$$W = (m/l) \, dx \, g(x)$$

Total work done is :

$$W = \int_0^{l/3} (m/l) \, dx \, g(x)$$

$$W = mgl/18$$

Question 39

A uniform chain of length L and mass M overhangs a horizontal table with its two third part on the table. The friction coefficient between the table and the chain is μ . Find the work done by the friction during the period the chain slips off the table.

Solution 39

Work done for small element is:

$$dW = \mu R \, dx$$

$$dW = \mu (M/L \, dx) \, g(x)$$

Total work done is :

$$W = \int_0^{2L/3} \mu M/L \, g \, (x) \, dx$$

$$W = (2\mu MgL)/9$$

Question 40

A block of mass $1 \, \text{kg}$ is placed at the point A of a rough track shown in figure (8-E6). If slightly pushed towards right, it stops at the point B of the track. Calculate the work done by the frictional force on the block during its transit from A to B.

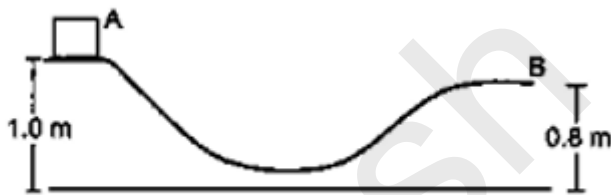


Figure 8-E6

Solution 40

Work done = change in P.E.

$$W = mgh - mgH$$

$$W = 1 \times 10 \times (0.8 - 1)$$

$$W = -2 \, \text{J}$$

Question 41

A block of mass $5.0 \, \text{kg}$ is suspended from the end of a vertical spring which is stretched by $10 \, \text{cm}$ under the load of the block. The block is given a sharp impulse from below so that it acquires an upward speed of $2.0 \, \text{m/s}$. How high will it rise? Take $g = 10 \, \text{m/s}^2$.

Solution 41

$$k = (mg)/x = 50/0.1 = 500 \, \text{N/m}$$

Total energy T.E. = $\frac{1}{2} m v^2 + \frac{1}{2} k x^2$ (1)
and

T.E. (at height h) = $\frac{1}{2} k (h - x)^2 + mgh$ (2)

Equating 1 and 2

$$\frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k (h - x)^2 + mgh$$

$$\frac{1}{2} \times 5 \times 2^2 + \frac{1}{2} \times 500 \times 0.1^2 = \frac{1}{2} \times 500 (h - 0.1^2) + 5 \times 10 \times h$$

$$h = 0.2 \text{ m}$$

$$h = 20 \text{ cm}$$

Question 42

A block of mass 250 g is kept on a vertical spring of spring constant 100 N/m fixed from below. The spring is now compressed to have a length 10 cm shorter than its natural length and the system is released from this position. How high does the block rise? Take $g = 10 \text{ m/s}^2$.

Solution 42

By law of conservation of energy:

$$\frac{1}{2} k x^2 = mgh$$

$$\frac{1}{2} \times 100 \times 0.1 = mgh = 0.25 \times 10 \times h$$

$$h = 0.2 \text{ m}$$

$$h = 20 \text{ cm}$$

Question 43

Figure (8-E7) shows a spring fixed at the bottom end of an incline of inclination 37° . A small block of mass 2 kg starts slipping down the incline from a point 4.8 m away from the spring. The block compresses the spring by 20 cm, stops momentarily and then rebounds through a distance of 1 m up the incline. Find (a) the friction coefficient between the plane and the block and (b) the spring constant of the spring. Take $g = 10 \text{ m/s}^2$.

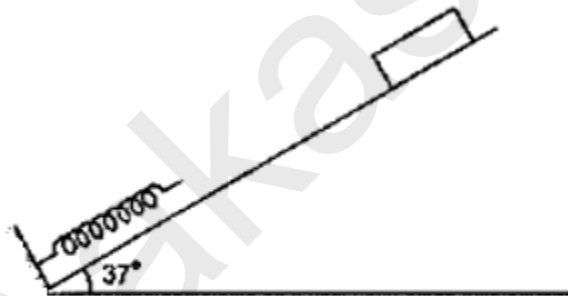


Figure 8-E7

Solution 43

By work energy principle

(i) for downward motion,

$$0 - 0 = (-mg \sin \theta \times s) - \mu R s - \frac{1}{2} k x^2$$

$$80\mu + 0.02R = 60 \text{ (1)}$$

(ii) for upward motion

$$0 - 0 = (-mg \sin \theta \times s) - \mu R s + \frac{1}{2} k x^2$$

$$-16 \mu + 0.02 k = 12 \dots\dots (2)$$

Solving 1 and 2 we get,

$$\mu = 0.5$$

and

$$k = 1000 \text{ N/m}$$

Question 44

A block of mass m moving at a speed v compresses a spring through a distance x before its speed is halved. Find the spring constant of the spring.

Solution 44

Energy at A = Energy at B

$$\frac{1}{2} m v_a^2 = \frac{1}{2} m v_b^2 + k x^2$$

$$m v^2 = \frac{m v^2}{4} + k x^2$$

$$k = \frac{3 m v^2}{2 x^2}$$

Question 45

Consider the situation shown in figure (8-E8). Initially the spring is unstretched when the system is released from rest. Assuming no friction in the pulley, find the maximum elongation of the spring.

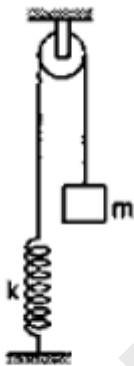


Figure 8-E8

Solution 45

From figure

$$m g x = \frac{1}{2} k x^2$$

$$x = \frac{2 m g}{k}$$

Question 46

A block of mass m is attached to two unstretched springs of spring constants k_1 and k_2 as shown in figure (8-E9). The block is displaced towards right through a distance x and is released. Find the speed of the block as it passes through the mean position shown.

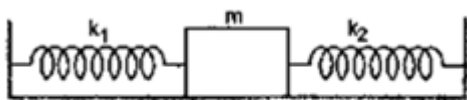


Figure 8-E9

Solution 46

By law of conservation of energy

$$\frac{1}{2} m v^2 = \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 x^2$$

$$\text{So, } v = x \sqrt{(k_1 + k_2)/m}$$

Question 47

A block of mass m sliding on a smooth horizontal surface with a velocity v meets a long horizontal spring fixed at one end and having spring constant k as shown in figure (8-E10). Find the maximum compression of the spring. Will the velocity of the block be the same as when it comes back to the original position shown?



Figure 8-E10

Solution 47

(a) By law of conservation of energy:

$$\frac{1}{2} m v^2 = \frac{1}{2} k x^2$$

$$\text{Max } x = v \sqrt{m/k}$$

(b) Velocity of the block will not be the same when it comes back to the original position, it will be of smaller magnitude and opposite direction.

Question 48

A small block of mass 100 g is pressed against a horizontal spring fixed at one end to compress the spring through 5.0 cm (figure 8-E11). The spring constant is 100 N/m. When released, the block moves horizontally till it leaves the spring. Where will it hit the ground 2 m below the spring?

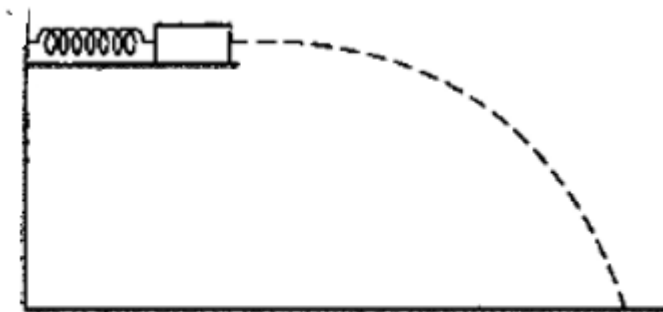


Figure 8-E11

Solution 48

By law of conservation of energy:

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$v = 1.58 \text{ m/s}^2$$

Now, projectile motion

$$y = (u \sin \theta)t - \frac{1}{2}gt^2$$

$$\theta = 0^\circ, y = -2$$

$$t = 0.63 \text{ sec}$$

and

$$x = (u \cos \theta)t = 1.58 \times 0.63$$

$$x = 1 \text{ m}$$

Question 49

A small heavy block is attached to the lower end of a light rod of length l which can be rotated about its clamped upper end. What minimum horizontal velocity should the block be given so that it moves in a complete vertical circle?

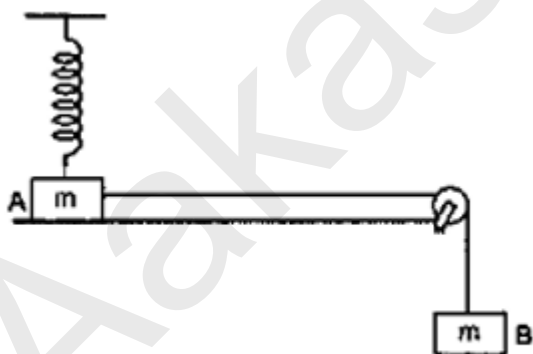


Figure 8-E12

Solution 49

Minimum velocity at A = v

Minimum velocity at B = 0

By law of conservation of energy

$$\frac{1}{2} m v^2 = mg (2l)$$

$$v = 2\sqrt{gl}$$

Question 50

Figure (8-E12) shows two blocks A and B, each having a mass of 320 g connected by a light string passing over a smooth light pulley. The horizontal surface on which the block A can slide is smooth. The block A is attached to a spring of spring constant 40 N/m whose other end is fixed to a support 40 cm above the horizontal surface. Initially, the spring is vertical and upstretched when the system is released to move. Find the velocity of the block A at the instant it breaks off the surface below it. Take $g = 10 \text{ m/s}^2$.

Solution 50

from figure

$$kx \cos\theta = mg$$

$$40 \times x \times (0.4/[0.4 + x]) = 0.32 \times 10$$

$$x = 0.1\text{m}$$

and

$$S = \sqrt{(h + x)^2 - h^2} = \sqrt{[(0.4+0.1)^2 - 0.4^2]}$$

$$S = 0.3\text{m}$$

Change in K.E. = Work done

$$\frac{1}{2} m v^2 = \frac{1}{2} k x^2 + mgs$$

$$\frac{1}{2} \times 0.32 \times v^2 = \frac{1}{2} \times 40 \times 0.1^2 + 0.32 \times 10 \times 0.3$$

$$v = 1.5 \text{ m/s}$$

Question 51

One end of a spring of natural length h and spring constant k is fixed at the ground and the other is fitted with a smooth ring of mass m which is allowed to slide on a horizontal rod fixed at a height h (figure 8-E13). Initially, the spring makes an angle of 37° with the vertical when the system is released from rest. Find the speed of the ring when the spring becomes vertical.

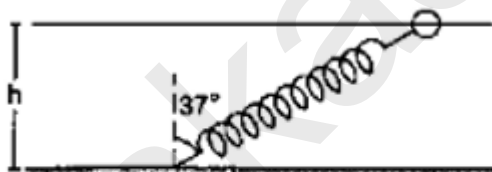


Figure 8-E13

Solution 51

$$\cos\theta = \cos 37^\circ = BC/AC = 4/5$$

$$AC = (h + x) = \frac{5h}{4}$$

$$\text{and } x = AC - h = \frac{5h}{4} - h = \frac{h}{4}$$

By work energy principle,

$$\frac{1}{2} k x^2 = \frac{1}{2} m v^2$$

$$v = \frac{h}{4} \sqrt{\frac{k}{m}}$$

Question 52

Figure (8-E14) shows a light rod of length 1 rigidly attached to a small heavy block at one end and a hook at the other end. The system is released from rest with the rod in a horizontal position. There is a fixed smooth ring at a depth h below the initial position of the hook and the hook gets into the ring as it reaches there. What should be the minimum value of h so that the block moves in a complete circle about the ring?

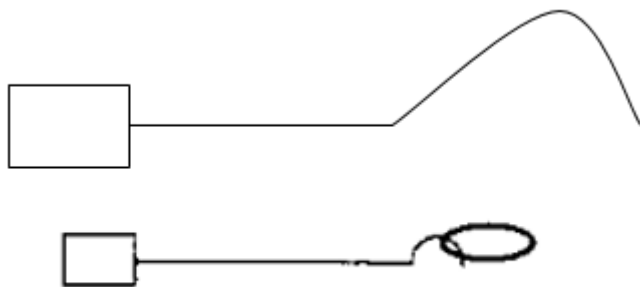


Figure 8-E14

Solution 52

$$v_{\min} = \sqrt{2gl}$$

Total energy at A = Total energy at B

$$mgh = \frac{1}{2} m v^2 \quad [v = \sqrt{2gl}]$$

$$h = l$$

Question 53

The bob of a pendulum at rest is given a sharp hit to impart a horizontal velocity $\sqrt{10gl}$, where l is the length of the pendulum. Find the tension in the string when (a) the string is horizontal, (b) the bob is at its highest point and (c) the string makes an angle of 60° with the upward vertical.

Solution 53

(a) By law of conservation of energy,

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_2^2 + mgl$$

$$\frac{1}{2} \times m \times (\sqrt{10gl})^2 = \frac{1}{2} m v_2^2 + mgl$$

$$v_2 = \sqrt{8gl}$$

and

$$T_B = (mv_2^2)/R = [m \times (\sqrt{8gl})^2]/l$$

$$T_B = 8mg$$

(b) By law of conservation at A and C

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_3^2 + mgh$$

$$\frac{1}{2} \times m \times (\sqrt{10gl})^2 = \frac{1}{2} m v_3^2 + mg \times 2l$$

$$v_3 = \sqrt{6gl}$$

and

Tension

$$T_c = \left[\frac{mv_3^2}{l} \right] - mg$$

$$T_c = 5mg$$

(c) By law of conservation of energy at A and D

$$\frac{1}{2} mv_1^2 = \frac{1}{2} m v_4^2 + mgh$$

$$\frac{1}{2} (\sqrt{logl})^2 = \frac{1}{2} m v_4^2 + gl (1 + \cos 60^\circ)$$

$$v_4 = \sqrt{7gl}$$

and

Tension

$$T_D = \frac{mv^2}{l} - mg \cos 60^\circ$$

by putting $v = \sqrt{7gl}$ we get,

$$T_D = 6.5mg$$

Question 54

A simple pendulum consists of a 50 cm long string connected to a 100 g ball. The ball is pulled aside so that the string makes an angle of 37° with the vertical and is then released. Find the tension in the string when the bob is at its lowest position.



Figure 8-E15

Solution 54

from figure,

$$\cos \theta = \frac{AC}{AB}$$

and

$$AC = AB \cos \theta$$

$$AC = 0.4$$

and

$$CD = AD - AC$$

$$CD = 0.1 \text{ m}$$

Energy is same at B and D

$$\frac{1}{2} m v^2 = mgh$$

$$\frac{1}{2} v^2 = 10 \times 0.1$$

$$v = \sqrt{2} \text{ m/s}$$

$$\text{Tension } T = \frac{mv^2}{r} + mg$$

$$T = \frac{(0.1 \times 2)}{0.5} + 0.1 \times 10$$

$$T = 1.4 \text{ N}$$

Question 55

Figure (8-E15) shows a smooth track, a part of which is a circle of radius R . A block of mass m is pushed against a spring of spring constant k fixed at the left end and is then released. Find the initial

compression of the spring so that the block presses the track with a force mg when it reaches the point P, where the radius of the track is horizontal.

Solution 55

According to figure,

$$(mv^2)/R = mg$$

$$v = \sqrt{gR}$$

Energy is same at A and P

$$\frac{1}{2} kx^2 = \frac{mgR + 2mgR}{2}$$

$$x = \sqrt{\frac{3mgR}{K}}$$

Question 56

The bob of a stationary pendulum is given a sharp hit to impart it a horizontal speed of $\sqrt{3gl}$. Find the angle rotated by the string before it becomes slack.

Solution 56

Change in K.E. is given as

$$\frac{1}{2} m v^2 - \frac{1}{2} m u^2 = -mgh$$

$$v^2 = 3gl - 2gl(1 + \cos\theta) \dots\dots\dots 1$$

and

$$(mv^2)/l = mg \cos\theta$$

$$v^2 = gl \cos\theta \dots\dots\dots 2$$

from equation 1 and 2

$$3gl - 2gl - 2gl \cos\theta = gl \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{1}{3}\right)$$

Thus,

$$\text{Angle} = 180^\circ - \cos^{-1}\left(\frac{1}{3}\right)$$

$$\text{Angle} = \cos^{-1}(-1/3)$$

Question 57

A heavy particle is suspended by a 1.5 m long string. It is given a horizontal velocity of $\sqrt{57}$ m/s. (a) Find the angle made by the string with the upward vertical, when it becomes slack. (b) Find the speed of the particle at this instant. (c) Find the maximum height reached by the particle over the point of suspension. Take $g = 10 \text{ m/s}^2$.

Solution 57

$$(a) mg \cos\theta = (mv^2)/l$$

$$v = (\sqrt{gl \cos\theta}) \dots\dots 1$$

Change in K.E. is given as :

$$\frac{1}{2} m v^2 - \frac{1}{2} m u^2 = mgh$$

$$\frac{1}{2} v^2 - \frac{1}{2} (\sqrt{57})^2 = g(1.5[1 + \cos\theta])$$

$$v = (\sqrt{57 - 3g(1 + \cos\theta)}) \dots\dots\dots 2$$

from equation 1 and 2

$$1.5 \times g \cos\theta = 57 - 3g(1 + \cos\theta)$$

$$\theta = 53^\circ$$

(b) $v = \sqrt{57 - 3g(1 + \cos\theta)}$ $\theta = 53^\circ$
 $v = 3 \text{ m/s}$

(c) After point B projectile motion will take place

$$H = OE + DC$$

$$H = 1.5 \cos\theta + \frac{(u^2 \sin^2\theta)}{2 \times g}$$

$$H = 1.5 \times \frac{3}{5} + \frac{(9 \times 0.8^2)}{2 \times 10}$$

$$H = 1.2 \text{ m}$$

Question 58

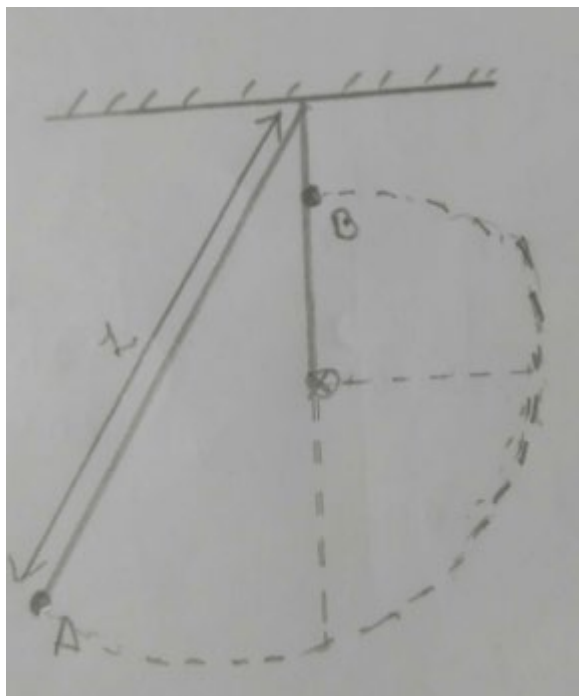
A simple pendulum of length L having a bob of mass m is deflected from its rest position by an angle θ and released (figure 8-E16). The string hits a peg which is fixed at a distance x below the point of suspension and the bob starts going in a circle centred at the peg. (a) Assuming that initially the bob has a height less than the peg, show that the maximum height reached by the bob equals its initial height. (b) If the pendulum is released with $\theta = 90^\circ$ and $x = L/2$ find the maximum height reached by the bob above its lowest position before the string becomes slack. (c) Find the minimum value of x/L for which the bob goes in a complete circle about the peg when the pendulum is released from $\theta = 90^\circ$.



Figure 8-E16

Solution 58

Total energy at A and B are same



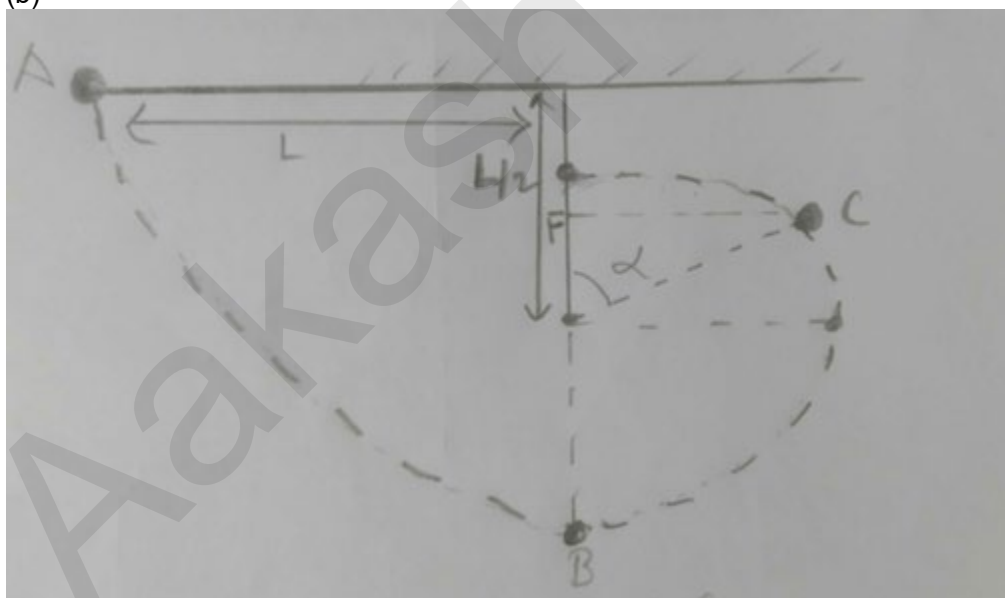
$$K.E_A + P.E_A = K.E_B + P.E_B$$

we get,

$$P.E_A = P.E_B$$

Thus maximum height is same as initial height.

(b)



$$\text{At C, } \frac{1}{2} m v_c^2 - 0 = mg (L/2)(1 - \cos\alpha)$$

$$V_c = (\sqrt{gL (1 - \cos\alpha)}) \dots\dots 1$$

Again,

$$V_c = (\sqrt{gL \cos\alpha}) \dots\dots 2$$

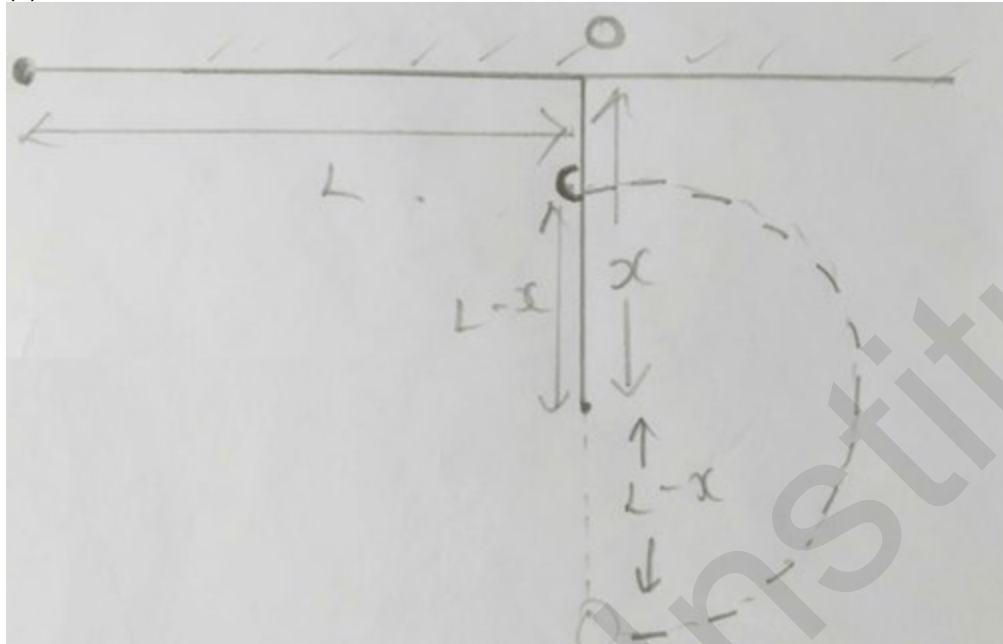
equating 1 and 2

$$\cos\alpha = \frac{2}{3} \dots\dots 3$$

$$\text{Now, } BF = \frac{L}{2} + \frac{L}{2} \cos \alpha$$

$$BF = (5L/6)$$

(c)



$$(mv_c^2)/L-x = mg$$

$$v_c = (\sqrt{g(L-x)}) \dots 1$$

Also,

$$\frac{1}{2} mv_c^2 - 0 = mg(OC)$$

$$v_c = (\sqrt{2g(2x-L)}) \dots 2$$

Equating 1 and 2

$$g(L-x) = 2g(2x-L)$$

$$x/L = 0.6$$

Question 59

A particle slides on the surface of a fixed smooth sphere starting from the topmost point. Find the angle rotated by the radius through the particle, when it leaves contact with the sphere.

Solution 59

From figure,

$$(mv^2)/R = mg \cos \theta$$

$$v = (\sqrt{gR \cos \theta}) \dots 1$$

and

Change in K.E. = Work done

$$\frac{1}{2} m v^2 - 0 = mg(R - R \cos \theta)$$

$$v = (\sqrt{2gR[1 - \cos \theta]}) \dots 2$$

from 1 and 2

$$gR \cos \theta = 2gR(1 - \cos \theta)$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Question 60

A particle of mass m is kept on a fixed, smooth sphere of radius R at a position, where the radius through the particle makes an angle of 30° with the vertical. The particle is released from this position. (a) What is the force exerted by the sphere on the particle just after the release? (b) Find the distance travelled by the particle before it leaves contact with the sphere.

Solution 60

(a) Net force = $mg \cos\theta = \sqrt{3}/2 \, mg$

(b) $N = 0$

and

$$\frac{mv^2}{R} = mg \cos\theta$$

$$v = (\sqrt{gR \cos\theta}) \dots\dots 1$$

and

$$\frac{1}{2}$$

$$mv^2 = mgR (\cos 30^\circ - \cos\theta)$$

$$v = (\sqrt{2gR \sqrt{3}/2 - \cos\theta}) \dots\dots 2$$

from equation 1 and 2 we get,

$$\frac{1}{2}$$

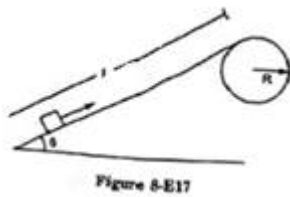
$$\theta = \cos^{-1}(\sqrt{3})$$

and distance, $l = R [\theta - \pi/6]$

$$l = 0.43R$$

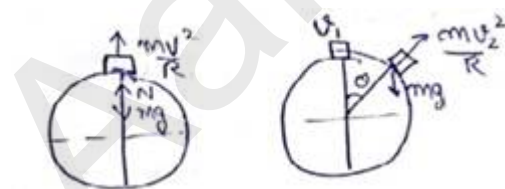
Question 61

A particle of mass m is kept on the top of a smooth sphere of radius R . It is given a sharp impulse which imparts it a horizontal speed v . (a) Find the normal force between the sphere and the particle just after the impulse. (b) What should be the minimum value of v for which the particle does not slip on the sphere? (c) Assuming the velocity v to be half the minimum calculated in part, (b) Find the angle made by the radius through the particle with the vertical when it leaves the sphere.



Solution 61

(a) from free body diagram



$$N = mg - (mv^2)/R$$

(b) When particle moves with maximum velocity then,

$$mg = (mv^2)/R$$

$$v = \sqrt{gR}$$

(c) From figure,

$$v_1 = (\sqrt{gR})/2 \dots\dots 1$$

and

$$(mv_2^2)/R = mg \cos\theta$$

$$v_2 = (\sqrt{gR \cos\theta}) \dots\dots 2$$

Also, change in K.E. = Work done

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = mgR(1 - \cos\theta) \dots\dots 3$$

from equation 2 and 3

$$gR \cos\theta = v_1^2 + 2gR(1 - \cos\theta)$$

$$\theta = \cos^{-1}\left(\frac{3}{4}\right)$$

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Question 62

Figure (8-E17) shows a smooth track which consists of a straight inclined part of length 1 joining smoothly with the circular part. A particle of mass m is projected up the incline from its bottom. (a) Find the minimum projection-speed v , for which the particle reaches the top of the track. (b) Assuming that the projection-speed is $2v_0$ and that the block does not lose contact with the track before reaching its top, find the force acting on it when it reaches the top. (c) Assuming that the projection-speed is only slightly greater than $2v_0$, where will the block lose contact with the track?

Solution 62

(a) $F_{AB} = mg \sin\theta$

$$W_B = F_{AB}S$$

$$W_B = mg \sin\theta \times l \dots\dots 1$$

and

$$W_C = mgR(1 - \cos\theta) \dots\dots 2$$

Total work done

$$W = W_B + W_C$$

and

change in K.E. = Work done

$$v_0 = [\sqrt{2g(R(1 - \cos\theta) + l \sin\theta)}]$$

(b) By work energy equation

change in K.E. = Work done

$$v_c = [\sqrt{6g(l \sin\theta + R(1 - \cos\theta))}]$$

and

Force,

$$F = (mv_c^2)/R$$

$$F = 6mg(l/R \sin\theta + 1 - \cos\theta)$$

(c) $(mv^2)/R = mg \cos\theta$

$$v = \sqrt{gR \cos\theta} \dots\dots 1$$

Also,

$$v = [\sqrt{2gR(1 - \cos\theta)}] \dots\dots 2$$

equating 1 and 2 we get,

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Question 63

A chain of length 1 and mass m lies on the surface of a smooth sphere of radius $R > 1$ with one end tied to the top of the sphere. (a) Find the gravitational potential energy of the chain with reference level

at the centre of the sphere. (b) Suppose the chain is released and slides down the sphere. Find the kinetic energy of the chain, when it has slid through an angle θ . (c) Find the tangential acceleration dv/dt of the chain when the chain starts sliding down.

Solution 63

$$\begin{aligned} \text{(a) G.P.E} &= dm \, g \, R \cos\theta \\ &= (mgR \cos\theta \, d\theta)/l \quad [dm = m/l \, R \, d\theta] \\ \text{Total G.P.E} &= \int_0^{l/R} (mgR^2)/l \cos\theta \, d\theta \quad [\theta = l/R] \\ \text{G.P.E} &= (mgR^2)/l \sin(l/R) \end{aligned}$$

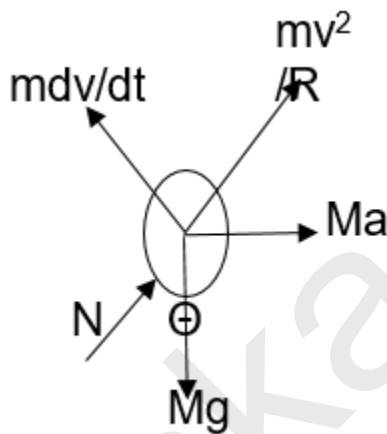
$$\begin{aligned} \text{(b) K.E.} &= \text{change in P.E} \\ \text{K.E.} &= (mgR^2)/l [\sin(l/R) + \sin\theta - \sin(\theta + l/R)] \end{aligned}$$

$$\begin{aligned} \text{(c) From above equation} \\ \text{K.E.} &= (mgR^2)/l [\sin(l/R) + \sin\theta - \sin(\theta + l/R)] \\ \text{as } \theta &= 0 \\ \frac{dv}{dt} &= (gR/l)[1 - \cos(l/R)] \end{aligned}$$

Question 64

A smooth sphere of radius R is made to translate in a straight line with a constant acceleration a . A particle kept on the top of the sphere is released from there at zero velocity with respect to the sphere. Find the speed of the particle with respect to the sphere as a function of the angle θ it slides.

Solution 64



According to figure,

$$\begin{aligned} m \frac{dv}{dt} &= ma \cos\theta + mg \sin\theta \\ V dv &= a R \cos\theta \, d\theta + g R \sin\theta \, d\theta \quad [v = R \, d\theta/dt] \\ \text{Integ} \\ V &= [2R (a \sin\theta + g \cos\theta)]^{1/2} \end{aligned}$$