RD Sharma Solutions For Class 10 Maths Chapter 6 Trigonometric Identities

Prove the following trigonometric identities:

1. $(1 – \cos^2 A) \operatorname{cosec}^2 A = 1$

Solution:

Taking the L.H.S,

$(1 – \cos^2 A) \operatorname{cosec}^2 A$

$= (\sin^2 A) \operatorname{cosec}^2 A$ [$\because \sin^2 A + \cos^2 A = 1 \Rightarrow 1 – \sin^2 A = \cos^2 A$]

$= 1^2$

= 1 = R.H.S

– Hence Proved

2. $(1 + \cot^2 A) \sin^2 A = 1$

Solution:

By using the identity,

$\operatorname{cosec}^2 A – \cot^2 A = 1 \Rightarrow \operatorname{cosec}^2 A = \cot^2 A + 1$

Taking,

L.H.S = $(1 + \cot^2 A) \sin^2 A$

$= \operatorname{cosec}^2 A \sin^2 A$

$= (\operatorname{cosec} A \sin A)^2$

$= ((1/\sin A) \times \sin A)^2$

$= (1)^2$

= 1

= R.H.S

– Hence Proved

3. $\tan^2 \theta \cos^2 \theta = 1 - \cos^2 \theta$

Solution:

We know that,

$\sin^2 \theta + \cos^2 \theta = 1$

Taking,

L.H.S = $\tan^2 \theta \cos^2 \theta$

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$= (\tan\theta \times \cos\theta)^2$

$= (\sin\theta)^2$

$= \sin^2\theta$

$= 1 - \cos^2\theta$

= R.H.S

– Hence Proved

4. $\text{cosec}\,\theta\sqrt{(1 - \cos^2\theta)} = 1$

Solution:

Using identity,

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$

Taking L.H.S,

$\text{L.H.S} = \text{cosec}\,\theta\sqrt{(1 - \cos^2\theta)}$

$= \text{cosec}\,\theta\sqrt{(\sin^2\theta)}$

$= \text{cosec}\,\theta \times \sin\theta$

$= 1$

= R.H.S

– Hence Proved

5. $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1) = 1$

Solution:

Using identities,

$(\sec^2\theta - \tan^2\theta) = 1$ and $(\text{cosec}^2\theta - \cot^2\theta) = 1$

We have,

$\text{L.H.S} = (\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$

$= \tan^2\theta \times \cot^2\theta$

$= (\tan\theta \times \cot\theta)^2$

$= (\tan\theta \times 1/\tan\theta)^2$

$= 1^2$

$= 1$

= R.H.S

– Hence Proved

6. $\tan\theta + 1/\tan\theta = \sec\theta\,\text{cosec}\,\theta$

Solution:

We have,

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L.H.S = tan θ + 1/ tan θ

= ($\tan^2$ θ + 1)/ tan θ

= $\sec^2$ θ / tan θ [∵ $\sec^2$ θ − $\tan^2$ θ = 1]

= (1/$\cos^2$ θ) x 1/ (sin θ/cos θ) [∵ tan θ = sin θ / cos θ]

= cos θ/ (sin θ x $\cos^2$ θ)

= 1/ cos θ x 1/ sin θ

= sec θ x cosec θ

= sec θ cosec θ

= R.H.S

– Hence Proved

7. cos θ/ (1 – sin θ) = (1 + sin θ)/ cos θ

Solution:

We know that,

$\sin^2$ θ + $\cos^2$ θ = 1

So, by multiplying both the numerator and the denominator by (1+ sin θ), we get

$$\frac{\cos\theta}{1-\sin\theta}$$

$$=\frac{\cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$=\frac{\cos\theta}{(1+\sin\theta)(1-\sin^2\theta)}$$

$$=\frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$

$$=\frac{(1+\sin\theta)}{\cos\theta}$$

L.H.S =

= R.H.S

– Hence Proved

8. **cos θ/ (1 + sin θ) = (1 – sin θ)/ cos θ**

Solution:

We know that,

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$\sin^2\theta + \cos^2\theta = 1$

So, by multiplying both the numerator and the denominator by (1- sin θ), we get

$$\frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{\cos\theta(1-\sin\theta)}{(1-\sin^2\theta)}$$

$$= \frac{\cos\theta(1-\sin\theta)}{(\cos^2\theta)}$$

$$= \frac{(1-\sin\theta)}{\cos\theta}$$

$$= \frac{(1-\sin\theta)}{\cos\theta}$$

L.H.S =

= R.H.S

– Hence Proved

9. $\cos^2\theta + 1/(1 + \cot^2\theta) = 1$

Solution:

We already know that,

$\text{cosec}^2\theta - \cot^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$

Taking L.H.S,

$$\text{L.H.S} = \cos^2 A + \frac{1}{1+\cot^2 A}$$

$$= \cos^2 A + \frac{1}{\text{cosec}^2 A}$$

$$= \cos^2 A + \left(\frac{1}{\text{cosec}\, A}\right)^2$$

$= \cos^2 A + \sin^2 A$

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= 1

= R.H.S

– Hence Proved

10. $\sin^2 A + 1/(1 + \tan^2 A) = 1$

Solution:

We already know that,

$\sec^2\theta - \tan^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$

Taking L.H.S,

$$\text{L.H.S} = \sin A^2 + \frac{1}{1 + \tan^2 A}$$

$$= \sin A^2 + \frac{1}{\sec^2 A}$$

$$= \sin A^2 + \left(\frac{1}{\sec A}\right)^2$$

$= \sin^2 A + \cos^2 A$

= 1

= R.H.S

– Hence Proved

11.

$$\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \operatorname{cosec}\theta - \cot\theta$$

Solution:

We know that, $\sin^2\theta + \cos^2\theta = 1$

Taking the L.H.S,

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$$\text{L.H.S} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

= cosec θ – cot θ

= R.H.S

– Hence Proved

12. 1 – cos θ/ sin θ = sin θ/ 1 + cos θ

Solution:

We know that,

$\sin^2\theta + \cos^2\theta = 1$

So, by multiplying both the numerator and the denominator by (1+ cos θ), we get

$$\text{L.H.S} = \frac{1-\cos^2\theta}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{(\sin^2\theta)}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{(\sin\theta)}{(1+\cos\theta)}$$

= R.H.S

– Hence Proved

13. sin θ/ (1 – cos θ) = cosec θ + cot θ

Solution:

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Taking L.H.S,

$$L.H.S = \frac{\sin\theta}{1-\cos\theta}$$

On multiplying by its conjugates, we have

$$= \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$$

Since, $(1 - \cos^2\theta) = \sin^2\theta$

$$= \frac{\sin\theta + (\sin\theta \times \cos\theta)}{\sin^2\theta}$$

$$= \frac{\sin\theta}{\sin^2\theta} + \frac{\sin\theta \times \cos\theta}{\sin^2\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

= cosec θ + cot θ

= R.H.S

– Hence Proved

14. (1 – sin θ) / (1 + sin θ) = (sec θ – tan θ)^2

Solution:

Taking the L.H.S,

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$$L.H.S = \frac{1-\sin\theta}{1+\sin\theta}$$

On multiplying by its conjugate, we have

$$= \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

Since, $1 - \sin^2\theta = \cos^2\theta$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$= (\sec\theta - \tan\theta)^2$

= R.H.S

– Hence Proved

15. $$\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta} = \cot\theta$$

Solution:

Taking L.H.S,

$$L.H.S = \frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta}$$

Here, $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$$= \frac{\operatorname{cosec}^2\theta \times \tan\theta}{\sec^2\theta}$$

$$= \frac{1}{\sin^2\theta} \times \frac{\cos^2\theta}{1} \times \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

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= cot θ

= R.H.S

– Hence Proved

16. $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

Solution:

Taking L.H.S,

L.H.S = $\tan^2\theta - \sin^2\theta$

$$\text{Since, } \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$$= \sin^2\theta\left[\frac{1}{\cos^2\theta} - 1\right]$$

$$= \sin^2\theta\left[\frac{1-\cos^2\theta}{\cos^2\theta}\right]$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta$$

= $\tan^2\theta \sin^2\theta$

= R.H.S

– Hence Proved

17. $(\text{cosec}\,\theta + \sin\theta)(\text{cosec}\,\theta - \sin\theta) = \cot^2\theta + \cos^2\theta$

Solution:

Taking L.H.S = $(\text{cosec}\,\theta + \sin\theta)(\text{cosec}\,\theta - \sin\theta)$

On multiplying we get,

= $\text{cosec}^2\theta - \sin^2\theta$

= $(1 + \cot^2\theta) - (1 - \cos^2\theta)$ [Using $\text{cosec}^2\theta - \cot^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$]

= $1 + \cot^2\theta - 1 + \cos^2\theta$

= $\cot^2\theta + \cos^2\theta$

= R.H.S

– Hence Proved

18. $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$

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Solution:

Taking L.H.S = (sec θ + cos θ)(sec θ – cos θ)

On multiplying we get,

= $\sec^2 \theta - \sin^2 \theta$

= $(1 + \tan^2 \theta) - (1 - \sin^2 \theta)$ [Using $\sec^2 \theta - \tan^2 \theta = 1$ and $\sin^2 \theta + \cos^2 \theta = 1$]

= $1 + \tan^2 \theta - 1 + \sin^2 \theta$

= $\tan^2 \theta + \sin^2 \theta$

= R.H.S

– Hence Proved

19. sec A(1- sin A) (sec A + tan A) = 1

Solution:

Taking L.H.S = sec A(1 – sin A)(sec A + tan A)

Substituting sec A = 1/cos A and tan A =sin A/cos A in the above we have,

L.H.S = 1/cos A (1 – sin A)(1/cos A + sin A/cos A)

= $1 - \sin^2 A / \cos^2 A$ [After taking L.C.M]

= $\cos^2 A / \cos^2 A$ [$\because 1 - \sin^2 A = \cos^2 A$]

= 1

= R.H.S

– Hence Proved

20. (cosec A – sin A)(sec A – cos A)(tan A + cot A) = 1

Solution:

Taking L.H.S = (cosec A – sin A)(sec A – cos A)(tan A + cot A)

Putting, $\operatorname{cosec} A = \frac{1}{\sin A}, \sec A = \frac{1}{\cos A}, \tan A = \frac{\sin A}{\cos A}, \cot A = \frac{\cos A}{\sin A}$

Subsituting the above in the L.H.S, we get

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right) \left(\frac{1 - \cos^2 A}{\cos A}\right) \left(\frac{\sin^2 A + \cos^2 A}{\sin A \ \cos A}\right)$$

= $(\cos^2 A / \sin A)(\sin^2 A / \cos A)(1 / \sin A \cos A)$ [$\because \sin^2 \theta + \cos^2 \theta = 1$]

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= (sin A cos A) (1/ cos A sin A)

= 1

= R.H.S

– Hence Proved

21. $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) = 1$

Solution:

Taking L.H.S = $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

And, we know $\sin^2\theta + \cos^2\theta = 1$ and $\sec^2\theta - \tan^2\theta = 1$

So,

L.H.S = $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

= $(1 + \tan^2\theta)\{(1 - \sin\theta)(1 + \sin\theta)\}$

= $(1 + \tan^2\theta)(1 - \sin^2\theta)$

= $\sec^2\theta\ (\cos^2\theta)$

= $(1/\cos^2\theta) \times \cos^2\theta$

= 1

= R.H.S

– Hence Proved

22. $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$

Solution:

We know that,

$\cot^2 A = \cos^2 A/\sin^2 A$ and $\tan^2 A = \sin^2 A/\cos^2 A$

Substituting the above in L.H.S, we get

L.H.S = $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A$

= $\{\sin^2 A\ (\cos^2 A/\sin^2 A)\} + \{\cos^2 A\ (\sin^2 A/\cos^2 A)\}$

= $\cos^2 A + \sin^2 A$

= 1 [$\because \sin^2\theta + \cos^2\theta = 1$]

= R.H.S

– Hence Proved

(i) $\cot\theta - \tan\theta = \dfrac{2\cos 2\theta - 1}{\sin\theta * \cos\theta}$

(ii) $\tan\theta - \cot\theta = \left(\dfrac{2\sin^2\theta - 1}{\sin\theta * \cos\theta}\right)$

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23.

Solution:

(i) Taking the L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

L.H.S = cot θ – tan θ

$$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta \times \cos\theta}$$

$$= \frac{\cos^2\theta - (1 - \cos^2\theta)}{\sin\theta \times \cos\theta}$$

$$= \frac{\cos^2\theta - 1 - \cos^2\theta}{\sin\theta \times \cos\theta}$$

$$= \left(\frac{2\cos^2\theta - 1}{\sin\theta \times \cos\theta}\right)$$

= R.H.S

– Hence Proved

(ii) Taking the L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

L.H.S = tan θ – cot θ

$$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta - (1 - \sin^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta - (1 + \sin^2\theta)}{\sin\theta\cos\theta}$$

$$= \left(\frac{2\sin^2\theta - 1}{\sin\theta \quad \cos\theta}\right)$$

= R.H.S

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– Hence Proved

24. ($\cos^2\theta/\sin\theta$) – cosec θ + sin θ = 0

Solution:

Taking L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

$$L.H.S = \frac{\cos^2\theta}{\sin\theta} - \text{cosec}\,\theta + \sin\theta$$

$$= \left(\frac{\cos^2\theta}{\sin\theta} - \text{cosec}\,\theta\right) + \sin\theta$$

$$= \left(\frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta}\right) + \sin\theta$$

$$= \left(\frac{\cos^2\theta - 1}{\sin\theta}\right) + \sin\theta$$

$$= \left(\frac{-\sin^2\theta}{\sin\theta}\right) + \sin\theta$$

= – sin θ + sin θ

= 0

= R.H.S

- Hence proved

25. $\frac{1}{1+\sin A} + \frac{1}{1-\sin A} = 2\sec^2 A$

Solution:

Taking L.H.S,

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$$\text{LHS} = \frac{1}{1+\sin A} + \frac{1}{1-\sin A}$$

$$= \frac{(1-\sin A)+(1+\sin A)}{(1+\sin A)(1-\sin A)}$$

$$= \frac{1-\sin A+1+\sin A}{1-\sin^2 A} \quad \because (1+\sin A)(1-\sin A) = 1-\sin^2 A$$

$$= \frac{2}{1-\sin^2 A}$$

$$= \frac{2}{\cos^2 A} \quad [\because 1-\sin^2 A = \cos A]$$

$= 2 \sec^2 A$

= R.H.S

- Hence proved

$$\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$$

26.

Solution:

Taking the LHS and using $\sin^2\theta + \cos^2\theta = 1$, we have

$$\text{LHS} = \frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1+\sin^2\theta+2\sin\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$= 2/\cos\theta$

$= 2\sec\theta$

= R.H.S

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- Hence proved

$$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}=\frac{1+\sin^2\theta}{1-\sin^2\theta}$$

27.

Solution:

Taking the LHS and using $\sin^2\theta + \cos^2\theta = 1$, we have

$$\text{L. H. S} = \frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}$$

$$= \frac{(1+2\sin\theta+\sin^2\theta)+(1-2\sin\theta+\sin^2\theta)}{2\cos^2\theta}$$

$$= \frac{1+2\sin\theta+\sin^2\theta+1-2\sin\theta+\sin^2\theta}{2\cos^2\theta}$$

$$= \frac{2+2\sin^2\theta}{2\cos^2\theta}$$

$$= \frac{2(1+\sin^2\theta)}{2(1-\sin^2\theta)}$$

$$= \frac{(1+\sin^2\theta)}{(1-\sin^2\theta)}$$

= R.H.S

- Hence proved

$$\frac{1+\tan^2\theta}{1+\cot^2\theta} = \tan^2\theta$$

28.

Solution:

Taking L.H.S,

$$\frac{1+\tan^2\theta}{1+\cot^2\theta}$$

Using $\sec^2\theta - \tan^2\theta = 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$

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$$= \frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{\sec^2\theta}{\text{cosec}^2\theta}$$

$$= \frac{1}{\cos^2\theta}\ \frac{}{1}\sin^2\theta = \tan^2\theta$$

= R.H.S

- Hence proved

$$\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$$

29.

Solution:

Taking L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

Multiplying by (1 - cos θ) to numerator and denominator

$$\text{LHS} = \frac{1+\sec\theta}{\sec\theta}$$

$$= \frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\cos\theta+1}{\cos\theta}\cdot\cos\theta$$

$$= 1+\cos\theta$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$$

$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{\sin^2\theta}{1-\cos\theta}$$

= R.H.S

- Hence proved

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$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta$$

30.

Solution:

Taking LHS, we have

$$\text{LHS}=\frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\cot\theta}{1-\tan\theta}$$

$$=\frac{\tan^2\theta}{\tan\theta-1}+\frac{\cot\theta}{1-\tan\theta}$$

$$=\frac{1}{1-\tan\theta}\left[\frac{1}{\tan\theta}-\tan^2\theta\right]$$

$$=\frac{1}{1-\tan\theta}\left[\frac{1-\tan^3\theta}{\tan\theta}\right]$$

$$=\frac{1}{1-\tan\theta}\,\frac{(1-\tan\theta)(1+\tan\theta+\tan^2\theta)}{\tan\theta}\ \left[\text{Since, } a^3-b^3=(a-b)(a^2+ab+b^2)\right]$$

$$=\frac{1+\tan\theta+\tan^2\theta}{\tan\theta}$$

$$=\frac{1}{\tan\theta}+\frac{\tan\theta}{\tan\theta}+\frac{\tan^2\theta}{\tan\theta}$$

= 1 + tan θ + cot θ

= R.H.S

- Hence proved

31. $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

Solution:

From trig. Identities we have,

$\sec^2\theta - \tan^2\theta = 1$

On cubing both sides,

$(\sec^2\theta - \tan^2\theta)^3 = 1$

$\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta(\sec^2\theta - \tan^2\theta) = 1$

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[Since, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$]
$\sec^6\theta - \tan^6\theta - 3\sec^2\theta \tan^2\theta = 1$

$\Rightarrow \sec^6\theta = \tan^6\theta + 3\sec^2\theta \tan^2\theta + 1$

Hence, L.H.S = R.H.S

- Hence proved

32. $\text{cosec}^6\theta = \cot^6\theta + 3\cot^2\theta \text{ cosec}^2\theta + 1$

Solution:

From trig. Identities we have,

$\text{cosec}^2\theta - \cot^2\theta = 1$

On cubing both sides,

$(\text{cosec}^2\theta - \cot^2\theta)^3 = 1$

$\text{cosec}^6\theta - \cot^6\theta - 3\text{cosec}^2\theta \cot^2\theta (\text{cosec}^2\theta - \cot^2\theta) = 1$

[Since, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$]
$\text{cosec}^6\theta - \cot^6\theta - 3\text{cosec}^2\theta \cot^2\theta = 1$

$\Rightarrow \text{cosec}^6\theta = \cot^6\theta + 3 \text{ cosec}^2\theta \cot^2\theta + 1$

Hence, L.H.S = R.H.S

- Hence proved

$$\frac{(1 + \tan^2\theta)\cot\theta}{\text{cosec}^2\theta} = \tan\theta$$ **33.**

Solution:

Taking L.H.S and using $\sec^2\theta - \tan^2\theta = 1 \Rightarrow 1 + \tan^2\theta = \sec^2\theta$

$$\text{LHS} = \frac{\sec^2\theta \cdot \cot\theta}{\text{cosec}^2\theta}$$

$$= \frac{1 \cdot \sin^2\theta}{\cos^2\theta} \cdot \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

= R.H.S

- Hence proved

$$\frac{1 + \cos A}{\sin^2 A} = \frac{1}{1 - \cos A}$$ **34.**

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Solution:

Taking L.H.S and using the identity $\sin^2 A + \cos^2 A = 1$, we get

$\sin^2 A = 1 - \cos^2 A$

$\Rightarrow \sin^2 A = (1 - \cos A)(1 + \cos A)$

$$\text{LHS} = \frac{1 + \cos A}{(1 - \cos A)(1 + \cos A)}$$

$$= \frac{1}{(1 - \cos A)}$$

- Hence proved

$$\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1 + \sin A)^2}$$

35.

Solution:

We have,

$$\text{LHS} = \frac{\sec A - \tan A}{\sec A + \tan A}$$

Rationalizing the denominator and numerator with (sec A + tan A) and using $\sec^2 \theta - \tan^2 \theta = 1$ we get,

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$$= \frac{\sec^2 A - \tan^2 A}{(\sec A + \tan A)^2}$$

$$= \frac{1}{(\sec A + \tan A)^2}$$

$$= \frac{1}{(\sec^2 A + \tan^2 A + 2 \sec A \tan A)}$$

$$= \frac{1}{\left(\frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} + \frac{2\sin A}{\cos A}\right)}$$

$$\Rightarrow \frac{\cos^2 A}{1 + \sin^2 A + 2\sin A}$$

$$= \frac{\cos^2 A}{(1 + \sin A)^2}$$

= R.H.S

- Hence proved

$$1 + \frac{\cos A}{\sin A} = \frac{\sin A}{1 - \cos A}$$

36.

Solution:

We have,

$$\text{LHS} = \frac{1 + \cos A}{\sin A}$$

On multiplying numerator and denominator by (1 – cos A), we get

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$$= \frac{(1+\cos A)(1-\cos A)}{\sin A(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{\sin A(1-\cos A)}$$

$$= \frac{\sin^2 A}{\sin A(1-\cos A)}$$

$$= \frac{\sin A}{1-\cos A}$$

= R.H.S

- Hence proved

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

37. (i)

Solution:

Taking L.H.S and rationalizing the numerator and denominator with √(1 + sin A), we get

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \sqrt{\frac{(1+\sin A)}{\cos A}}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

= R.H.S

- Hence proved

$$\sqrt{\frac{(1-\cos A)}{(1+\cos A)}} + \sqrt{\frac{(1+\cos A)}{(1-\cos A)}} = 2\,\mathrm{cosec}\, A$$

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(ii)

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}} + \sqrt{\frac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(1-\cos^2 A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(\sin^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(\sin^2 A)}}$$

$$= \frac{(1-\cos A)}{(\sin A)} + \frac{(1+\cos A)}{(\sin A)}$$

$$= \frac{(1-\cos A+1+\cos A)}{(\sin A)}$$

$$= \frac{(2)}{(\sin A)}$$

= 2 cosec A

= R.H.S

- Hence proved

38. Prove that:

$$\sqrt{\frac{(\sec\theta-1)}{(\sec\theta+1)}} + \sqrt{\frac{(\sec\theta+1)}{(\sec\theta-1)}} = 2\ \text{cosec}\ \theta$$

(i)

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

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$$= \sqrt{\frac{(\sec\theta - 1)(\sec\theta - 1)}{(\sec\theta + 1)(\sec\theta - 1)}} + \sqrt{\frac{(\sec\theta + 1)(\sec\theta + 1)}{(\sec\theta - 1)(\sec\theta + 1)}}$$

$$= \sqrt{\frac{(\sec\theta - 1)^2}{(\sec^2\theta - 1)}} + \sqrt{\frac{(\sec\theta + 1)^2}{(\sec^2\theta - 1)}}$$

$$= \sqrt{\frac{(\sec\theta - 1)^2}{\tan^2\theta}} + \sqrt{\frac{(\sec\theta + 1)^2}{\tan^2\theta}}$$

$$= \frac{(\sec\theta - 1)}{\tan\theta} + \frac{(\sec\theta + 1)}{\tan\theta}$$

$$= \frac{(\sec\theta - 1 + \sec\theta + 1)}{\tan\theta}$$

$$= \frac{(2\cos\theta)}{\cos\theta\sin\theta}$$

$$= \frac{2}{\sin\theta}$$

= 2 cosec θ

= R.H.S

- Hence proved

(ii) $\sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)}} = 2\sec\theta$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

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$$= \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}} + \sqrt{\frac{(1-\sin\theta)^2}{(1-\sin^2\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{(\cos^2\theta)}} + \sqrt{\frac{(1-\sin\theta)^2}{(\cos^2\theta)}}$$

$$= \frac{(1+\sin\theta)}{(\cos\theta)} + \frac{(1-\sin\theta)}{(\cos\theta)}$$

$$= \sqrt{\frac{(1+\sin\theta+1-\sin\theta)}{(\cos\theta)}}$$

$$= \frac{(2)}{(\cos\theta)} = 2\sec\theta$$

= R.H.S

- Hence proved

(iii) $\sqrt{\frac{(1+\cos\theta)}{(1-\cos\theta)}} + \sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)}} = 2\,\mathrm{cosec}\,\theta$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

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$$= \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}} + \sqrt{\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}} + \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos^2\theta)}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{(\sin^2\theta)}} + \sqrt{\frac{(1+\cos\theta)^2}{(\sin^2\theta)}}$$

$$= \frac{(1-\cos\theta)}{(\sin\theta)} + \frac{(1+\cos\theta)}{(\sin\theta)}$$

$$= \frac{(1-\cos\theta+1+\cos\theta)}{(\sin\theta)}$$

$$= \frac{(2)}{(\sin\theta)}$$

= 2 cosec θ

= R.H.S

- Hence proved

$$\frac{\sec\theta-1}{\sec\theta+1} = \left(\frac{\sin\theta}{1+\cos\theta}\right)^2$$

(iv)

Solution:

Taking L.H.S, we have

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$$= \frac{\sec\theta - 1}{\sec\theta + 1} = \frac{1 - \cos\theta}{1 + \cos\theta}$$

On multiplying numerator and denominator by $1 + \cos\theta$, we get

$$= \frac{(1 - \cos\theta)(1 + \cos\theta)}{(1 + \cos\theta)(1 + \cos\theta)}$$

$$= \frac{(1 - \cos^2\theta)}{(1 + \cos\theta)^2}$$

$$= \frac{\sin^2\theta}{(1 + \cos\theta)^2}$$

$$= \left(\frac{\sin\theta}{1 + \cos\theta}\right)^2$$

= R.H.S

- Hence proved

$$(\sec A - \tan A)^2 = \frac{1 - \sin A}{1 + \sin A}$$

39.

Solution:

Taking LHS = $(\sec A - \tan A)^2$, we have

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$$= \left[\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right]^2$$

$$= \frac{(1 - \sin A)^2}{\cos^2 A}$$

$$= \frac{(1 - \sin A)^2}{1 - \sin^2 A}$$

$$= \frac{(1 - \sin A)^2}{(1 + \sin A)(1 - \sin A)}$$

$$= \frac{(1 - \sin A)}{(1 + \sin A)}$$

= R.H.S

- Hence proved

$$\frac{1 - \cos A}{1 + \cos A} = (\cot A - \operatorname{cosec} A)^2$$

40.

Solution:

Taking L.H.S and rationalizing the numerator and denominator with (1 – cos A), we get

$$= \frac{(1 - \cos A)(1 - \cos A)}{(1 + \cos A)(1 - \cos A)}$$

$$= \frac{(1 - \cos A)^2}{(1 - \cos^2 A)}$$

$$= \frac{(1 - \cos A)^2}{(\sin^2 A)}$$

$$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2$$

= (cosec A – cot A)^2

= (cot A – cosec A)^2

= R.H.S

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- Hence proved

$$\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2 \operatorname{cosec} A \cot A$$

41.

Solution:

Considering L.H.S and taking L.C.M and on simplifying we have,

$$= \frac{\sec A + 1 + \sec A - 1}{(\sec A + 1)(\sec A - 1)}$$

$$= \frac{2 \sec A}{(\sec^2 A - 1)}$$

$$= \frac{2 \sec A}{(\tan^2 A)}$$

$$= \frac{2 \cos^2 A}{(\cos A \sin^2 A)}$$

$$= \frac{2 \cos A}{(\sin^2 A)}$$

$$= \frac{2 \cos A}{(\sin A)(\sin A)}$$

= 2 cosec A cot A = RHS

- Hence proved

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$$

42.

Solution:

Taking LHS, we have

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$$= \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$$

$$= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A - \sin A}$$

= cos A + sin A

= RHS

- Hence proved

$$\frac{(\text{cosec } A)}{(\text{cosec } A - 1)} + \frac{(\text{cosec } A)}{(\text{cosec } A + 1)} = 2 \sec^2 A$$ **43.**

Solution:

Considering L.H.S and taking L.C.M and on simplifying we have,

$$= \frac{(\text{cosec } A)(\text{cosec } A + 1 + \text{cosec } A - 1)}{(\text{cosec}^2 A - 1)}$$

$$= \frac{(2 \text{ cosec}^2 A)}{\cot^2 A}$$

$$= \frac{2 \sin^2 A}{\sin^2 A. \cos^2 A}$$

$$= \frac{2}{\cos^2 A}$$

= 2 $\sec^2$ A

= RHS

- Hence proved

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RD Sharma Solutions for Class 10 Maths Chapter 6 Exercise 6.2 Page No: 6.54

1. If cos θ = 4/5, find all other trigonometric ratios of angle θ.

Solution:

We have,

cos θ = 4/5

And we know that,

sin θ = $\sqrt{(1 – \cos^2 \theta)}$

⇒ sin θ = $\sqrt{(1 – (4/5)^2)}$

= $\sqrt{(1 – (16/25))}$

= $\sqrt{[(25 – 16)/25]}$

= $\sqrt{(9/25)}$

= 3/5

∴ sin θ = 3/5

Since, cosec θ = 1/ sin θ

= 1/ (3/5)

⇒ cosec θ = 5/3

And, sec θ = 1/ cos θ

= 1/ (4/5)

⇒ cosec θ = 5/4

Now,

tan θ = sin θ/ cos θ

= (3/5)/ (4/5)

⇒ tan θ = 3/4

And, cot θ = 1/ tan θ

= 1/ (3/4)

⇒ cot θ = 4/3

2. If sin θ = 1/$\sqrt{2}$, find all other trigonometric ratios of angle θ.

Solution:

We have,

sin θ = 1/$\sqrt{2}$

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And we know that,

$\cos \theta = \sqrt{(1 - \sin^2 \theta)}$

$\Rightarrow \cos \theta = \sqrt{(1 - (1/\sqrt{2})^2)}$

$= \sqrt{(1 - (1/2))}$

$= \sqrt{[(2 - 1)/2]}$

$= \sqrt{(1/2)}$

$= 1/\sqrt{2}$

$\therefore \cos \theta = 1/\sqrt{2}$

Since, $\text{cosec } \theta = 1/ \sin \theta$

$= 1/ (1/\sqrt{2})$

$\Rightarrow \text{cosec } \theta = \sqrt{2}$

And, $\sec \theta = 1/ \cos \theta$

$= 1/ (1/\sqrt{2})$

$\Rightarrow \sec \theta = \sqrt{2}$

Now,

$\tan \theta = \sin \theta/ \cos \theta$

$= (1/\sqrt{2})/ (1/\sqrt{2})$

$\Rightarrow \tan \theta = 1$

And, $\cot \theta = 1/ \tan \theta$

$= 1/ (1)$

$\Rightarrow \cot \theta = 1$

$$\text{If } \tan\theta = \frac{1}{\sqrt{2}}, \text{find the value of } \frac{\text{cosec}^2\,\theta - \sec^2\theta}{\text{cosec}^2\theta + \cot^2\theta}.$$

3.

Solution:

Given,

$\tan \theta = 1/\sqrt{2}$

By using $\sec^2 \theta - \tan^2 \theta = 1$,

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$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{1+\frac{1}{2}} = \sqrt{\frac{3}{2}}$$

And,

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

From identity, we have

$$\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+2} = \sqrt{3}$$

Substituting the values, we get

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \cot^2\theta} = \frac{\left(\sqrt{3}\right)^2 - \left(\sqrt{\frac{3}{2}}\right)^2}{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2}$$

$$= \frac{3-\frac{3}{2}}{3+2} = \frac{\frac{3}{2}}{5} = \frac{3}{10}$$

If $\tan\theta = \frac{3}{4}$, find the value of $\frac{1-\cos\theta}{1+\cos\theta}$

4.

Solution:

Given,

tan θ = 3/4

By using $\sec^2\theta - \tan^2\theta = 1$,

$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\left(\frac{3}{4}\right)^2} = \sqrt{1+\frac{9}{16}} = \sqrt{\frac{16+9}{16}} = \sqrt{\frac{25}{16}}$$

sec θ = 5/4

Since, sec θ = 1/ cos θ

⇒ cos θ = 1/ sec θ

= 1/ (5/4)

= 4/5

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$$\frac{\text{cosec}^2\,\theta - \sec^2\theta}{\text{cosec}^2\theta + \cot^2\theta} = \frac{1-\frac{4}{5}}{1+\frac{4}{5}} = \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{9}$$

So,

$$\text{If } \tan\theta = \frac{12}{5}, \text{find the value of } \frac{1+\sin\theta}{1-\sin\theta}$$

5.

Solution:

Given, tan θ = 12/5

Since, cot θ = 1/ tan θ = 1/ (12/5) = 5/12

Now, by using $\text{cosec}^2\theta - \cot^2\theta = 1$

$\text{cosec}\,\theta = \sqrt{(1 + \cot^2\theta)}$

$= \sqrt{(1 + (5/12)^2)}$

$= \sqrt{(1 + 25/144)}$

$= \sqrt{(169/ 25)}$

⇒ cosec θ = 13/5

Now, we know that

sin θ = 1/ cosec θ

= 1/ (13/5)

⇒ sin θ = 5/13

Putting value of sin θ in the expression we have,

$$= \frac{1+\frac{12}{13}}{1-\frac{12}{13}} = \frac{\frac{13+12}{18}}{\frac{13-12}{18}}$$

= 25/ 1

= 25

$$\text{If } \cot\theta = \frac{1}{\sqrt{3}}, \text{find the value of } \frac{1-\cos^2\theta}{2-\sin^2\theta}$$

6.

Solution:

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Given,

cot θ = 1/√3

Using $\text{cosec}^2\theta - \cot^2\theta = 1$, we can find cosec θ

$\text{cosec}\ \theta = \sqrt{(1 + \cot^2\theta)}$

$= \sqrt{(1 + (1/\sqrt{3})^2)}$

$= \sqrt{(1 + (1/3))} = \sqrt{((3 + 1)/3)}$

$= \sqrt{(4/3)}$

⇒ cosec θ = 2/√3

So, sin θ = 1/ cosec θ = 1/ (2/√3)

⇒ sin θ = √3/2

And, we know that

$\cos\theta = \sqrt{(1 - \sin^2\theta)}$

$= \sqrt{(1 - (\sqrt{3}/2)^2)}$

$= \sqrt{(1 - (3/4))}$

= $\sqrt{((4 - 3)/4)}$

$= \sqrt{(1/4)}$

⇒ cos θ = 1/2

Now, using cos θ and sin θ in the expression, we have

$$= \frac{1 - \left(\frac{1}{2}\right)^2}{2 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1 - \frac{1}{4}}{2 - \frac{3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}}$$

= 3/5

If $\text{cosec}\ A = \sqrt{2}$, find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$.

7.

Solution:

Given,

$\text{cosec}\ A = \sqrt{2}$

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Using $\text{cosec}^2 A - \cot^2 A = 1$, we find cot A

$$\cot A = \sqrt{\text{cosec}^2 A - 1} \quad = \sqrt{(\sqrt{2})^2 - 1} \quad = \sqrt{2-1} \quad = 1$$

So, tan A = 1/ cot A
= 1/ 1 = 1

And, sin A = 1/ cosec A = 1/ $\sqrt{2}$

$$\cos A = \sqrt{1 - \sin^2 A} \quad = \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

On substituting we get,

$$= \frac{2\left[\frac{1}{\sqrt{2}}\right]^2 + 3[1]^2}{4\left[[1] - \left[\frac{1}{\sqrt{2}}\right]^2\right]}$$

$$= \frac{2 \times \frac{1}{2} + 3}{4\left[1 - \frac{1}{2}\right]} \Rightarrow \frac{1+3}{4.\frac{1}{2}}$$

= 4/2

= 2

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