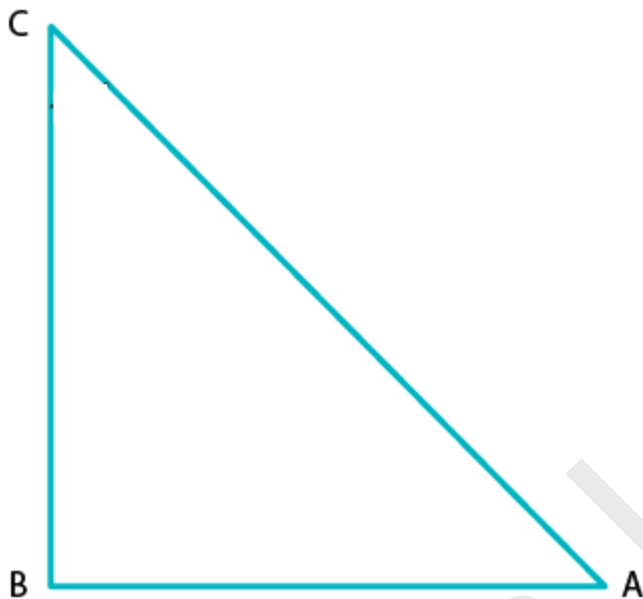


RD Sharma Solutions For Class 10 Maths Chapter 5 Trigonometric Ratios

1. In each of the following, one of the six trigonometric ratios is given. Find the values of the other trigonometric ratios.



(i) $\sin A = \frac{2}{3}$

Solution:

We have,

$$\sin A = \frac{2}{3} \dots\dots\dots (1)$$

As we know, by sin definition;

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{2}{3} \dots\dots(2)$$

By comparing eq. (1) and (2), we have

Opposite side = 2 and Hypotenuse = 3

Now, on using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the values of perpendicular side (BC) and hypotenuse (AC) and for the base side as (AB), we get

$$\Rightarrow 3^2 = AB^2 + 2^2$$

$$AB^2 = 3^2 - 2^2$$

$$AB^2 = 9 - 4$$

$$AB^2 = 5$$

$$AB = \sqrt{5}$$

$$\text{Hence, Base} = \sqrt{5}$$

By definition,

$$\cos A = \text{Base/Hypotenuse}$$

$$\Rightarrow \cos A = \sqrt{5}/3$$

$$\text{Since, cosec } A = 1/\sin A = \text{Hypotenuse/Perpendicular}$$

$$\Rightarrow \text{cosec } A = 3/2$$

$$\text{And, sec } A = \text{Hypotenuse/Base}$$

$$\Rightarrow \sec A = 3/\sqrt{5}$$

$$\text{And, tan } A = \text{Perpendicular/Base}$$

$$\Rightarrow \tan A = 2/\sqrt{5}$$

$$\text{And, cot } A = 1/\tan A = \text{Base/Perpendicular}$$

$$\Rightarrow \cot A = \sqrt{5}/2$$

$$\text{(ii) } \cos A = 4/5$$

Solution:

We have,

$$\cos A = 4/5 \dots\dots\dots (1)$$

As we know, by cos definition

$$\cos A = \text{Base/Hypotenuse} \dots (2)$$

By comparing eq. (1) and (2), we get

$$\text{Base} = 4 \text{ and Hypotenuse} = 5$$

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and hypotenuse (AC) and for the perpendicular (BC), we get

$$5^2 = 4^2 + BC^2$$

$$BC^2 = 5^2 - 4^2$$

$$BC^2 = 25 - 16$$

$$BC^2 = 9$$

$$BC = 3$$

$$\text{Hence, Perpendicular} = 3$$

By definition,

$$\sin A = \text{Perpendicular/Hypotenuse}$$

$$\Rightarrow \sin A = 3/5$$

Then, $\operatorname{cosec} A = 1/\sin A$

$$\Rightarrow \operatorname{cosec} A = 1 / (3/5) = 5/3 = \text{Hypotenuse/Perpendicular}$$

And, $\sec A = 1/\cos A$

$$\Rightarrow \sec A = \text{Hypotenuse/Base}$$

$$\sec A = 5/4$$

And, $\tan A = \text{Perpendicular/Base}$

$$\Rightarrow \tan A = 3/4$$

Next, $\cot A = 1/\tan A = \text{Base/Perpendicular}$

$$\therefore \cot A = 4/3$$

(iii) $\tan \theta = 11/1$

Solution:

We have, $\tan \theta = 11 \dots \dots \dots (1)$

By definition,

$$\tan \theta = \text{Perpendicular/ Base} \dots (2)$$

On Comparing eq. (1) and (2), we get;

$$\text{Base} = 1 \text{ and Perpendicular} = 5$$

Now, using Pythagoras theorem in ΔABC .

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and perpendicular (BC) to get hypotenuse(AC), we get;

$$AC^2 = 1^2 + 11^2$$

$$AC^2 = 1 + 121$$

$$AC^2 = 122$$

$$AC = \sqrt{122}$$

$$\text{Hence, hypotenuse} = \sqrt{122}$$

By definition,

$$\sin = \text{Perpendicular/Hypotenuse}$$

$$\Rightarrow \sin \theta = 11/\sqrt{122}$$

$$\text{And, } \operatorname{cosec} \theta = 1/\sin \theta$$

$$\Rightarrow \operatorname{cosec} \theta = \sqrt{122}/11$$

$$\text{Next, } \cos \theta = \text{Base/ Hypotenuse}$$

$$\Rightarrow \cos \theta = 1/\sqrt{122}$$

$$\text{And, } \sec \theta = 1/\cos \theta$$

$$\Rightarrow \sec \theta = \sqrt{122}/1 = \sqrt{122}$$

And, $\cot \theta = 1/\tan \theta$

$$\therefore \cot \theta = 1/11$$

(iv) $\sin \theta = 11/15$

Solution:

We have, $\sin \theta = 11/15$ (1)

By definition,

$$\sin \theta = \text{Perpendicular} / \text{Hypotenuse} \dots (2)$$

On Comparing eq. (1) and (2), we get;

$$\text{Perpendicular} = 11 \text{ and Hypotenuse} = 15$$

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base (AB), we have

$$15^2 = AB^2 + 11^2$$

$$AB^2 = 15^2 - 11^2$$

$$AB^2 = 225 - 121$$

$$AB^2 = 104$$

$$AB = \sqrt{104}$$

$$AB = \sqrt{(2 \times 2 \times 2 \times 13)}$$

$$AB = 2\sqrt{(2 \times 13)}$$

$$AB = 2\sqrt{26}$$

$$\text{Hence, Base} = 2\sqrt{26}$$

By definition,

$$\cos \theta = \text{Base} / \text{Hypotenuse}$$

$$\therefore \cos \theta = 2\sqrt{26} / 15$$

$$\text{And, } \operatorname{cosec} \theta = 1/\sin \theta$$

$$\therefore \operatorname{cosec} \theta = 15/11$$

$$\text{And, } \sec \theta = \text{Hypotenuse} / \text{Base}$$

$$\therefore \sec \theta = 15 / 2\sqrt{26}$$

$$\text{And, } \tan \theta = \text{Perpendicular} / \text{Base}$$

$$\therefore \tan \theta = 11 / 2\sqrt{26}$$

$$\text{And, } \cot \theta = \text{Base} / \text{Perpendicular}$$

$$\therefore \cot \theta = 2\sqrt{26} / 11$$

(v) $\tan \alpha = 5/12$

Solution:

We have, $\tan \alpha = 5/12 \dots (1)$

By definition,

$$\tan \alpha = \text{Perpendicular/Base} \dots (2)$$

On Comparing eq. (1) and (2), we get

Base = 12 and Perpendicular side = 5

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and the perpendicular (BC) to get hypotenuse (AC), we have

$$AC^2 = 12^2 + 5^2$$

$$AC^2 = 144 + 25$$

$$AC^2 = 169$$

$$AC = 13 \text{ [After taking sq root on both sides]}$$

Hence, Hypotenuse = 13

By definition,

$$\sin \alpha = \text{Perpendicular/Hypotenuse}$$

$$\therefore \sin \alpha = 5/13$$

And, $\operatorname{cosec} \alpha = \text{Hypotenuse/Perpendicular}$

$$\therefore \operatorname{cosec} \alpha = 13/5$$

And, $\cos \alpha = \text{Base/Hypotenuse}$

$$\therefore \cos \alpha = 12/13$$

And, $\sec \alpha = 1/\cos \alpha$

$$\therefore \sec \alpha = 13/12$$

And, $\tan \alpha = \sin \alpha / \cos \alpha$

$$\therefore \tan \alpha = 5/12$$

Since, $\cot \alpha = 1/\tan \alpha$

$$\therefore \cot \alpha = 12/5$$

(vi) $\sin \theta = \sqrt{3}/2$

Solution:

We have, $\sin \theta = \sqrt{3}/2 \dots (1)$

By definition,

$$\sin \theta = \text{Perpendicular/ Hypotenuse} \dots (2)$$

On Comparing eq. (1) and (2), we get;

$$\text{Perpendicular} = \sqrt{3} \text{ and Hypotenuse} = 2$$

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of perpendicular (BC) and hypotenuse (AC) and get the base (AB), we get;

$$2^2 = AB^2 + (\sqrt{3})^2$$

$$AB^2 = 2^2 - (\sqrt{3})^2$$

$$AB^2 = 4 - 3$$

$$AB^2 = 1$$

$$AB = 1$$

Thus, Base = 1

By definition,

$$\cos \theta = \text{Base/Hypotenuse}$$

$$\therefore \cos \theta = 1/2$$

$$\text{And, cosec } \theta = 1/\sin \theta$$

$$\text{Or cosec } \theta = \text{Hypotenuse/Perpendicular}$$

$$\therefore \text{cosec } \theta = 2/\sqrt{3}$$

$$\text{And, sec } \theta = \text{Hypotenuse/Base}$$

$$\therefore \sec \theta = 2/1$$

$$\text{And, tan } \theta = \text{Perpendicular/Base}$$

$$\therefore \tan \theta = \sqrt{3}/1$$

$$\text{And, cot } \theta = \text{Base/Perpendicular}$$

$$\therefore \cot \theta = 1/\sqrt{3}$$

$$\text{(vii) } \cos \theta = 7/25$$

Solution:

$$\text{We have, } \cos \theta = 7/25 \dots\dots\dots (1)$$

By definition,

$$\cos \theta = \text{Base/Hypotenuse}$$

On Comparing eq. (1) and (2), we get;

$$\text{Base} = 7 \text{ and Hypotenuse} = 25$$

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC)

$$25^2 = 7^2 + BC^2$$

$$BC^2 = 25^2 - 7^2$$

$$BC^2 = 625 - 49$$

$$BC^2 = 576$$

$$BC = \sqrt{576}$$

$$BC = 24$$

Hence, Perpendicular side = 24

By definition,

$\sin \theta = \text{perpendicular/Hypotenuse}$

$$\therefore \sin \theta = 24/25$$

Since, $\operatorname{cosec} \theta = 1/\sin \theta$

Also, $\operatorname{cosec} \theta = \text{Hypotenuse/Perpendicular}$

$$\therefore \operatorname{cosec} \theta = 25/24$$

Since, $\sec \theta = 1/\operatorname{cosec} \theta$

Also, $\sec \theta = \text{Hypotenuse/Base}$

$$\therefore \sec \theta = 25/7$$

Since, $\tan \theta = \text{Perpendicular/Base}$

$$\therefore \tan \theta = 24/7$$

Now, $\cot \theta = 1/\tan \theta$

So, $\cot \theta = \text{Base/Perpendicular}$

$$\therefore \cot \theta = 7/24$$

(viii) $\tan \theta = 8/15$

Solution:

We have, $\tan \theta = 8/15$ (1)

By definition,

$\tan \theta = \text{Perpendicular/Base}$ (2)

On Comparing eq. (1) and (2), we get;

Base = 15 and Perpendicular = 8

Now, using Pythagoras theorem in ΔABC

$$AC^2 = 15^2 + 8^2$$

$$AC^2 = 225 + 64$$

$$AC^2 = 289$$

$$AC = \sqrt{289}$$

$$AC = 17$$

Hence, Hypotenuse = 17

By definition,

Since, $\sin \theta = \text{perpendicular}/\text{Hypotenuse}$

$$\therefore \sin \theta = 8/17$$

Since, $\operatorname{cosec} \theta = 1/\sin \theta$

Also, $\operatorname{cosec} \theta = \text{Hypotenuse}/\text{Perpendicular}$

$$\therefore \operatorname{cosec} \theta = 17/8$$

Since, $\cos \theta = \text{Base}/\text{Hypotenuse}$

$$\therefore \cos \theta = 15/17$$

Since, $\sec \theta = 1/\cos \theta$

Also, $\sec \theta = \text{Hypotenuse}/\text{Base}$

$$\therefore \sec \theta = 17/15$$

Since, $\cot \theta = 1/\tan \theta$

Also, $\cot \theta = \text{Base}/\text{Perpendicular}$

$$\therefore \cot \theta = 15/8$$

(ix) $\cot \theta = 12/5$

Solution:

We have, $\cot \theta = 12/5$ (1)

By definition,

$$\cot \theta = 1/\tan \theta$$

$$\cot \theta = \text{Base}/\text{Perpendicular} \dots\dots (2)$$

On Comparing eq. (1) and (2), we have

Base = 12 and Perpendicular side = 5

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and perpendicular (BC) to get the hypotenuse (AC);

$$AC^2 = 12^2 + 5^2$$

$$AC^2 = 144 + 25$$

$$AC^2 = 169$$

$$AC = \sqrt{169}$$

$$AC = 13$$

Hence, Hypotenuse = 13

By definition,

Since, $\sin \theta = \text{perpendicular}/\text{Hypotenuse}$

$$\therefore \sin \theta = 5/13$$

Since, $\operatorname{cosec} \theta = 1/\sin \theta$

Also, $\operatorname{cosec} \theta = \text{Hypotenuse}/\text{Perpendicular}$

$$\therefore \operatorname{cosec} \theta = 13/5$$

Since, $\cos \theta = \text{Base}/\text{Hypotenuse}$

$$\therefore \cos \theta = 12/13$$

Since, $\sec \theta = 1/\cos \theta$

Also, $\sec \theta = \text{Hypotenuse}/\text{Base}$

$$\therefore \sec \theta = 13/12$$

Since, $\tan \theta = 1/\cot \theta$

Also, $\tan \theta = \text{Perpendicular}/\text{Base}$

$$\therefore \tan \theta = 5/12$$

$$\textbf{(x) } \sec \theta = 13/5$$

Solution:

We have, $\sec \theta = 13/5 \dots \dots \dots (1)$

By definition,

$$\sec \theta = \text{Hypotenuse}/\text{Base} \dots \dots \dots (2)$$

On Comparing eq. (1) and (2), we get

$$\text{Base} = 5 \text{ and Hypotenuse} = 13$$

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

And. putting the value of base side (AB) and hypotenuse (AC) to get the perpendicular side (BC)

$$13^2 = 5^2 + BC^2$$

$$BC^2 = 13^2 - 5^2$$

$$BC^2 = 169 - 25$$

$$BC^2 = 144$$

$$BC = \sqrt{144}$$

$$BC = 12$$

Hence, Perpendicular = 12

By definition,

Since, $\sin \theta = \text{perpendicular}/\text{Hypotenuse}$

$$\therefore \sin \theta = 12/13$$

$$\text{Since, cosec } \theta = 1/\sin \theta$$

$$\text{Also, cosec } \theta = \text{Hypotenuse/Perpendicular}$$

$$\therefore \text{cosec } \theta = 13/12$$

$$\text{Since, } \cos \theta = 1/\sec \theta$$

$$\text{Also, } \cos \theta = \text{Base/Hypotenuse}$$

$$\therefore \cos \theta = 5/13$$

$$\text{Since, } \tan \theta = \text{Perpendicular/Base}$$

$$\therefore \tan \theta = 12/5$$

$$\text{Since, } \cot \theta = 1/\tan \theta$$

$$\text{Also, } \cot \theta = \text{Base/Perpendicular}$$

$$\therefore \cot \theta = 5/12$$

$$\text{(xi) cosec } \theta = \sqrt{10}$$

Solution:

$$\text{We have, cosec } \theta = \sqrt{10}/1 \dots\dots\dots (1)$$

By definition,

$$\text{cosec } \theta = \text{Hypotenuse/ Perpendicular} \dots\dots\dots(2)$$

$$\text{And, cosec } \theta = 1/\sin \theta$$

On comparing eq.(1) and(2), we get

$$\text{Perpendicular side} = 1 \text{ and Hypotenuse} = \sqrt{10}$$

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base side (AB)

$$(\sqrt{10})^2 = AB^2 + 1^2$$

$$AB^2 = (\sqrt{10})^2 - 1^2$$

$$AB^2 = 10 - 1$$

$$AB = \sqrt{9}$$

$$AB = 3$$

So, Base side = 3

By definition,

$$\text{Since, } \sin \theta = \text{Perpendicular/Hypotenuse}$$

$$\therefore \sin \theta = 1/\sqrt{10}$$

$$\text{Since, } \cos \theta = \text{Base/Hypotenuse}$$

$$\therefore \cos \theta = 3/\sqrt{10}$$

Since, $\sec \theta = 1/\cos \theta$

Also, $\sec \theta = \text{Hypotenuse}/\text{Base}$

$$\therefore \sec \theta = \sqrt{10}/3$$

Since, $\tan \theta = \text{Perpendicular}/\text{Base}$

$$\therefore \tan \theta = 1/3$$

Since, $\cot \theta = 1/\tan \theta$

$$\therefore \cot \theta = 3/1$$

(xii) $\cos \theta = 12/15$

Solution:

We have; $\cos \theta = 12/15$ (1)

By definition,

$$\cos \theta = \text{Base}/\text{Hypotenuse} \dots \dots \dots (2)$$

By comparing eq. (1) and (2), we get;

$$\text{Base} = 12 \text{ and Hypotenuse} = 15$$

Now, using Pythagoras theorem in ΔABC , we get

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC);

$$15^2 = 12^2 + BC^2$$

$$BC^2 = 15^2 - 12^2$$

$$BC^2 = 225 - 144$$

$$BC^2 = 81$$

$$BC = \sqrt{81}$$

$$BC = 9$$

So, Perpendicular = 9

By definition,

Since, $\sin \theta = \text{perpendicular}/\text{Hypotenuse}$

$$\therefore \sin \theta = 9/15 = 3/5$$

Since, $\text{cosec } \theta = 1/\sin \theta$

Also, $\text{cosec } \theta = \text{Hypotenuse}/\text{Perpendicular}$

$$\therefore \text{cosec } \theta = 15/9 = 5/3$$

Since, $\sec \theta = 1/\cos \theta$

Also, $\sec \theta = \text{Hypotenuse}/\text{Base}$

$$\therefore \sec \theta = 15/12 = 5/4$$

Since, $\tan \theta = \text{Perpendicular/Base}$

$$\therefore \tan \theta = 9/12 = 3/4$$

Since, $\cot \theta = 1/\tan \theta$

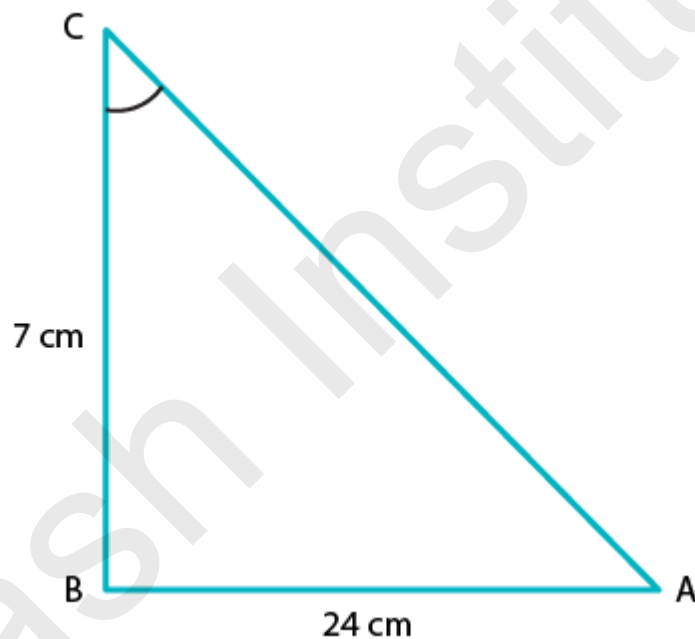
Also, $\cot \theta = \text{Base/Perpendicular}$

$$\therefore \cot \theta = 12/9 = 4/3$$

2. In a $\triangle ABC$, right angled at B, $AB = 24 \text{ cm}$, $BC = 7 \text{ cm}$. Determine

(i) $\sin A$, $\cos A$ (ii) $\sin C$, $\cos C$

Solution:



(i) Given: In $\triangle ABC$, $AB = 24 \text{ cm}$, $BC = 7 \text{ cm}$ and $\angle ABC = 90^\circ$

To find: $\sin A$, $\cos A$

By using Pythagoras theorem in $\triangle ABC$ we have

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 24^2 + 7^2$$

$$AC^2 = 576 + 49$$

$$AC^2 = 625$$

$$AC = \sqrt{625}$$

$$AC = 25$$

Hence, Hypotenuse = 25

By definition,

$\sin A = \text{Perpendicular side opposite to angle A} / \text{Hypotenuse}$

$$\sin A = BC / AC$$

$$\sin A = 7 / 25$$

And,

$\cos A = \text{Base side adjacent to angle A} / \text{Hypotenuse}$

$$\cos A = AB / AC$$

$$\cos A = 24 / 25$$

(ii) Given: In $\triangle ABC$, $AB = 24$ cm and $BC = 7$ cm and $\angle ABC = 90^\circ$

To find: $\sin C$, $\cos C$

By using Pythagoras theorem in $\triangle ABC$ we have

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 24^2 + 7^2$$

$$AC^2 = 576 + 49$$

$$AC^2 = 625$$

$$AC = \sqrt{625}$$

$$AC = 25$$

Hence, Hypotenuse = 25

By definition,

$\sin C = \text{Perpendicular side opposite to angle C} / \text{Hypotenuse}$

$$\sin C = AB / AC$$

$$\sin C = 24 / 25$$

And,

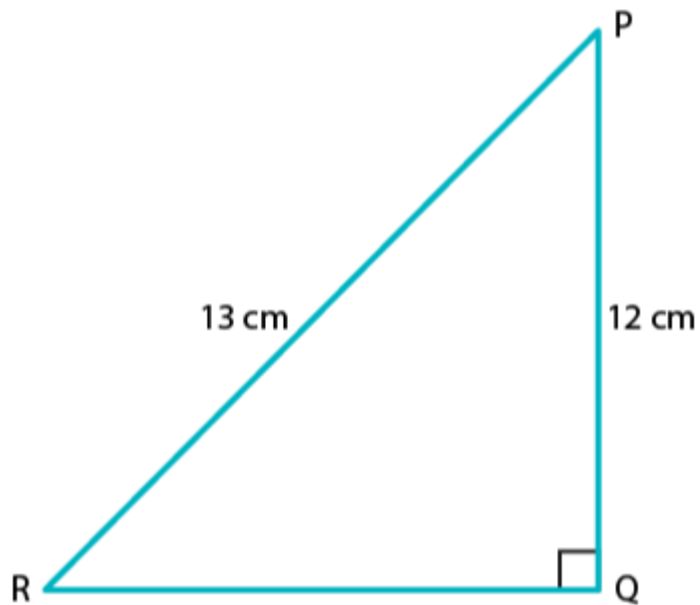
$\cos C = \text{Base side adjacent to angle C} / \text{Hypotenuse}$

$$\cos C = BC / AC$$

$$\cos C = 7 / 25$$

3. In fig. 5.37, find $\tan P$ and $\cot R$. Is $\tan P = \cot R$?

Solution:



By using Pythagoras theorem in $\triangle PQR$, we have

$$PR^2 = PQ^2 + QR^2$$

Putting the length of given side PR and PQ in the above equation

$$13^2 = 12^2 + QR^2$$

$$QR^2 = 13^2 - 12^2$$

$$QR^2 = 169 - 144$$

$$QR^2 = 25$$

$$QR = \sqrt{25} = 5$$

By definition,

$\tan P = \frac{\text{Perpendicular side opposite to } P}{\text{Base side adjacent to angle } P}$

$$\tan P = \frac{QR}{PQ}$$

$$\tan P = \frac{5}{12} \dots\dots\dots (1)$$

And,

$\cot R = \frac{\text{Base}}{\text{Perpendicular}}$

$$\cot R = \frac{QR}{PQ}$$

$$\cot R = \frac{5}{12} \dots\dots (2)$$

When comparing equation (1) and (2), we can see that R.H.S of both the equation is equal.

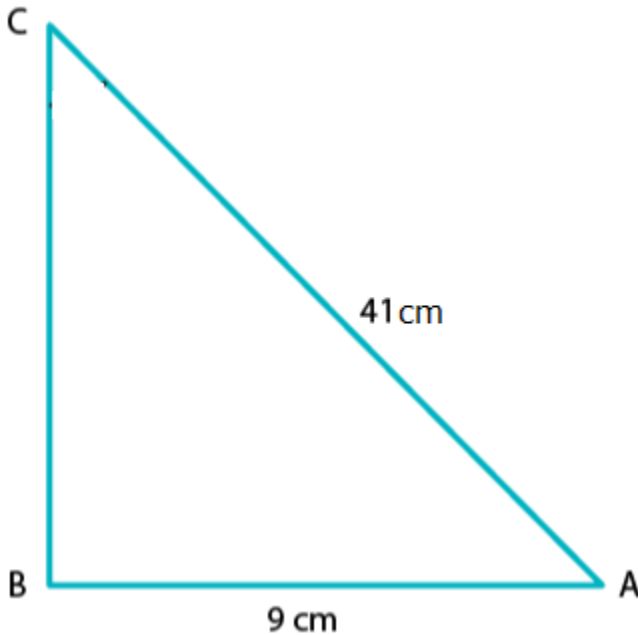
Therefore, L.H.S of both equations should also be equal.

$$\therefore \tan P = \cot R$$

Yes, $\tan P = \cot R = \frac{5}{12}$

4. If $\sin A = \frac{9}{41}$, compute $\cos A$ and $\tan A$.

Solution:



Given that, $\sin A = 9/41$ (1)

Required to find: $\cos A$, $\tan A$

By definition, we know that

$\sin A = \text{Perpendicular} / \text{Hypotenuse}$ (2)

On Comparing eq. (1) and (2), we get;

Perpendicular side = 9 and Hypotenuse = 41

Let's construct $\triangle ABC$ as shown below,

And, here the length of base AB is unknown.

Thus, by using Pythagoras theorem in $\triangle ABC$, we get;

$$AC^2 = AB^2 + BC^2$$

$$41^2 = AB^2 + 9^2$$

$$AB^2 = 41^2 - 9^2$$

$$AB^2 = 168 - 81$$

$$AB = 1600$$

$$AB = \sqrt{1600}$$

$$AB = 40$$

$\Rightarrow$ Base of triangle ABC, $AB = 40$

We know that,

$\cos A = \text{Base} / \text{Hypotenuse}$

$$\cos A = AB/AC$$

$$\cos A = 40/41$$

And,

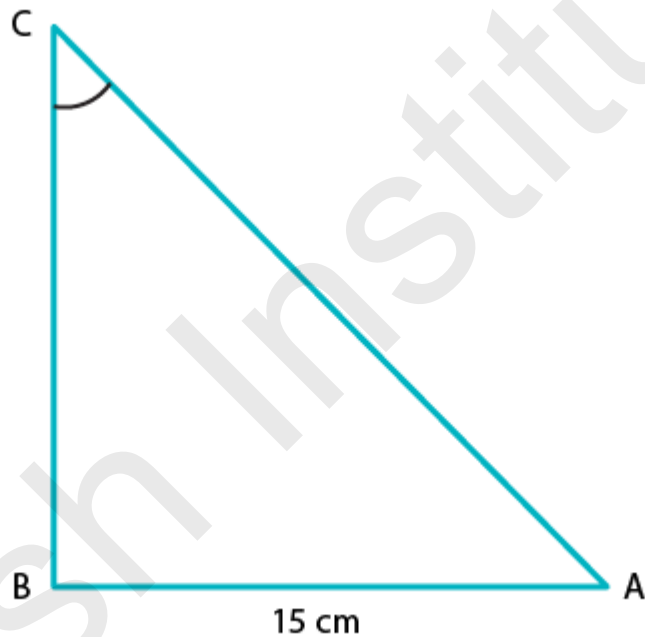
$$\tan A = \text{Perpendicular} / \text{Base}$$

$$\tan A = BC/AB$$

$$\tan A = 9/40$$

5. Given $15\cot A = 8$, find $\sin A$ and $\sec A$.

Solution



We have, $15\cot A = 8$

Required to find: $\sin A$ and $\sec A$

As, $15 \cot A = 8$

$$\Rightarrow \cot A = 8/15 \dots\dots(1)$$

And we know,

$$\cot A = 1/\tan A$$

Also by definition,

$$\cot A = \text{Base side adjacent to } \angle A / \text{Perpendicular side opposite to } \angle A \dots\dots (2)$$

On comparing equation (1) and (2), we get;

Base side adjacent to $\angle A = 8$

Perpendicular side opposite to $\angle A = 15$

So, by using Pythagoras theorem to $\triangle ABC$, we have

$$AC^2 = AB^2 + BC^2$$

Substituting values for sides from the figure

$$AC^2 = 8^2 + 15^2$$

$$AC^2 = 64 + 225$$

$$AC^2 = 289$$

$$AC = \sqrt{289}$$

$$AC = 17$$

Therefore, hypotenuse = 17

By definition,

$\sin A = \text{Perpendicular/Hypotenuse}$

$$\Rightarrow \sin A = BC/AC$$

$\sin A = 15/17$ (using values from the above)

Also,

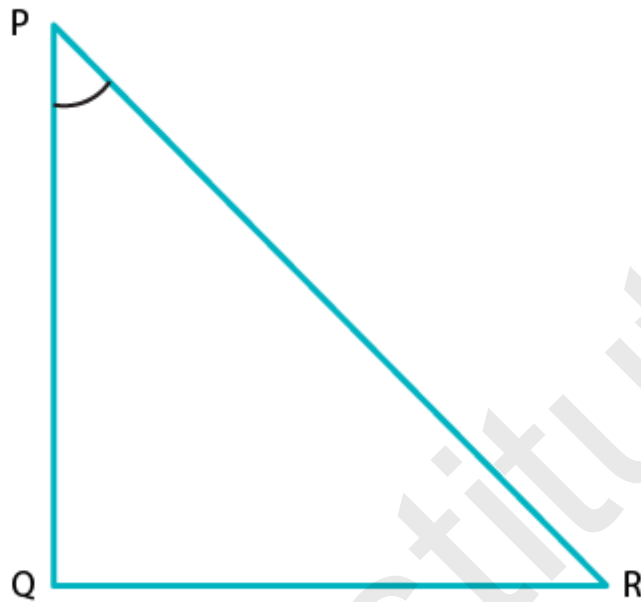
$$\sec A = 1/\cos A$$

$$\Rightarrow \sec A = \text{Hypotenuse/ Base side adjacent to } \angle A$$

$$\therefore \sec A = 17/8$$

6. In $\triangle PQR$, right-angled at Q, PQ = 4cm and RQ = 3 cm. Find the value of $\sin P$, $\sin R$, $\sec P$ and $\sec R$.

Solution:



Given:

$\triangle PQR$ is right-angled at Q.

$PQ = 4\text{cm}$

$RQ = 3\text{cm}$

Required to find: $\sin P$, $\sin R$, $\sec P$, $\sec R$

Given $\triangle PQR$,

By using Pythagoras theorem to $\triangle PQR$, we get

$$PR^2 = PQ^2 + RQ^2$$

Substituting the respective values,

$$PR^2 = 4^2 + 3^2$$

$$PR^2 = 16 + 9$$

$$PR^2 = 25$$

$$PR = \sqrt{25}$$

$$PR = 5$$

$$\Rightarrow \text{Hypotenuse} = 5$$

By definition,

$\sin P = \frac{\text{Perpendicular side opposite to angle } P}{\text{Hypotenuse}}$

$$\sin P = \frac{RQ}{PR}$$

$$\Rightarrow \sin P = \frac{3}{5}$$

And,

$\sin R = \text{Perpendicular side opposite to angle R} / \text{Hypotenuse}$

$$\sin R = PQ / PR$$

$$\Rightarrow \sin R = 4/5$$

And,

$$\sec P = 1/\cos P$$

$\sec P = \text{Hypotenuse} / \text{Base side adjacent to } \angle P$

$$\sec P = PR / PQ$$

$$\Rightarrow \sec P = 5/4$$

Now,

$$\sec R = 1/\cos R$$

$\sec R = \text{Hypotenuse} / \text{Base side adjacent to } \angle R$

$$\sec R = PR / RQ$$

$$\Rightarrow \sec R = 5/3$$

7. If $\cot \theta = 7/8$, evaluate

(i) $(1+\sin \theta)(1-\sin \theta) / (1+\cos \theta)(1-\cos \theta)$

(ii) $\cot^2 \theta$

Solution:

(i) Required to evaluate:

$$\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}, \text{ given } \cot \theta = 7/8$$

Taking the numerator, we have

$$(1+\sin \theta)(1-\sin \theta) = 1 - \sin^2 \theta \text{ [Since, } (a+b)(a-b) = a^2 - b^2]$$

Similarly,

$$(1+\cos \theta)(1-\cos \theta) = 1 - \cos^2 \theta$$

We know that,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow 1 - \cos^2 \theta = \sin^2 \theta$$

And,

$$1 - \sin^2 \theta = \cos^2 \theta$$

Thus,

$$(1+\sin \theta)(1-\sin \theta) = 1 - \sin^2 \theta = \cos^2 \theta$$

$$(1+\cos \theta)(1-\cos \theta) = 1 - \cos^2 \theta = \sin^2 \theta$$

$$\Rightarrow \frac{(1 + \sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$$

$$= \cos^2 \theta / \sin^2 \theta$$

$$= (\cos \theta / \sin \theta)^2$$

And, we know that $(\cos \theta / \sin \theta) = \cot \theta$

$$\Rightarrow \frac{(1 + \sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$$

$$= (\cot \theta)^2$$

$$= (7/8)^2$$

$$= 49/64$$

(ii) Given,

$$\cot \theta = 7/8$$

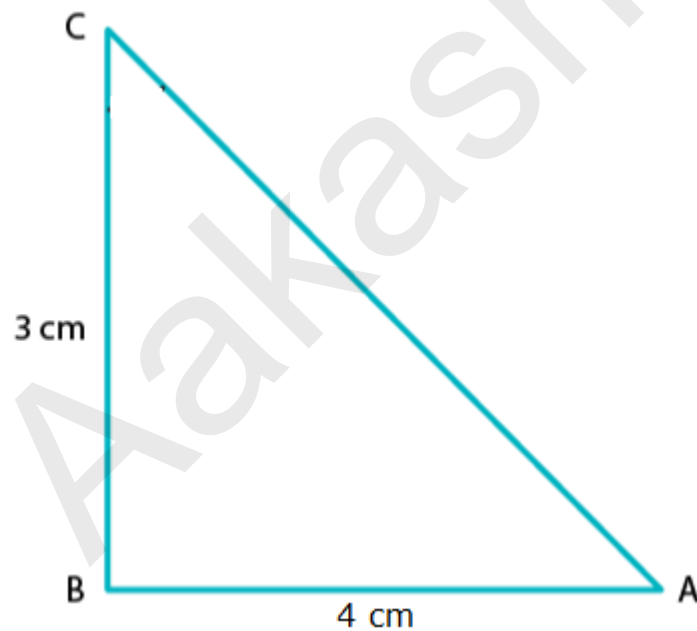
So, by squaring on both sides we get

$$(\cot \theta)^2 = (7/8)^2$$

$$\therefore \cot^2 \theta = 49/64$$

8. If $3\cot A = 4$, check whether $(1 - \tan^2 A) / (1 + \tan^2 A) = (\cos^2 A - \sin^2 A)$ or not.

Solution:



Given,

$$3\cot A = 4$$

$$\Rightarrow \cot A = 4/3$$

By definition,

$$\tan A = 1/ \cot A = 1/ (4/3)$$

$$\Rightarrow \tan A = 3/4$$

Thus,

Base side adjacent to $\angle A = 4$

Perpendicular side opposite to $\angle A = 3$

In $\triangle ABC$, Hypotenuse is unknown

Thus, by applying Pythagoras theorem in $\triangle ABC$

We get

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 4^2 + 3^2$$

$$AC^2 = 16 + 9$$

$$AC^2 = 25$$

$$AC = \sqrt{25}$$

$$AC = 5$$

Hence, hypotenuse = 5

Now, we can find that

$$\sin A = \text{opposite side to } \angle A / \text{Hypotenuse} = 3/5$$

And,

$$\cos A = \text{adjacent side to } \angle A / \text{Hypotenuse} = 4/5$$

Taking the LHS,

$$\text{L.H.S} = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

Putting value of $\tan A$

We get,

$$\text{L.H.S} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

Take L.C.M of both numerator and denominator;

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{7}{25}$$

Thus, $\text{LHS} = 7/25$

Now, taking RHS

$$\text{R.H.S} = \cos^2 A - \sin^2 A$$

Putting value of sin A and cos A

$$\text{R.H.S} = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$\cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25}$$

$$\cos^2 A - \sin^2 A = \frac{16-9}{25}$$

$$\cos^2 A - \sin^2 A = \frac{7}{25}$$

Therefore,

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

Hence Proved

9. If $\tan \theta = a/b$, find the value of $(\cos \theta + \sin \theta) / (\cos \theta - \sin \theta)$

Solution:

Given,

$$\tan \theta = a/b$$

And, we know by definition that

$$\tan \theta = \text{opposite side} / \text{adjacent side}$$

Thus, by comparison

$$\text{Opposite side} = a \text{ and adjacent side} = b$$

To find the hypotenuse, we know that by Pythagoras theorem that

$$\text{Hypotenuse}^2 = \text{opposite side}^2 + \text{adjacent side}^2$$

$$\Rightarrow \text{Hypotenuse} = \sqrt{a^2 + b^2}$$

So, by definition

$$\sin \theta = \text{opposite side} / \text{Hypotenuse}$$

$$\sin \theta = a / \sqrt{a^2 + b^2}$$

And,

$$\cos \theta = \text{adjacent side} / \text{Hypotenuse}$$

$$\cos \theta = b / \sqrt{a^2 + b^2}$$

Now,

After substituting for $\cos \theta$ and $\sin \theta$, we have

$$\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} = \frac{(a+b)/\sqrt{(a^2+b^2)}}{(a-b)/\sqrt{(a^2+b^2)}}$$

$$\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} = \frac{(a+b)}{(a-b)}$$

$\therefore$

Hence Proved.

$$\frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta}$$

10. If $3 \tan \theta = 4$, find the value of

Solution:

Given, $3 \tan \theta = 4$

$$\Rightarrow \tan \theta = 4/3$$

$$\frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta}$$

From, let's divide the numerator and denominator by $\cos \theta$.

We get,

$$(4 - \tan \theta) / (2 + \tan \theta)$$

$$\Rightarrow (4 - (4/3)) / (2 + (4/3)) \text{ [using the value of } \tan \theta]$$

$$\Rightarrow (12 - 4) / (6 + 4) \text{ [After taking LCM and cancelling it]}$$

$$\Rightarrow 8/10 = 4/5$$

$$\frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta}$$

$$\therefore = 4/5$$

$$\frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \cos \theta}$$