

RD Sharma Solutions for Class 11 Maths Chapter 5 Trigonometric Functions

Prove the following identities:

1. $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

Solution:

Let us consider LHS: $\sec^4 x - \sec^2 x$

$$(\sec^2 x)^2 - \sec^2 x$$

By using the formula, $\sec^2 \theta = 1 + \tan^2 \theta$.

$$(1 + \tan^2 x)^2 - (1 + \tan^2 x)$$

$$1 + 2\tan^2 x + \tan^4 x - 1 - \tan^2 x$$

$$\tan^4 x + \tan^2 x$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

2. $\sin^6 x + \cos^6 x = 1 - 3 \sin^2 x \cos^2 x$

Solution:

Let us consider LHS: $\sin^6 x + \cos^6 x$

$$(\sin^2 x)^3 + (\cos^2 x)^3$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$(\sin^2 x + \cos^2 x)[(\sin^2 x)^2 + (\cos^2 x)^2 - \sin^2 x \cos^2 x]$$

By using the formula, $\sin^2 x + \cos^2 x = 1$ and $a^2 + b^2 = (a + b)^2 - 2ab$

$$1 \times [(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x - \sin^2 x \cos^2 x]$$

$$1^2 - 3\sin^2 x \cos^2 x$$

$$1 - 3\sin^2 x \cos^2 x$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

3. $(\operatorname{cosec} x - \sin x)(\sec x - \cos x)(\tan x + \cot x) = 1$

Solution:

Let us consider LHS: $(\operatorname{cosec} x - \sin x)(\sec x - \cos x)(\tan x + \cot x)$

By using the formulas

$$\operatorname{cosec} \theta = 1/\sin \theta;$$

$$\sec \theta = 1/\cos \theta;$$

$$\tan \theta = \sin \theta / \cos \theta;$$

$$\cot \theta = \cos \theta / \sin \theta$$

Now,

$$\left(\frac{1}{\sin x} - \sin x\right) \left(\frac{1}{\cos x} - \cos x\right) \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$\frac{1 - \sin^2 x}{\sin x} \times \frac{1 - \cos^2 x}{\cos x} \times \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

By using the formula,

$$\sin^2 x + \cos^2 x = 1;$$

$$\frac{\cos^2 x}{\sin x} \times \frac{\sin^2 x}{\cos x} \times \frac{1}{\sin x \cos x}$$

$$1 = \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$\mathbf{4. \operatorname{cosec} x (\sec x - 1) - \cot x (1 - \cos x) = \tan x - \sin x}$$

Solution:

Let us consider LHS: $\operatorname{cosec} x (\sec x - 1) - \cot x (1 - \cos x)$

By using the formulas

$$\operatorname{cosec} \theta = 1/\sin \theta;$$

$$\sec \theta = 1/\cos \theta;$$

$$\tan \theta = \sin \theta / \cos \theta;$$

$$\cot \theta = \cos \theta / \sin \theta$$

Now,

$$\frac{1}{\sin x} \left(\frac{1}{\cos x} - 1 \right) - \frac{\cos x}{\sin x} (1 - \cos x)$$

$$\frac{1}{\sin x} \left(\frac{1 - \cos x}{\cos x} \right) - \frac{\cos x}{\sin x} (1 - \cos x)$$

$$\left(\frac{1 - \cos x}{\sin x} \right) \left(\frac{1}{\cos x} - \cos x \right)$$

$$\left(\frac{1 - \cos x}{\sin x} \right) \left(\frac{1 - \cos^2 x}{\cos x} \right)$$

By using the formula, $1 - \cos^2 x = \sin^2 x$;

$$\left(\frac{1 - \cos x}{\sin x} \right) \left(\frac{\sin^2 x}{\cos x} \right)$$

$$(1 - \cos x) \left(\frac{\sin x}{\cos x} \right)$$

$$\frac{\sin x}{\cos x} - \sin x$$

$$\tan x - \sin x$$

= RHS

∴ LHS = RHS

Hence Proved.

5.

$$\frac{1 - \sin x \cos x}{\cos x (\sec x - \operatorname{cosec} x)} \cdot \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} = \sin x$$

Solution:

Let us consider the LHS:

$$\frac{1 - \sin x \cos x}{\cos x (\sec x - \operatorname{cosec} x)} \cdot \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x}$$

By using the formula,

$$\operatorname{cosec} \theta = 1/\sin \theta;$$

$$\sec \theta = 1/\cos \theta;$$

Now,

$$\frac{1 - \sin x \cos x}{\cos x \left(\frac{1}{\cos x} - \frac{1}{\sin x} \right)} \times \frac{(\sin x)^2 - (\cos x)^2}{(\sin x)^3 + (\cos x)^3}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\frac{1 - \sin x \cos x}{\cos x \left(\frac{\sin x - \cos x}{\cos x \sin x} \right)} \times \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}$$

$$\frac{\sin x (1 - \sin x \cos x)}{\sin x - \cos x} \times \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}$$

$$\frac{\sin x (1 - \sin x \cos x)}{1} \times \frac{1}{[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}$$

By using the formula, $\sin^2 x + \cos^2 x = 1$.

$$\sin x (1 - \sin x \cos x) \times \frac{1}{(1 - \sin x \cos x)}$$

$$\frac{1 - \sin x \cos x}{\cos x \left(\frac{1}{\cos x} - \frac{1}{\sin x} \right)} \times \frac{(\sin x)^2 - (\cos x)^2}{(\sin x)^3 + (\cos x)^3}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\frac{1 - \sin x \cos x}{\cos x \left(\frac{\sin x - \cos x}{\cos x \sin x} \right)} \times \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}$$

$$\frac{\sin x (1 - \sin x \cos x)}{\sin x - \cos x} \times \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}$$

$$\frac{\sin x (1 - \sin x \cos x)}{1} \times \frac{1}{[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}$$

By using the formula, $\sin^2 x + \cos^2 x = 1$.

$$\sin x (1 - \sin x \cos x) \times \frac{1}{(1 - \sin x \cos x)}$$

$\sin x$

= RHS

$\therefore$ LHS = RHS

Hence Proved.

6.

$$\frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x} = (\sec x \operatorname{cosec} x + 1)$$

Solution:

Let us consider the LHS:

$$\frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x}$$

By using the formula,

$$\tan \theta = \sin \theta / \cos \theta;$$

$$\cot \theta = \cos \theta / \sin \theta$$

Now,

$$\frac{\frac{\sin x}{\cos x}}{1 - \frac{\cos x}{\sin x}} + \frac{\frac{\cos x}{\sin x}}{1 - \frac{\sin x}{\cos x}}$$

$$\frac{\frac{\sin x}{\cos x}}{\frac{\sin x - \cos x}{\sin x}} + \frac{\frac{\cos x}{\sin x}}{\frac{\cos x - \sin x}{\cos x}}$$

$$\frac{\sin^2 x}{\cos x (\sin x - \cos x)} - \frac{\cos^2 x}{\sin x (\sin x - \cos x)}$$

$$\frac{\sin^3 x - \cos^3 x}{\sin x \cos x (\sin x - \cos x)}$$

By using the formula, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$\frac{(\sin x - \cos x) [(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x \cos x (\sin x - \cos x)}$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\frac{[1 + \sin x \cos x]}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} + \frac{\sin x \cos x}{\sin x \cos x}$$

$$\frac{1}{\sin x} \times \frac{1}{\cos x} + 1$$

By using the formula,

$$\operatorname{cosec} \theta = 1/\sin \theta,$$

$$\sec \theta = 1/\cos \theta;$$

$$\operatorname{cosec} x \times \sec x + 1$$

$$\sec x \operatorname{cosec} x + 1$$

=RHS

$\therefore \text{LHS} = \text{RHS}$

Hence Proved.

7.

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 2$$

Solution:

Let us consider LHS:

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x}$$

By using the formula $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

$$\frac{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}{\sin x + \cos x} + \frac{(\sin x - \cos x)[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x - \cos x}$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$1 - \sin x \cos x + 1 + \sin x \cos x$$

$$2$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved.

$$8. (\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2 = 1$$

Solution:

Let us consider LHS:

$$(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2$$

Expanding the above equation we get,

$$[(\sec x \sec y)^2 + (\tan x \tan y)^2 + 2(\sec x \sec y)(\tan x \tan y)] - [(\sec x \tan y)^2 + (\tan x \sec y)^2 + 2(\sec x \tan y)(\tan x \sec y)]$$

$$[\sec^2 x \sec^2 y + \tan^2 x \tan^2 y + 2(\sec x \sec y)(\tan x \tan y)] - [\sec^2 x \tan^2 y + \tan^2 x \sec^2 y + 2(\sec^2 x \tan^2 y)(\tan x \sec y)]$$

$$\sec^2 x \sec^2 y - \sec^2 x \tan^2 y + \tan^2 x \tan^2 y - \tan^2 x \sec^2 y$$

$$\sec^2 x (\sec^2 y - \tan^2 y) + \tan^2 x (\tan^2 y - \sec^2 y)$$

$$\sec^2 x (\sec^2 y - \tan^2 y) - \tan^2 x (\sec^2 y - \tan^2 y)$$

We know, $\sec^2 x - \tan^2 x = 1$.

$$\sec^2 x \times 1 - \tan^2 x \times 1$$

$$\sec^2 x - \tan^2 x$$

1 = RHS

∴ LHS = RHS

Hence proved.

9.

$$\frac{\cos x}{1 - \sin x} = \frac{1 + \cos x + \sin x}{1 + \cos x - \sin x}$$

Solution:

Let us Consider RHS:

$$\begin{aligned} & \frac{1 + \cos x + \sin x}{1 + \cos x - \sin x} \\ & \frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) - (\sin x)} \\ & \frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) - (\sin x)} \times \frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) + (\sin x)} \end{aligned}$$

$$\frac{[(1 + \cos x) + (\sin x)]^2}{(1 + \cos x)^2 - (\sin x)^2}$$

$$\frac{[(1 + \cos x) + (\sin x)]^2}{(1 + \cos x)^2 - (\sin x)^2}$$

$$\frac{(1 + \cos x)^2 + (\sin x)^2 + 2(1 + \cos x)(\sin x)}{(1 + \cos^2 x + 2 \cos x) - (\sin^2 x)}$$

$$\frac{(1 + \cos x)^2 + (\sin x)^2 + 2(1 + \cos x)(\sin x)}{(1 + \cos^2 x + 2 \cos x) - (\sin^2 x)}$$

$$\frac{(1 + \cos x)^2 + (\sin x)^2 + 2(1 + \cos x)(\sin x)}{(1 + \cos^2 x + 2 \cos x) - (\sin^2 x)}$$

$$\frac{1 + \cos^2 x + 2 \cos x + \sin^2 x + 2 \sin x + 2 \sin x \cos x}{1 + \cos^2 x + 2 \cos x - \sin^2 x}$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\frac{1 + 1 + 2 \cos x + 2 \sin x + 2 \sin x \cos x}{(1 - \sin^2 x) + \cos^2 x + 2 \cos x}$$

We know, $1 - \cos^2 x = \sin^2 x$.

$$\frac{2 + 2 \cos x + 2 \sin x + 2 \sin x \cos x}{\cos^2 x + \cos^2 x + 2 \cos x}$$

$$\frac{2 + 2 \cos x + 2 \sin x + 2 \sin x \cos x}{2 \cos^2 x + 2 \cos x}$$

$$\frac{2 + 2 \cos x + 2 \sin x + 2 \sin x \cos x}{\cos^2 x + \cos^2 x + 2 \cos x}$$

$$\frac{1 + \cos x + \sin x + \sin x \cos x}{\cos x (\cos x + 1)}$$

$$\frac{1(1 + \cos x) + \sin x (\cos x + 1)}{\cos x (\cos x + 1)}$$

$$\frac{(1 + \sin x)(\cos x + 1)}{\cos x (\cos x + 1)}$$

$$\frac{1 + \sin x}{\cos x} \times \frac{\cos x}{\cos x}$$

$$\frac{(1 + \sin x) \cos x}{\cos^2 x}$$

We know, $1 - \sin^2 x = \cos^2 x$.

$$\frac{(1 + \sin x) \cos x}{1 - \sin^2 x}$$

$$\frac{(1 + \sin x) \cos x}{(1 - \sin x)(1 + \sin x)}$$

$$\frac{\cos x}{1 - \sin x}$$

= LHS

∴ LHS = RHS

Hence Proved.

10.

$$\frac{\tan^3 x}{1 + \tan^2 x} + \frac{\cot^3 x}{1 + \cot^2 x} = \frac{1 - 2 \sin^2 x \cos^2 x}{\sin x \cos x}$$

Solution:

Let us consider LHS:

$$\frac{\tan^3 x}{1 + \tan^2 x} + \frac{\cot^3 x}{1 + \cot^2 x}$$

By using the formulas,

$1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \operatorname{cosec}^2 x$

$$\frac{\tan^3 x}{\sec^2 x} + \frac{\cot^3 x}{\operatorname{cosec}^2 x}$$

$$\frac{\frac{\sin^3 x}{\cos^3 x}}{\frac{1}{\cos^2 x}} + \frac{\frac{\cos^3 x}{\sin^3 x}}{\frac{1}{\sin^2 x}}$$

$$\frac{\sin^3 x}{\cos x} + \frac{\cos^3 x}{\sin x}$$

$$\frac{\sin^4 x + \cos^4 x}{\cos x \sin x}$$

$$\frac{(\sin^2 x)^2 + (\cos^2 x)^2}{\cos x \sin x}$$

We know, $a^2 + b^2 = (a + b)^2 - 2ab$

$$\frac{(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x}{\sin x \cos x}$$

$$\frac{1^2 - 2 \sin^2 x \cos^2 x}{\sin x \cos x}$$

$$\frac{1 - 2 \sin^2 x \cos^2 x}{\sin x \cos x}$$

= RHS

∴ LHS = RHS

Hence Proved.

11.

$$1 - \frac{\sin^2 x}{1 + \cot x} - \frac{\cos^2 x}{1 + \tan x} = \sin x \cos x$$

Solution:

Let us consider LHS:

$$1 - \frac{\sin^2 x}{1 + \cot x} - \frac{\cos^2 x}{1 + \tan x}$$

By using the formula,

$$\tan \theta = \sin \theta / \cos \theta;$$

$$\cot \theta = \cos \theta / \sin \theta$$

Now,

$$\begin{aligned} & 1 - \frac{\sin^2 x}{1 + \frac{\cos x}{\sin x}} - \frac{\cos^2 x}{1 + \frac{\sin x}{\cos x}} \\ & 1 - \frac{\sin^3 x}{\sin x + \cos x} - \frac{\cos^3 x}{\sin x + \cos x} \\ & \frac{\sin x + \cos x - (\sin^3 x + \cos^3 x)}{\sin x + \cos x} \end{aligned}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\begin{aligned} & \frac{\sin x + \cos x - ((\sin x + \cos x)(\sin x)^2 + (\cos x)^2 - \sin x \cos x))}{\sin x + \cos x} \\ & \frac{(\sin x + \cos x)(1 - \sin^2 x - \cos^2 x + \sin x \cos x)}{\sin x + \cos x} \end{aligned}$$

$$1 - (\sin^2 x + \cos^2 x) + \sin x \cos x$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$1 - 1 + \sin x \cos x$$

$$\sin x \cos x$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

12.

$$\left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{\operatorname{cosec}^2 x - \sin^2 x} \right) \sin^2 x \cos^2 x = \frac{1 - \sin^2 x \cos^2 x}{2 + \sin^2 x \cos^2 x}$$

Solution:

Let us consider LHS:

$$\left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{\operatorname{cosec}^2 x - \sin^2 x} \right) \sin^2 x \cos^2 x$$

By using the formula,

$$\operatorname{cosec} \theta = 1/\sin \theta,$$

$$\sec \theta = 1/\cos \theta;$$

$$\left(\frac{1}{\frac{1}{\cos^2 x} - \cos^2 x} + \frac{1}{\frac{1}{\sin^2 x} - \sin^2 x} \right) \sin^2 x \cos^2 x$$

$$\left(\frac{\cos^2 x}{1 - \cos^4 x} + \frac{\sin^2 x}{1 - \sin^4 x} \right) \sin^2 x \cos^2 x$$

$$\left(\frac{\cos^2 x (1 - \sin^4 x) + \sin^2 x (1 - \cos^4 x)}{(1 - \cos^4 x)(1 - \sin^4 x)} \right) \sin^2 x \cos^2 x$$

$$\left(\frac{\cos^2 x - \cos^2 x \sin^4 x + \sin^2 x - \sin^2 x \cos^4 x}{(1 + \sin^2 x)(1 - \sin^2 x)(1 + \cos^2 x)(1 - \cos^2 x)} \right) \sin^2 x \cos^2 x$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\left(\frac{1 - \cos^2 x \sin^4 x - \sin^2 x \cos^4 x}{(1 + \sin^2 x) \cos^2 x (1 + \cos^2 x) \sin^2 x} \right) \sin^2 x \cos^2 x$$

$$\left(\frac{1 - \cos^2 x \sin^2 x (\sin^2 x + \cos^2 x)}{(1 + \sin^2 x)(1 + \cos^2 x)} \right)$$

$$\frac{1 - \sin^2 x \cos^2 x}{2 + \sin^2 x \cos^2 x}$$

= RHS

∴ LHS = RHS

Hence proved.

$$13. (1 + \tan \alpha \tan \beta)^2 + (\tan \alpha - \tan \beta)^2 = \sec^2 \alpha \sec^2 \beta$$

Solution:

Let us consider LHS: $(1 + \tan \alpha \tan \beta)^2 + (\tan \alpha - \tan \beta)^2$

$$1 + \tan^2 \alpha \tan^2 \beta + 2 \tan \alpha \tan \beta + \tan^2 \alpha + \tan^2 \beta - 2 \tan \alpha \tan \beta$$

$$1 + \tan^2 \alpha \tan^2 \beta + \tan^2 \alpha + \tan^2 \beta$$

$$\tan^2 \alpha (\tan^2 \beta + 1) + 1 (1 + \tan^2 \beta)$$

$$(1 + \tan^2 \beta) (1 + \tan^2 \alpha)$$

We know, $1 + \tan^2 \theta = \sec^2 \theta$

So,

$$\sec^2 \alpha \sec^2 \beta$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

EXERCISE 5.2 PAGE NO: 5.25

1. Find the values of the other five trigonometric functions in each of the following:

(i) $\cot x = 12/5$, x in quadrant III

(ii) $\cos x = -1/2$, x in quadrant II

(iii) $\tan x = 3/4$, x in quadrant III

(iv) $\sin x = 3/5$, x in quadrant I

Solution:

(i) $\cot x = 12/5$, x in quadrant III

In third quadrant, $\tan x$ and $\cot x$ are positive. $\sin x$, $\cos x$, $\sec x$, $\csc x$ are negative.

By using the formulas,

$$\tan x = 1/\cot x$$

$$= 1/(12/5)$$

$$= 5/12$$

$$\csc x = -\sqrt{1 + \cot^2 x}$$

$$= -\sqrt{1 + (12/5)^2}$$

$$= -\sqrt{(25+144)/25}$$

$$= -\sqrt{(169/25)}$$

$$= -13/5$$

$$\sin x = 1/\csc x$$

$$= 1/(-13/5)$$

$$= -5/13$$

$$\cos x = -\sqrt{1 - \sin^2 x}$$

$$= -\sqrt{1 - (-5/13)^2}$$

$$= -\sqrt{(169-25)/169}$$

$$= -\sqrt{(144/169)}$$

$$= -12/13$$

$$\sec x = 1/\cos x$$

$$= 1/(-12/13)$$

$$= -13/12$$

$$\therefore \sin x = -5/13, \cos x = -12/13, \tan x = 5/12, \operatorname{cosec} x = -13/5, \sec x = -13/12$$

(ii) $\cos x = -1/2$, x in quadrant II

In second quadrant, $\sin x$ and $\operatorname{cosec} x$ are positive. $\tan x$, $\cot x$, $\cos x$, $\sec x$ are negative.

By using the formulas,

$$\sin x = \sqrt{1 - \cos^2 x}$$

$$= \sqrt{1 - (-1/2)^2}$$

$$= \sqrt{4-1}/4$$

$$= \sqrt{3/4}$$

$$= \sqrt{3}/2$$

$$\tan x = \sin x / \cos x$$

$$= (\sqrt{3}/2) / (-1/2)$$

$$= -\sqrt{3}$$

$$\cot x = 1/\tan x$$

$$= 1/-\sqrt{3}$$

$$= -1/\sqrt{3}$$

$$\operatorname{cosec} x = 1/\sin x$$

$$= 1/(\sqrt{3}/2)$$

$$= 2/\sqrt{3}$$

$$\sec x = 1/\cos x$$

$$= 1/(-1/2)$$

$$= -2$$

$$\therefore \sin x = \sqrt{3}/2, \tan x = -\sqrt{3}, \operatorname{cosec} x = 2/\sqrt{3}, \cot x = -1/\sqrt{3}, \sec x = -2$$

(iii) $\tan x = 3/4$, x in quadrant III

In third quadrant, $\tan x$ and $\cot x$ are positive. $\sin x$, $\cos x$, $\sec x$, $\operatorname{cosec} x$ are negative.

By using the formulas,

$$\sin x = \sqrt{1 - \cos^2 x}$$

$$= -\sqrt{1 - (-4/5)^2}$$

$$= -\sqrt{(25-16)/25}$$

$$= -\sqrt{9/25}$$

$$= -3/5$$

$$\cos x = 1/\sec x$$

$$= 1/(-5/4)$$

$$= -4/5$$

$$\cot x = 1/\tan x$$

$$= 1/(3/4)$$

$$= 4/3$$

$$\operatorname{cosec} x = 1/\sin x$$

$$= 1/(-3/5)$$

$$= -5/3$$

$$\sec x = -\sqrt{(1 + \tan^2 x)}$$

$$= -\sqrt{(1+(3/4)^2)}$$

$$= -\sqrt{(16+9)/16}$$

$$= -\sqrt{(25/16)}$$

$$= -5/4$$

$$\therefore \sin x = -3/5, \cos x = -4/5, \operatorname{cosec} x = -5/3, \sec x = -5/4, \cot x = 4/3$$

(iv) $\sin x = 3/5$, x in quadrant I

In first quadrant, all trigonometric ratios are positive.

So, by using the formulas,

$$\tan x = \sin x/\cos x$$

$$= (3/5)/(4/5)$$

$$= 3/4$$

$$\operatorname{cosec} x = 1/\sin x$$

$$= 1/(3/5)$$

$$= 5/3$$

$$\cos x = \sqrt{(1-\sin^2 x)}$$

$$= \sqrt{(1 - (3/5)^2)}$$

$$= \sqrt{(25-9)/25}$$

$$= \sqrt{(16/25)}$$

$$= 4/5$$

$$\sec x = 1/\cos x$$

$$= 1/(4/5)$$

$$= 5/4$$

$$\cot x = 1/\tan x$$

$$= 1/(3/4)$$

$$= 4/3$$

$$\therefore \cos x = 4/5, \tan x = 3/4, \operatorname{cosec} x = 5/3, \sec x = 5/4, \cot x = 4/3$$

2. If $\sin x = 12/13$ and lies in the second quadrant, find the value of $\sec x + \tan x$.

Solution:

Given:

$\sin x = 12/13$ and x lies in the second quadrant.

We know, in second quadrant, $\sin x$ and $\operatorname{cosec} x$ are positive and all other ratios are negative.

By using the formulas,

$$\begin{aligned}\cos x &= \sqrt{1 - \sin^2 x} \\ &= -\sqrt{1 - (12/13)^2} \\ &= -\sqrt{1 - (144/169)} \\ &= -\sqrt{(169 - 144)/169} \\ &= -\sqrt{25/169} \\ &= -5/13\end{aligned}$$

We know,

$$\tan x = \sin x / \cos x$$

$$\sec x = 1 / \cos x$$

Now,

$$\tan x = (12/13) / (-5/13)$$

$$= -12/5$$

$$\sec x = 1 / (-5/13)$$

$$= -13/5$$

$$\sec x + \tan x = -13/5 + (-12/5)$$

$$= (-13 - 12)/5$$

$$= -25/5$$

$$= -5$$

$$\therefore \sec x + \tan x = -5$$

3. If $\sin x = 3/5$, $\tan y = 1/2$ and $\pi/2 < x < \pi < y < 3\pi/2$ find the value of $8 \tan x - \sqrt{5} \sec y$.

Solution:

Given:

$$\sin x = 3/5, \tan y = 1/2 \text{ and } \pi/2 < x < \pi < y < 3\pi/2$$

We know that, x is in second quadrant and y is in third quadrant.

In second quadrant, $\cos x$ and $\tan x$ are negative.

In third quadrant, $\sec y$ is negative.

By using the formula,

$$\cos x = -\sqrt{1-\sin^2 x}$$

$$\tan x = \sin x / \cos x$$

Now,

$$\cos x = -\sqrt{1-\sin^2 x}$$

$$= -\sqrt{1 - (3/5)^2}$$

$$= -\sqrt{1 - 9/25}$$

$$= -\sqrt{(25-9)/25}$$

$$= -\sqrt{16/25}$$

$$= -4/5$$

$$\tan x = \sin x / \cos x$$

$$= (3/5)/(-4/5)$$

$$= 3/5 \times -5/4$$

$$= -3/4$$

$$\text{We know that } \sec y = -\sqrt{1+\tan^2 y}$$

$$= -\sqrt{1 + (1/2)^2}$$

$$= -\sqrt{1 + 1/4}$$

$$= -\sqrt{(4+1)/4}$$

$$= -\sqrt{5/4}$$

$$= -\sqrt{5}/2$$

$$\text{Now, } 8 \tan x - \sqrt{5} \sec y = 8(-3/4) - \sqrt{5}(-\sqrt{5}/2)$$

$$= -6 + 5/2$$

$$= (-12+5)/2$$

$$= -7/2$$

$$\therefore 8 \tan x - \sqrt{5} \sec y = -7/2$$

4. If $\sin x + \cos x = 0$ and x lies in the fourth quadrant, find $\sin x$ and $\cos x$.

Solution:

Given:

$$\sin x + \cos x = 0 \text{ and } x \text{ lies in fourth quadrant.}$$

$$\sin x = -\cos x$$

$$\sin x / \cos x = -1$$

$$\text{So, } \tan x = -1 \text{ (since, } \tan x = \sin x / \cos x \text{)}$$

We know that, in fourth quadrant, $\cos x$ and $\sec x$ are positive and all other ratios are negative.

By using the formulas,

$$\sec x = \sqrt{1 + \tan^2 x}$$

$$\cos x = 1/\sec x$$

$$\sin x = -\sqrt{1 - \cos^2 x}$$

Now,

$$\sec x = \sqrt{1 + \tan^2 x}$$

$$= \sqrt{1 + (-1)^2}$$

$$= \sqrt{2}$$

$$\cos x = 1/\sec x$$

$$= 1/\sqrt{2}$$

$$\sin x = -\sqrt{1 - \cos^2 x}$$

$$= -\sqrt{1 - (1/\sqrt{2})^2}$$

$$= -\sqrt{1 - (1/2)}$$

$$= -\sqrt{(2-1)/2}$$

$$= -\sqrt{1/2}$$

$$= -1/\sqrt{2}$$

$$\therefore \sin x = -1/\sqrt{2} \text{ and } \cos x = 1/\sqrt{2}$$

5. If $\cos x = -3/5$ and $\pi < x < 3\pi/2$ find the values of other five trigonometric functions and hence

evaluate $\frac{\operatorname{cosec} x + \cot x}{\sec x - \tan x}$

Solution:

Given:

$$\cos x = -3/5 \text{ and } \pi < x < 3\pi/2$$

We know that in the third quadrant, $\tan x$ and $\cot x$ are positive and all other ratios are negative.

By using the formulas,

$$\sin x = -\sqrt{1 - \cos^2 x}$$

$$\tan x = \sin x / \cos x$$

$$\cot x = 1/\tan x$$

$$\sec x = 1/\cos x$$

$$\operatorname{Cosec} x = 1/\sin x$$

Now,

$$\sin x = -\sqrt{1 - \cos^2 x}$$

$$= -\sqrt{1 - (-3/5)^2}$$

$$= -\sqrt{(1-9/25)}$$

$$= -\sqrt{((25-9)/25)}$$

$$= -\sqrt{(16/25)}$$

$$= -4/5$$

$$\tan x = \sin x / \cos x$$

$$= (-4/5) / (-3/5)$$

$$= -4/5 \times -5/3$$

$$= 4/3$$

$$\cot x = 1/\tan x$$

$$= 1/(4/3)$$

$$= 3/4$$

$$\sec x = 1/\cos x$$

$$= 1/(-3/5)$$

$$= -5/3$$

$$\operatorname{cosec} x = 1/\sin x$$

$$= 1/(-4/5)$$

$$= -5/4$$

$$\frac{\operatorname{cosec} x + \cot x}{\sec x - \tan x}$$

$$\therefore \frac{\sec x - \tan x}{\sec x - \tan x}$$

$$= [(-5/4) + (3/4)] / [(-5/3) - (4/3)]$$

$$= [(-5+3)/4] / [(-5-4)/3]$$

$$= [-2/4] / [-9/3]$$

$$= [-1/2] / [-3]$$

$$= 1/6$$

EXERCISE 5.3 PAGE NO: 5.39

1. Find the values of the following trigonometric ratios:

(i) $\sin 5\pi/3$

(ii) $\sin 17\pi$

(iii) $\tan 11\pi/6$

(iv) $\cos (-25\pi/4)$

(v) $\tan 7\pi/4$

(vi) $\sin 17\pi/6$

(vii) $\cos 19\pi/6$

(viii) $\sin (-11\pi/6)$

(ix) $\operatorname{cosec} (-20\pi/3)$

(x) $\tan (-13\pi/4)$

(xi) $\cos 19\pi/4$

(xii) $\sin 41\pi/4$

(xiii) $\cos 39\pi/4$

(xiv) $\sin 151\pi/6$

Solution:

(i) $\sin 5\pi/3$

$$5\pi/3 = (5\pi/3 \times 180)^\circ$$

$$= 300^\circ$$

$$= (90 \times 3 + 30)^\circ$$

Since, 300° lies in IV quadrant in which sine function is negative.

$$\sin 5\pi/3 = \sin (300)^\circ$$

$$= \sin (90 \times 3 + 30)^\circ$$

$$= -\cos 30^\circ$$

$$= -\sqrt{3}/2$$

(ii) $\sin 17\pi$

$$\sin 17\pi = \sin 3060^\circ$$

$$= \sin (90 \times 34 + 0)^\circ$$

Since, 3060° lies in the negative direction of x-axis i.e., on boundary line of II and III quadrants.

$$\sin 17\pi = \sin (90 \times 34 + 0)^\circ$$

$$= -\sin 0^\circ$$

$$= 0$$

(iii) $\tan 11\pi/6$

$$\tan 11\pi/6 = (11/6 \times 180)^\circ$$

$$= 330^\circ$$

Since, 330° lies in the IV quadrant in which tangent function is negative.

$$\tan 11\pi/6 = \tan (300)^\circ$$

$$= \tan (90 \times 3 + 60)^\circ$$

$$= -\cot 60^\circ$$

$$= -1/\sqrt{3}$$

$$\text{(iv) } \cos (-25\pi/4)$$

$$\cos (-25\pi/4) = \cos (-1125)^\circ$$

$$= \cos (1125)^\circ$$

Since, 1125° lies in the I quadrant in which cosine function is positive.

$$\cos (1125)^\circ = \cos (90 \times 12 + 45)^\circ$$

$$= \cos 45^\circ$$

$$= 1/\sqrt{2}$$

$$\text{(v) } \tan 7\pi/4$$

$$\tan 7\pi/4 = \tan 315^\circ$$

$$= \tan (90 \times 3 + 45)^\circ$$

Since, 315° lies in the IV quadrant in which tangent function is negative.

$$\tan 315^\circ = \tan (90 \times 3 + 45)^\circ$$

$$= -\cot 45^\circ$$

$$= -1$$

$$\text{(vi) } \sin 17\pi/6$$

$$\sin 17\pi/6 = \sin 510^\circ$$

$$= \sin (90 \times 5 + 60)^\circ$$

Since, 510° lies in the II quadrant in which sine function is positive.

$$\sin 510^\circ = \sin (90 \times 5 + 60)^\circ$$

$$= \cos 60^\circ$$

$$= 1/2$$

$$\text{(vii) } \cos 19\pi/6$$

$$\cos 19\pi/6 = \cos 570^\circ$$

$$= \cos (90 \times 6 + 30)^\circ$$

Since, 570° lies in III quadrant in which cosine function is negative.

$$\cos 570^\circ = \cos (90 \times 6 + 30)^\circ$$

$$= -\cos 30^\circ$$

$$= -\sqrt{3}/2$$

$$\text{(viii) } \sin (-11\pi/6)$$

$$\sin (-11\pi/6) = \sin (-330)^\circ$$

$$= -\sin (90 \times 3 + 60)^\circ$$

Since, 330° lies in the IV quadrant in which the sine function is negative.

$$\sin (-330^\circ) = -\sin (90 \times 3 + 60)^\circ$$

$$= -(-\cos 60^\circ)$$

$$= -(-1/2)$$

$$= 1/2$$

$$\text{(ix) cosec } (-20\pi/3)$$

$$\text{cosec } (-20\pi/3) = \text{cosec } (-1200)^\circ$$

$$= -\text{cosec } (1200)^\circ$$

$$= -\text{cosec } (90 \times 13 + 30)^\circ$$

Since, 1200° lies in the II quadrant in which cosec function is positive.

$$\text{cosec } (-1200)^\circ = -\text{cosec } (90 \times 13 + 30)^\circ$$

$$= -\sec 30^\circ$$

$$= -2/\sqrt{3}$$

$$\text{(x) tan } (-13\pi/4)$$

$$\tan (-13\pi/4) = \tan (-585)^\circ$$

$$= -\tan (90 \times 6 + 45)^\circ$$

Since, 585° lies in the III quadrant in which the tangent function is positive.

$$\tan (-585)^\circ = -\tan (90 \times 6 + 45)^\circ$$

$$= -\tan 45^\circ$$

$$= -1$$

$$\text{(xi) cos } 19\pi/4$$

$$\cos 19\pi/4 = \cos 855^\circ$$

$$= \cos (90 \times 9 + 45)^\circ$$

Since, 855° lies in the II quadrant in which the cosine function is negative.

$$\cos 855^\circ = \cos (90 \times 9 + 45)^\circ$$

$$= -\sin 45^\circ$$

$$= -1/\sqrt{2}$$

$$\text{(xii) sin } 41\pi/4$$

$$\sin 41\pi/4 = \sin 1845^\circ$$

$$= \sin (90 \times 20 + 45)^\circ$$

Since, 1845° lies in the I quadrant in which the sine function is positive.

$$\sin 1845^\circ = \sin (90 \times 20 + 45)^\circ$$

$$= \sin 45^\circ$$

$$= 1/\sqrt{2}$$

(xiii) $\cos 39\pi/4$

$$\cos 39\pi/4 = \cos 1755^\circ$$

$$= \cos (90 \times 19 + 45)^\circ$$

Since, 1755° lies in the IV quadrant in which the cosine function is positive.

$$\cos 1755^\circ = \cos (90 \times 19 + 45)^\circ$$

$$= \sin 45^\circ$$

$$= 1/\sqrt{2}$$

(xiv) $\sin 151\pi/6$

$$\sin 151\pi/6 = \sin 4530^\circ$$

$$= \sin (90 \times 50 + 30)^\circ$$

Since, 4530° lies in the III quadrant in which the sine function is negative.

$$\sin 4530^\circ = \sin (90 \times 50 + 30)^\circ$$

$$= -\sin 30^\circ$$

$$= -1/2$$

2. prove that:

(i) $\tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ = 0$

(ii) $\sin 8\pi/3 \cos 23\pi/6 + \cos 13\pi/3 \sin 35\pi/6 = 1/2$

(iii) $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ = 1/2$

(iv) $\tan (-125^\circ) \cot (-405^\circ) - \tan (-765^\circ) \cot (675^\circ) = 0$

(v) $\cos 570^\circ \sin 510^\circ + \sin (-330^\circ) \cos (-390^\circ) = 0$

(vi) $\tan 11\pi/3 - 2 \sin 4\pi/6 - 3/4 \operatorname{cosec}^2 \pi/4 + 4 \cos^2 17\pi/6 = (3 - 4\sqrt{3})/2$

(vii) $3 \sin \pi/6 \sec \pi/3 - 4 \sin 5\pi/6 \cot \pi/4 = 1$

Solution:

(i) $\tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ = 0$

Let us consider LHS:

$$\tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$$

$$\tan (90^\circ \times 2 + 45^\circ) \cot (90^\circ \times 4 + 45^\circ) + \tan (90^\circ \times 8 + 45^\circ) \cot (90^\circ \times 7 + 45^\circ)$$

We know that when n is odd, $\cot \rightarrow \tan$.

$$\tan 45^\circ \cot 45^\circ + \tan 45^\circ [-\tan 45^\circ]$$

$$\tan 45^\circ \cot 45^\circ - \tan 45^\circ \tan 45^\circ$$

$$1 \times 1 - 1 \times 1$$

$$1 - 1$$

$$0 = \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$\text{(ii)} \sin 8\pi/3 \cos 23\pi/6 + \cos 13\pi/3 \sin 35\pi/6 = 1/2$$

Let us consider LHS:

$$\sin 8\pi/3 \cos 23\pi/6 + \cos 13\pi/3 \sin 35\pi/6$$

$$\sin 480^\circ \cos 690^\circ + \cos 780^\circ \sin 1050^\circ$$

$$\sin (90^\circ \times 5 + 30^\circ) \cos (90^\circ \times 7 + 60^\circ) + \cos (90^\circ \times 8 + 60^\circ) \sin (90^\circ \times 11 + 60^\circ)$$

We know that when n is odd, $\sin \rightarrow \cos$ and $\cos \rightarrow \sin$.

$$\cos 30^\circ \sin 60^\circ + \cos 60^\circ [-\cos 60^\circ]$$

$$\sqrt{3}/2 \times \sqrt{3}/2 - 1/2 \times 1/2$$

$$3/4 - 1/4$$

$$2/4$$

$$1/2$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$\text{(iii)} \cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ = 1/2$$

Let us consider LHS:

$$\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ$$

$$\cos 24^\circ + \cos (90^\circ \times 1 - 35^\circ) + \cos (90^\circ \times 1 + 35^\circ) + \cos (90^\circ \times 2 + 24^\circ) + \cos (90^\circ \times 3 + 30^\circ)$$

We know that when n is odd, $\cos \rightarrow \sin$.

$$\cos 24^\circ + \sin 35^\circ - \sin 35^\circ - \cos 24^\circ + \sin 30^\circ$$

$$0 + 0 + 1/2$$

$$1/2$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$\text{(iv)} \tan (-125^\circ) \cot (-405^\circ) - \tan (-765^\circ) \cot (675^\circ) = 0$$

Let us consider LHS:

$$\tan (-125^\circ) \cot (-405^\circ) - \tan (-765^\circ) \cot (675^\circ)$$

We know that $\tan (-x) = -\tan (x)$ and $\cot (-x) = -\cot (x)$.

$$[-\tan (225^\circ)] [-\cot (405^\circ)] - [-\tan (765^\circ)] \cot (675^\circ)$$

$$\tan (225^\circ) \cot (405^\circ) + \tan (765^\circ) \cot (675^\circ)$$

$$\tan (90^\circ \times 2 + 45^\circ) \cot (90^\circ \times 4 + 45^\circ) + \tan (90^\circ \times 8 + 45^\circ) \cot (90^\circ \times 7 + 45^\circ)$$

$$\tan 45^\circ \cot 45^\circ + \tan 45^\circ [-\tan 45^\circ]$$

$$1 \times 1 + 1 \times (-1)$$

$$1 - 1$$

$$0$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$(v) \cos 570^\circ \sin 510^\circ + \sin (-330^\circ) \cos (-390^\circ) = 0$$

Let us consider LHS:

$$\cos 570^\circ \sin 510^\circ + \sin (-330^\circ) \cos (-390^\circ)$$

We know that $\sin (-x) = -\sin (x)$ and $\cos (-x) = +\cos (x)$.

$$\cos 570^\circ \sin 510^\circ + [-\sin (330^\circ)] \cos (390^\circ)$$

$$\cos 570^\circ \sin 510^\circ - \sin (330^\circ) \cos (390^\circ)$$

$$\cos (90^\circ \times 6 + 30^\circ) \sin (90^\circ \times 5 + 60^\circ) - \sin (90^\circ \times 3 + 60^\circ) \cos (90^\circ \times 4 + 30^\circ)$$

We know that $\cos$ is negative at $90^\circ + \theta$ i.e. in Q_2 and when n is odd, $\sin \rightarrow \cos$ and $\cos \rightarrow \sin$.

$$-\cos 30^\circ \cos 60^\circ - [-\cos 60^\circ] \cos 30^\circ$$

$$-\cos 30^\circ \cos 60^\circ + \cos 60^\circ \cos 30^\circ$$

$$0$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$(vi) \tan 11\pi/3 - 2 \sin 4\pi/6 - 3/4 \operatorname{cosec}^2 \pi/4 + 4 \cos^2 17\pi/6 = (3 - 4\sqrt{3})/2$$

Let us consider LHS:

$$\tan 11\pi/3 - 2 \sin 4\pi/6 - 3/4 \operatorname{cosec}^2 \pi/4 + 4 \cos^2 17\pi/6$$

$$\tan (11 \times 180^\circ)/3 - 2 \sin (4 \times 180^\circ)/6 - 3/4 \operatorname{cosec}^2 180^\circ/4 + 4 \cos^2 (17 \times 180^\circ)/6$$

$$\tan 660^\circ - 2 \sin 120^\circ - 3/4 (\operatorname{cosec} 45^\circ)^2 + 4 (\cos 510^\circ)^2$$

$$\tan (90^\circ \times 7 + 30^\circ) - 2 \sin (90^\circ \times 1 + 30^\circ) - 3/4 [\operatorname{cosec} 45^\circ]^2 + 4 [\cos (90^\circ \times 5 + 60^\circ)]^2$$

We know that $\tan$ and $\cos$ is negative at $90^\circ + \theta$ i.e. in Q_2 and when n is odd, $\tan \rightarrow \cot$, $\sin \rightarrow \cos$ and $\cos \rightarrow \sin$.

$$[-\cot 30^\circ] - 2 \cos 30^\circ - 3/4 [\operatorname{cosec} 45^\circ]^2 + [-\sin 60^\circ]^2$$

$$-\cot 30^\circ - 2 \cos 30^\circ - 3/4 [\operatorname{cosec} 45^\circ]^2 + [\sin 60^\circ]^2$$

$$-\sqrt{3} - 2\sqrt{3}/2 - 3/4 (\sqrt{2})^2 + 4 (\sqrt{3}/2)^2$$

$$-\sqrt{3} - \sqrt{3} - 6/4 + 12/4$$

$$(3 - 4\sqrt{3})/2$$

= RHS

∴ LHS = RHS

Hence proved.

(vii) $3 \sin \pi/6 \sec \pi/3 - 4 \sin 5\pi/6 \cot \pi/4 = 1$

Let us consider LHS:

$$3 \sin \pi/6 \sec \pi/3 - 4 \sin 5\pi/6 \cot \pi/4$$

$$3 \sin 180^\circ/6 \sec 180^\circ/3 - 4 \sin 5(180^\circ)/6 \cot 180^\circ/4$$

$$3 \sin 30^\circ \sec 60^\circ - 4 \sin 150^\circ \cot 45^\circ$$

$$3 \sin 30^\circ \sec 60^\circ - 4 \sin (90^\circ \times 1 + 60^\circ) \cot 45^\circ$$

We know that when n is odd, $\sin \rightarrow \cos$.

$$3 \sin 30^\circ \sec 60^\circ - 4 \cos 60^\circ \cot 45^\circ$$

$$3 (1/2) (2) - 4 (1/2) (1)$$

$$3 - 2$$

$$1$$

= RHS

∴ LHS = RHS

Hence proved.

3. Prove that:

(i)

$$\frac{\cos(2\pi + x) \operatorname{cosec}(2\pi + x) \tan(\pi/2 + x)}{\sec(\pi/2 + x) \cos x \cot(\pi + x)} = 1$$

(ii)

$$\frac{\operatorname{cosec}(90^\circ + x) + \cot(450^\circ + x)}{\operatorname{cosec}(90^\circ - x) + \tan(180^\circ - x)} + \frac{\tan(180^\circ + x) + \sec(180^\circ - x)}{\tan(360^\circ + x) - \sec(-x)} = 2$$

(iii)

$$\frac{\sin(\pi + x) \cos(\frac{\pi}{2} + x) \tan(\frac{3\pi}{2} - x) \cot(2\pi - x)}{\sin(2\pi - x) \cos(2\pi + x) \operatorname{cosec}(-x) \sin(\frac{3\pi}{2} - x)} = 1$$

(iv)

$$\left\{ 1 + \cot x - \sec\left(\frac{\pi}{2} + x\right) \right\} \left\{ 1 + \cot x + \sec\left(\frac{\pi}{2} + x\right) \right\} = 2 \cot x$$

(v)

$$\frac{\tan(\frac{\pi}{2} - x) \sec(\pi - x) \sin(-x)}{\sin(\pi + x) \cot(2\pi - x) \operatorname{cosec}(\frac{\pi}{2} - x)} = 1$$

Solution:

(i)

$$\frac{\cos(2\pi + x) \operatorname{cosec}(2\pi + x) \tan(\pi/2 + x)}{\sec(\pi/2 + x) \cos x \cot(\pi + x)} = 1$$

Let us consider LHS:

$$\frac{\cos(2\pi + x) \operatorname{cosec}(2\pi + x) \tan(\pi/2 + x)}{\sec(\pi/2 + x) \cos x \cot(\pi + x)}$$

$$\frac{\cos x \operatorname{cosec} x [-\cot x]}{[-\operatorname{cosec} x] \cos x \cot x}$$

$$\frac{-\cos x \operatorname{cosec} x \cot x}{-\operatorname{cosec} x \cos x \cot x}$$

$$1 = \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

(ii)

$$\frac{\operatorname{cosec}(90^\circ + x) + \cot(450^\circ + x)}{\operatorname{cosec}(90^\circ - x) + \tan(180^\circ - x)} + \frac{\tan(180^\circ + x) + \sec(180^\circ - x)}{\tan(360^\circ + x) - \sec(-x)} = 2$$

Let us consider LHS:

$$\frac{\operatorname{cosec}(90^\circ + x) + \cot(450^\circ + x)}{\operatorname{cosec}(90^\circ - x) + \tan(180^\circ - x)} + \frac{\tan(180^\circ + x) + \sec(180^\circ - x)}{\tan(360^\circ + x) - \sec(-x)}$$

$$\frac{\operatorname{cosec}(90^\circ + x) + \cot(90^\circ \times 5 + x)}{\operatorname{cosec}(90^\circ - x) + \tan(90^\circ \times 2 - x)} + \frac{\tan(90^\circ \times 2 + x) + \sec(90^\circ \times 2 - x)}{\tan(90^\circ \times 4 + x) - \sec(-x)}$$

We know that when n is odd, $\operatorname{cosec} \rightarrow \sec$ and also $\sec(-x) = \sec x$.

$$\frac{\sec x + \cot(90^\circ \times 5 + x)}{\operatorname{cosec}(90^\circ - x) + \tan(90^\circ \times 2 - x)} + \frac{\tan(90^\circ \times 2 + x) + \sec(90^\circ \times 2 - x)}{\tan(90^\circ \times 4 + x) - \sec(-x)}$$

$$\frac{\sec x - \tan x}{\sec x - \tan x} + \frac{\tan x - \sec x}{\tan x - \sec x}$$

$$1 + 1$$

2 = RHS

∴ LHS = RHS

Hence proved.

(iii)

$$\frac{\sin(\pi + x) \cos(\frac{\pi}{2} + x) \tan(\frac{3\pi}{2} - x) \cot(2\pi - x)}{\sin(2\pi - x) \cos(2\pi + x) \operatorname{cosec}(-x) \sin(\frac{3\pi}{2} - x)} = 1$$

Let us consider LHS:

$$\frac{\sin(\pi + x) \cos(\frac{\pi}{2} + x) \tan(\frac{3\pi}{2} - x) \cot(2\pi - x)}{\sin(2\pi - x) \cos(2\pi + x) \operatorname{cosec}(-x) \sin(\frac{3\pi}{2} - x)}$$
$$\frac{\sin(180^\circ - x) \cos(90^\circ + x) \tan(270^\circ - x) \cot(360^\circ - x)}{\sin(360^\circ - x) \cos(360^\circ + x) \operatorname{cosec}(-x) \sin(270^\circ - x)}$$

We know that $\operatorname{cosec}(-x) = -\operatorname{cosec} x$.

$$\frac{\sin(90^\circ \times 2 - x) \cos(90^\circ \times 1 + x) \tan(90^\circ \times 3 - x) \cot(90^\circ \times 4 - x)}{\sin(90^\circ \times 4 - x) \cos(90^\circ \times 4 + x) [-\operatorname{cosec}(x)] \sin(90^\circ \times 3 - x)}$$

We know that when n is odd, $\cos \rightarrow \sin$, $\tan \rightarrow \cot$ and $\sin \rightarrow \cos$.

$$\frac{(-\sin x)(-\sin x) \cot x (-\cot x)}{(-\sin x) \cos x (-\operatorname{cosec} x)(-\cos x)}$$

$$\frac{\sin^2 x \cot^2 x}{\sin x \operatorname{cosec} x \cos x \cos x}$$

$$\frac{\sin^2 x \times \frac{\cos^2 x}{\sin^2 x}}{\sin x \times \frac{1}{\sin x} \times \cos^2 x}$$

$$\frac{\cos^2 x}{\cos^2 x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

(iv)

$$\left\{1 + \cot x - \sec\left(\frac{\pi}{2} + x\right)\right\} \left\{1 + \cot x + \sec\left(\frac{\pi}{2} + x\right)\right\} = 2 \cot x$$

Let us consider LHS:

$$\left\{1 + \cot x - \sec\left(\frac{\pi}{2} + x\right)\right\} \left\{1 + \cot x + \sec\left(\frac{\pi}{2} + x\right)\right\}$$

$$\{1 + \cot x - (-\operatorname{cosec} x)\} \{1 + \cot x + (-\operatorname{cosec} x)\}$$

$$\{1 + \cot x + \operatorname{cosec} x\} \{1 + \cot x - \operatorname{cosec} x\}$$

$$\{(1 + \cot x) + (\operatorname{cosec} x)\} \{(1 + \cot x) - (\operatorname{cosec} x)\}$$

By using the formula, $(a + b)(a - b) = a^2 - b^2$

$$(1 + \cot x)^2 - (\operatorname{cosec} x)^2$$

$$1 + \cot^2 x + 2 \cot x - \operatorname{cosec}^2 x$$

We know that $1 + \cot^2 x = \operatorname{cosec}^2 x$

$$\operatorname{cosec}^2 x + 2 \cot x - \operatorname{cosec}^2 x$$

$$2 \cot x = \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

(v)

$$\frac{\tan(\frac{\pi}{2} - x) \sec(\pi - x) \sin(-x)}{\sin(\pi + x) \cot(2\pi - x) \operatorname{cosec}(\frac{\pi}{2} - x)} = 1$$

Let us consider LHS:

$$\frac{\tan(\frac{\pi}{2} - x) \sec(\pi - x) \sin(-x)}{\sin(\pi + x) \cot(2\pi - x) \operatorname{cosec}(\frac{\pi}{2} - x)}$$

$$\frac{\tan(90^\circ - x) \sec(180^\circ - x) \sin(-x)}{\sin(180^\circ + x) \cot(360^\circ - x) \operatorname{cosec}(90^\circ - x)}$$

We know that $\sin(-x) = -\sin x$.

$$\frac{\tan(90^\circ - x) \sec(180^\circ - x) [-\sin(x)]}{\sin(180^\circ + x) \cot(360^\circ - x) \operatorname{cosec}(90^\circ - x)}$$

We know that when n is odd, $\tan \rightarrow \cot$ and $\operatorname{cosec} \rightarrow \sec$.

$$\frac{(\cot x)(-\sec x)(-\sin x)}{(-\sin x)(-\cot x)(\sec x)}$$

$$\frac{\cot x \sec x \sin x}{\sin x \cot x \sec x}$$

$$1 = \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

4. Prove that: $\sin^2 \pi/18 + \sin^2 \pi/9 + \sin^2 7\pi/18 + \sin^2 4\pi/9 = 2$

Solution:

Let us consider LHS:

$$\sin^2 \pi/18 + \sin^2 \pi/9 + \sin^2 7\pi/18 + \sin^2 4\pi/9$$

$$\sin^2 \pi/18 + \sin^2 2\pi/18 + \sin^2 7\pi/18 + \sin^2 8\pi/18$$

$$\sin^2 \pi/18 + \sin^2 2\pi/18 + \sin^2 (\pi/2 - 2\pi/18) + \sin^2 (\pi/2 - \pi/18)$$

We know that when n is odd, $\sin \rightarrow \cos$.

$$\sin^2 \pi/18 + \sin^2 2\pi/18 + \cos^2 2\pi/18 + \cos^2 \pi/18$$

when rearranged,

$$\sin^2 \pi/18 + \cos^2 2\pi/18 + \sin^2 \pi/18 + \cos^2 2\pi/18$$

We know that $\sin^2 + \cos^2 x = 1$.

So,

$$1 + 1$$

$$2 = \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

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