

RD Sharma Solutions for Class 11 Maths Chapter 3 Functions

1. Define a function as a set of ordered pairs.

Solution:

Let A and B be two non-empty sets. A relation from A to B, i.e., a subset of $A \times B$, is called a function (or a mapping) from A to B, if

- (i) for each $a \in A$ there exists $b \in B$ such that $(a, b) \in f$
- (ii) $(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$

2. Define a function as a correspondence between two sets.

Solution:

Let A and B be two non-empty sets. Then a function 'f' from set A to B is a rule or method or correspondence which associates elements of set A to elements of set B such that:

- (i) all elements of set A are associated to elements in set B.
- (ii) an element of set A is associated to a unique element in set B.

3. What is the fundamental difference between a relation and a function? Is every relation a function?

Solution:

Let 'f' be a function and R be a relation defined from set X to set Y.

The domain of the relation R might be a subset of the set X, but the domain of the function f must be equal to X. This is because each element of the domain of a function must have an element associated with it, whereas this is not necessary for a relation.

In relation, one element of X might be associated with one or more elements of Y, while it must be associated with only one element of Y in a function.

Thus, not every relation is a function. However, every function is necessarily a relation.

4. Let $A = \{-2, -1, 0, 1, 2\}$ and $f: A \rightarrow Z$ be a function defined by $f(x) = x^2 - 2x - 3$. Find:

- (i) range of f i.e. $f(A)$
- (ii) pre-images of 6, -3 and 5

Solution:

Given:

$$A = \{-2, -1, 0, 1, 2\}$$

$$f: A \rightarrow Z \text{ such that } f(x) = x^2 - 2x - 3$$

- (i) Range of f i.e. $f(A)$

A is the domain of the function f. Hence, range is the set of elements $f(x)$ for all $x \in A$.

Substituting $x = -2$ in $f(x)$, we get

$$f(-2) = (-2)^2 - 2(-2) - 3$$

$$= 4 + 4 - 3$$

$$= 5$$

Substituting $x = -1$ in $f(x)$, we get

$$f(-1) = (-1)^2 - 2(-1) - 3$$

$$= 1 + 2 - 3$$

$$= 0$$

Substituting $x = 0$ in $f(x)$, we get

$$f(0) = (0)^2 - 2(0) - 3$$

$$= 0 - 0 - 3$$

$$= -3$$

Substituting $x = 1$ in $f(x)$, we get

$$f(1) = 1^2 - 2(1) - 3$$

$$= 1 - 2 - 3$$

$$= -4$$

Substituting $x = 2$ in $f(x)$, we get

$$f(2) = 2^2 - 2(2) - 3$$

$$= 4 - 4 - 3$$

$$= -3$$

Thus, the range of f is $\{-4, -3, 0, 5\}$.

(ii) pre-images of 6, -3 and 5

Let x be the pre-image of 6 $\Rightarrow f(x) = 6$

$$x^2 - 2x - 3 = 6$$

$$x^2 - 2x - 9 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-9)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4+36}}{2}$$

$$= \frac{2 \pm \sqrt{40}}{2}$$

$$= 1 \pm \sqrt{10}$$

However, $1 \pm \sqrt{10} \notin A$

Thus, there exists no pre-image of 6.

Now, let x be the pre-image of $-3 \Rightarrow f(x) = -3$

$$x^2 - 2x - 3 = -3$$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$x = 0 \text{ or } 2$$

Clearly, both 0 and 2 are elements of A.

Thus, 0 and 2 are the pre-images of -3.

Now, let x be the pre-image of 5 $\Rightarrow f(x) = 5$

$$x^2 - 2x - 3 = 5$$

$$x^2 - 2x - 8 = 0$$

$$x^2 - 4x + 2x - 8 = 0$$

$$x(x - 4) + 2(x - 4) = 0$$

$$(x + 2)(x - 4) = 0$$

$$x = -2 \text{ or } 4$$

However, $4 \notin A$ but $-2 \in A$

Thus, -2 is the pre-images of 5.

$\therefore \emptyset, \{0, 2\}, -2$ are the pre-images of 6, -3, 5

5. If a function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 3x - 2, & x < 0 \\ 1, & x = 0 \\ 4x + 1, & x > 0 \end{cases}$$

Find: $f(1), f(-1), f(0), f(2)$.

Solution:

Given:

Let us find $f(1), f(-1), f(0)$ and $f(2)$.

When $x > 0, f(x) = 4x + 1$

Substituting $x = 1$ in the above equation, we get

$$f(1) = 4(1) + 1$$

$$= 4 + 1$$

$$= 5$$

When $x < 0, f(x) = 3x - 2$

Substituting $x = -1$ in the above equation, we get

$$f(-1) = 3(-1) - 2$$

$$= -3 - 2$$

$$= -5$$

When $x = 0, f(x) = 1$

Substituting $x = 0$ in the above equation, we get

$$f(0) = 1$$

$$\text{When } x > 0, f(x) = 4x + 1$$

Substituting $x = 2$ in the above equation, we get

$$f(2) = 4(2) + 1$$

$$= 8 + 1$$

$$= 9$$

$$\therefore f(1) = 5, f(-1) = -5, f(0) = 1 \text{ and } f(2) = 9.$$

6. A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2$. Determine

(i) range of f

(ii) $\{x: f(x) = 4\}$

(iii) $\{y: f(y) = -1\}$

Solution:

Given:

$$f: \mathbb{R} \rightarrow \mathbb{R} \text{ and } f(x) = x^2.$$

(i) range of f

Domain of $f = \mathbb{R}$ (set of real numbers)

We know that the square of a real number is always positive or equal to zero.

$$\therefore \text{range of } f = \mathbb{R}^+ \cup \{0\}$$

(ii) $\{x: f(x) = 4\}$

Given:

$$f(x) = 4$$

$$\text{we know, } x^2 = 4$$

$$x^2 - 4 = 0$$

$$(x - 2)(x + 2) = 0$$

$$\therefore x = \pm 2$$

$$\therefore \{x: f(x) = 4\} = \{-2, 2\}$$

(iii) $\{y: f(y) = -1\}$

Given:

$$f(y) = -1$$

$$y^2 = -1$$

However, the domain of f is $\mathbb{R}$, and for every real number y , the value of y^2 is non-negative.

Hence, there exists no real y for which $y^2 = -1$.

$$\therefore \{y: f(y) = -1\} = \emptyset$$

7. Let $f: \mathbb{R}^+ \rightarrow \mathbb{R}$, where $\mathbb{R}^+$ is the set of all positive real numbers, be such that $f(x) = \log_e x$.

Determine

- (i) the image set of the domain of f**
- (ii) $\{x: f(x) = -2\}$**
- (iii) whether $f(xy) = f(x) + f(y)$ holds.**

Solution:

Given $f: \mathbb{R}^+ \rightarrow \mathbb{R}$ and $f(x) = \log_e x$.

(i) the image set of the domain of f

Domain of $f = \mathbb{R}^+$ (set of positive real numbers)

We know the value of logarithm to the base e (natural logarithm) can take all possible real values.

$\therefore$ The image set of $f = \mathbb{R}$

(ii) $\{x: f(x) = -2\}$

Given $f(x) = -2$

$$\log_e x = -2$$

$$\therefore x = e^{-2} \text{ [since, } \log_b a = c \Rightarrow a = b^c \text{]}$$

$$\therefore \{x: f(x) = -2\} = \{e^{-2}\}$$

(iii) Whether $f(xy) = f(x) + f(y)$ holds.

We have $f(x) = \log_e x \Rightarrow f(y) = \log_e y$

Now, let us consider $f(xy)$

$$f(xy) = \log_e(xy)$$

$$f(xy) = \log_e(x \times y) \text{ [since, } \log_b(a \times c) = \log_b a + \log_b c \text{]}$$

$$f(xy) = \log_e x + \log_e y$$

$$f(xy) = f(x) + f(y)$$

$\therefore$ the equation $f(xy) = f(x) + f(y)$ holds.

8. Write the following relations as sets of ordered pairs and find which of them are functions:

(i) $\{(x, y): y = 3x, x \in \{1, 2, 3\}, y \in \{3, 6, 9, 12\}\}$

(ii) $\{(x, y): y > x + 1, x = 1, 2 \text{ and } y = 2, 4, 6\}$

(iii) $\{(x, y): x + y = 3, x, y \in \{0, 1, 2, 3\}\}$

Solution:

(i) $\{(x, y): y = 3x, x \in \{1, 2, 3\}, y \in \{3, 6, 9, 12\}\}$

$$\text{When } x = 1, y = 3(1) = 3$$

$$\text{When } x = 2, y = 3(2) = 6$$

$$\text{When } x = 3, y = 3(3) = 9$$

$$\therefore R = \{(1, 3), (2, 6), (3, 9)\}$$

Hence, the given relation R is a function.

(ii) $\{(x, y): y > x + 1, x = 1, 2 \text{ and } y = 2, 4, 6\}$

When $x = 1$, $y > 1 + 1$ or $y > 2 \Rightarrow y = \{4, 6\}$

When $x = 2$, $y > 2 + 1$ or $y > 3 \Rightarrow y = \{4, 6\}$

$$\therefore R = \{(1, 4), (1, 6), (2, 4), (2, 6)\}$$

Hence, the given relation R is not a function.

(iii) $\{(x, y): x + y = 3, x, y \in \{0, 1, 2, 3\}\}$

When $x = 0$, $0 + y = 3 \Rightarrow y = 3$

When $x = 1$, $1 + y = 3 \Rightarrow y = 2$

When $x = 2$, $2 + y = 3 \Rightarrow y = 1$

When $x = 3$, $3 + y = 3 \Rightarrow y = 0$

$$\therefore R = \{(0, 3), (1, 2), (2, 1), (3, 0)\}$$

Hence, the given relation R is a function.

9. Let $f: \mathbf{R} \rightarrow \mathbf{R}$ and $g: \mathbf{C} \rightarrow \mathbf{C}$ be two functions defined as $f(x) = x^2$ and $g(x) = x^2$. Are they equal functions?

Solution:

Given:

$$f: \mathbf{R} \rightarrow \mathbf{R} \in f(x) = x^2 \text{ and } g: \mathbf{R} \rightarrow \mathbf{R} \in g(x) = x^2$$

f is defined from $\mathbf{R}$ to $\mathbf{R}$, the domain of $f = \mathbf{R}$.

g is defined from $\mathbf{C}$ to $\mathbf{C}$, the domain of $g = \mathbf{C}$.

Two functions are equal only when the domain and codomain of both the functions are equal.

In this case, the domain of $f \neq$ domain of g .

$\therefore f$ and g are not equal functions.

EXERCISE 3.2 PAGE NO: 3.11

1. If $f(x) = x^2 - 3x + 4$, then find the values of x satisfying the equation $f(x) = f(2x + 1)$.

Solution:

Given:

$$f(x) = x^2 - 3x + 4.$$

Let us find x satisfying $f(x) = f(2x + 1)$.

We have,

$$f(2x + 1) = (2x + 1)^2 - 3(2x + 1) + 4$$

$$= (2x)^2 + 2(2x)(1) + 1^2 - 6x - 3 + 4$$

$$= 4x^2 + 4x + 1 - 6x + 1$$

$$= 4x^2 - 2x + 2$$

$$\text{Now, } f(x) = f(2x + 1)$$

$$x^2 - 3x + 4 = 4x^2 - 2x + 2$$

$$4x^2 - 2x + 2 - x^2 + 3x - 4 = 0$$

$$3x^2 + x - 2 = 0$$

$$3x^2 + 3x - 2x - 2 = 0$$

$$3x(x + 1) - 2(x + 1) = 0$$

$$(x + 1)(3x - 2) = 0$$

$$x + 1 = 0 \text{ or } 3x - 2 = 0$$

$$x = -1 \text{ or } 3x = 2$$

$$x = -1 \text{ or } 2/3$$

$\therefore$ The values of x are -1 and $2/3$.

2. If $f(x) = (x - a)^2(x - b)^2$, find $f(a + b)$.

Solution:

Given:

$$F(x) = (x - a)^2(x - b)^2$$

Let us find $f(a + b)$.

We have,

$$f(a + b) = (a + b - a)^2(a + b - b)^2$$

$$f(a + b) = (b)^2(a)^2$$

$$\therefore f(a + b) = a^2b^2$$

3. If $y = f(x) = (ax - b) / (bx - a)$, show that $x = f(y)$.

Solution:

Given:

$$y = f(x) = (ax - b) / (bx - a) \Rightarrow f(y) = (ay - b) / (by - a)$$

Let us prove that $x = f(y)$.

We have,

$$y = (ax - b) / (bx - a)$$

By cross-multiplying,

$$y(bx - a) = ax - b$$

$$bxy - ay = ax - b$$

$$bxy - ax = ay - b$$

$$x(by - a) = ay - b$$

$$x = (ay - b) / (by - a) = f(y)$$

$$\therefore x = f(y)$$

Hence proved.

4. If $f(x) = 1 / (1 - x)$, show that $f[f\{f(x)\}] = x$.

Solution:

Given:

$$f(x) = 1 / (1 - x)$$

Let us prove that $f[f\{f(x)\}] = x$.

Firstly, let us solve for $f\{f(x)\}$.

$$f\{f(x)\} = f\{1/(1 - x)\}$$

$$= 1 / 1 - (1/(1 - x))$$

$$= 1 / [(1 - x - 1)/(1 - x)]$$

$$= 1 / (-x/(1 - x))$$

$$= (1 - x) / -x$$

$$= (x - 1) / x$$

$$\therefore f\{f(x)\} = (x - 1) / x$$

Now, we shall solve for $f[f\{f(x)\}]$

$$f[f\{f(x)\}] = f[(x-1)/x]$$

$$= 1 / [1 - (x-1)/x]$$

$$= 1 / [(x - (x-1))/x]$$

$$= 1 / [(x - x + 1)/x]$$

$$= 1 / (1/x)$$

$$\therefore f[f\{f(x)\}] = x$$

Hence proved.

5. If $f(x) = (x + 1) / (x - 1)$, show that $f[f(x)] = x$.

Solution:

Given:

$$f(x) = (x + 1) / (x - 1)$$

Let us prove that $f[f(x)] = x$.

$$f[f(x)] = f[(x+1)/(x-1)]$$

$$= [(x+1)/(x-1) + 1] / [(x+1)/(x-1) - 1]$$

$$= [(x+1) + (x-1)] / (x-1) / [(x+1) - (x-1)] / (x-1)$$

$$= [(x+1) + (x-1)] / [(x+1) - (x-1)]$$

$$= (x+1+x-1)/(x+1-x+1)$$

$$= 2x/2$$

$$= x$$

$$\therefore f[f(x)] = x$$

Hence proved.

6. If

$$f(x) = \begin{cases} x^2, & \text{when } x < 0 \\ x, & \text{when } 0 \leq x < 1 \\ \frac{1}{x}, & \text{when } x \geq 1 \end{cases}$$

Find:

(i) $f(1/2)$

(ii) $f(-2)$

(iii) $f(1)$

(iv) $f(\sqrt{3})$

(v) $f(\sqrt{-3})$

Solution:

(i) $f(1/2)$

When, $0 \leq x \leq 1$, $f(x) = x$

$$\therefore f(1/2) = 1/2$$

(ii) $f(-2)$

When, $x < 0$, $f(x) = x^2$

$$f(-2) = (-2)^2$$

$$= 4$$

$$\therefore f(-2) = 4$$

(iii) $f(1)$

When, $x \geq 1$, $f(x) = 1/x$

$$f(1) = 1/1$$

$$\therefore f(1) = 1$$

(iv) $f(\sqrt{3})$

We have $\sqrt{3} = 1.732 > 1$

When, $x \geq 1$, $f(x) = 1/x$

$$\therefore f(\sqrt{3}) = 1/\sqrt{3}$$

(v) $f(\sqrt{-3})$

We know $\sqrt{-3}$ is not a real number and the function $f(x)$ is defined only when $x \in \mathbb{R}$.

$\therefore f(\sqrt{-3})$ does not exist.

EXERCISE 3.3 PAGE NO: 3.18

1. Find the domain of each of the following real valued functions of real variable:

(i) $f(x) = 1/x$

(ii) $f(x) = 1/(x-7)$

(iii) $f(x) = (3x-2)/(x+1)$

(iv) $f(x) = (2x+1)/(x^2-9)$

(v) $f(x) = (x^2+2x+1)/(x^2-8x+12)$

Solution:

(i) $f(x) = 1/x$

We know, $f(x)$ is defined for all real values of x , except for the case when $x = 0$.

$\therefore$ Domain of $f = \mathbb{R} - \{0\}$

(ii) $f(x) = 1/(x-7)$

We know, $f(x)$ is defined for all real values of x , except for the case when $x - 7 = 0$ or $x = 7$.

$\therefore$ Domain of $f = \mathbb{R} - \{7\}$

(iii) $f(x) = (3x-2)/(x+1)$

We know, $f(x)$ is defined for all real values of x , except for the case when $x + 1 = 0$ or $x = -1$.

$\therefore$ Domain of $f = \mathbb{R} - \{-1\}$

(iv) $f(x) = (2x+1)/(x^2-9)$

We know, $f(x)$ is defined for all real values of x , except for the case when $x^2 - 9 = 0$.

$$x^2 - 9 = 0$$

$$x^2 - 3^2 = 0$$

$$(x + 3)(x - 3) = 0$$

$$x + 3 = 0 \text{ or } x - 3 = 0$$

$$x = \pm 3$$

$\therefore$ Domain of $f = \mathbb{R} - \{-3, 3\}$

(v) $f(x) = (x^2+2x+1)/(x^2-8x+12)$

We know, $f(x)$ is defined for all real values of x , except for the case when $x^2 - 8x + 12 = 0$.

$$x^2 - 8x + 12 = 0$$

$$x^2 - 2x - 6x + 12 = 0$$

$$x(x - 2) - 6(x - 2) = 0$$

$$(x - 2)(x - 6) = 0$$

$$x - 2 = 0 \text{ or } x - 6 = 0$$

$$x = 2 \text{ or } 6$$

$$\therefore \text{Domain of } f = \mathbb{R} - \{2, 6\}$$

2. Find the domain of each of the following real valued functions of real variable:

(i) $f(x) = \sqrt{x-2}$

(ii) $f(x) = 1/(\sqrt{x^2-1})$

(iii) $f(x) = \sqrt{9-x^2}$

(iv) $f(x) = \sqrt{(x-2)/(3-x)}$

Solution:

(i) $f(x) = \sqrt{x-2}$

We know the square of a real number is never negative.

$f(x)$ takes real values only when $x - 2 \geq 0$

$$x \geq 2$$

$$\therefore x \in [2, \infty)$$

$$\therefore \text{Domain } (f) = [2, \infty)$$

(ii) $f(x) = 1/(\sqrt{x^2-1})$

We know the square of a real number is never negative.

$f(x)$ takes real values only when $x^2 - 1 \geq 0$

$$x^2 - 1^2 \geq 0$$

$$(x + 1)(x - 1) \geq 0$$

$$x \leq -1 \text{ or } x \geq 1$$

$$\therefore x \in (-\infty, -1] \cup [1, \infty)$$

In addition, $f(x)$ is also undefined when $x^2 - 1 = 0$ because denominator will be zero and the result will be indeterminate.

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$\text{So, } x \in (-\infty, -1] \cup [1, \infty) - \{-1, 1\}$$

$$x \in (-\infty, -1) \cup (1, \infty)$$

$$\therefore \text{Domain } (f) = (-\infty, -1) \cup (1, \infty)$$

(iii) $f(x) = \sqrt{9-x^2}$

We know the square of a real number is never negative.

$f(x)$ takes real values only when $9 - x^2 \geq 0$

$$9 \geq x^2$$

$$x^2 \leq 9$$

$$x^2 - 9 \leq 0$$

$$x^2 - 3^2 \leq 0$$

$$(x + 3)(x - 3) \leq 0$$

$$x \geq -3 \text{ and } x \leq 3$$

$$x \in [-3, 3]$$

$$\therefore \text{Domain } (f) = [-3, 3]$$

$$\text{(iv) } f(x) = \sqrt{(x-2)/(3-x)}$$

We know the square root of a real number is never negative.

$f(x)$ takes real values only when $x - 2$ and $3 - x$ are both positive and negative.

(a) Both $x - 2$ and $3 - x$ are positive

$$x - 2 \geq 0$$

$$x \geq 2$$

$$3 - x \geq 0$$

$$x \leq 3$$

Hence, $x \geq 2$ and $x \leq 3$

$$\therefore x \in [2, 3]$$

(b) Both $x - 2$ and $3 - x$ are negative

$$x - 2 \leq 0$$

$$x \leq 2$$

$$3 - x \leq 0$$

$$x \geq 3$$

Hence, $x \leq 2$ and $x \geq 3$

However, the intersection of these sets is null set. Thus, this case is not possible.

$$\text{Hence, } x \in [2, 3] - \{3\}$$

$$x \in [2, 3]$$

$$\therefore \text{Domain } (f) = [2, 3]$$

3. Find the domain and range of each of the following real valued functions:

(i) $f(x) = (ax+b)/(bx-a)$

(ii) $f(x) = (ax-b)/(cx-d)$

(iii) $f(x) = \sqrt{x-1}$

$$(iv) f(x) = \sqrt{x-3}$$

$$(v) f(x) = (x-2)/(2-x)$$

$$(vi) f(x) = |x-1|$$

$$(vii) f(x) = -|x|$$

$$(viii) f(x) = \sqrt{9-x^2}$$

Solution:

$$(i) f(x) = (ax+b)/(bx-a)$$

$f(x)$ is defined for all real values of x , except for the case when $bx - a = 0$ or $x = a/b$.

$$\text{Domain } (f) = \mathbb{R} - (a/b)$$

$$\text{Let } f(x) = y$$

$$(ax+b)/(bx-a) = y$$

$$ax + b = y(bx - a)$$

$$ax + b = bxy - ay$$

$$ax - bxy = -ay - b$$

$$x(a - by) = -(ay + b)$$

$$\therefore x = -(ay+b)/(a-by)$$

$$\text{When } a - by = 0 \text{ or } y = a/b$$

Hence, $f(x)$ cannot take the value a/b .

$$\therefore \text{Range } (f) = \mathbb{R} - (a/b)$$

$$(ii) f(x) = (ax-b)/(cx-d)$$

$f(x)$ is defined for all real values of x , except for the case when $cx - d = 0$ or $x = d/c$. $\text{Domain } (f) = \mathbb{R} - (d/c)$

$$\text{Let } f(x) = y$$

$$(ax-b)/(cx-d) = y$$

$$ax - b = y(cx - d)$$

$$ax - b = cxy - dy$$

$$ax - cxy = b - dy$$

$$x(a - cy) = b - dy$$

$$\therefore x = (b-dy)/(a-cy)$$

$$\text{When } a - cy = 0 \text{ or } y = a/c,$$

Hence, $f(x)$ cannot take the value a/c .

$$\therefore \text{Range } (f) = \mathbb{R} - (a/c)$$

$$(iii) f(x) = \sqrt{x-1}$$

We know the square of a real number is never negative.

$f(x)$ takes real values only when $x - 1 \geq 0$

$$x \geq 1$$

$$\therefore x \in [1, \infty)$$

Thus, domain $(f) = [1, \infty)$

When $x \geq 1$, we have $x - 1 \geq 0$

Hence, $\sqrt{x-1} \geq 0 \Rightarrow f(x) \geq 0$

$$f(x) \in [0, \infty)$$

$$\therefore \text{Range } (f) = [0, \infty)$$

$$\text{(iv) } f(x) = \sqrt{x-3}$$

We know the square of a real number is never negative.

$f(x)$ takes real values only when $x - 3 \geq 0$

$$x \geq 3$$

$$\therefore x \in [3, \infty)$$

Domain $(f) = [3, \infty)$

When $x \geq 3$, we have $x - 3 \geq 0$

Hence, $\sqrt{x-3} \geq 0 \Rightarrow f(x) \geq 0$

$$f(x) \in [0, \infty)$$

$$\therefore \text{Range } (f) = [0, \infty)$$

$$\text{(v) } f(x) = (x-2)/(2-x)$$

$f(x)$ is defined for all real values of x , except for the case when $2 - x = 0$ or $x = 2$.

$$\text{Domain } (f) = \mathbb{R} - \{2\}$$

We have, $f(x) = (x-2)/(2-x)$

$$f(x) = -(2-x)/(2-x)$$

$$= -1$$

When $x \neq 2$, $f(x) = -1$

$$\therefore \text{Range } (f) = \{-1\}$$

$$\text{(vi) } f(x) = |x-1|$$

$$\text{we know } |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Now we have,

$$|x-1| = \begin{cases} -(x-1), & x-1 < 0 \\ x-1, & x-1 \geq 0 \end{cases}$$

$$\therefore f(x) = |x - 1| = \begin{cases} 1 - x, & x < 1 \\ x - 1, & x \geq 1 \end{cases}$$

Hence, $f(x)$ is defined for all real numbers x .

Domain $(f) = \mathbb{R}$

When, $x < 1$, we have $x - 1 < 0$ or $1 - x > 0$.

$$|x - 1| > 0 \Rightarrow f(x) > 0$$

When, $x \geq 1$, we have $x - 1 \geq 0$.

$$|x - 1| \geq 0 \Rightarrow f(x) \geq 0$$

$$\therefore f(x) \geq 0 \text{ or } f(x) \in [0, \infty)$$

$$\text{Range } (f) = [0, \infty)$$

$$\text{(vii) } f(x) = -|x|$$

$$\text{we know } |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Now we have,

$$-|x| = \begin{cases} -(-x), & x < 0 \\ -x, & x \geq 0 \end{cases}$$

$$\therefore f(x) = -|x| = \begin{cases} x, & x < 0 \\ -x, & x \geq 0 \end{cases}$$

Hence, $f(x)$ is defined for all real numbers x .

Domain $(f) = \mathbb{R}$

When, $x < 0$, we have $-|x| < 0$

$$f(x) < 0$$

When, $x \geq 0$, we have $-x \leq 0$.

$$-|x| \leq 0 \Rightarrow f(x) \leq 0$$

$$\therefore f(x) \leq 0 \text{ or } f(x) \in (-\infty, 0]$$

$$\text{Range } (f) = (-\infty, 0]$$

$$\text{(viii) } f(x) = \sqrt{9-x^2}$$

We know the square of a real number is never negative.

$f(x)$ takes real values only when $9 - x^2 \geq 0$

$$9 \geq x^2$$

$$x^2 \leq 9$$

$$x^2 - 9 \leq 0$$

$$x^2 - 3^2 \leq 0$$

$$(x + 3)(x - 3) \leq 0$$

$$x \geq -3 \text{ and } x \leq 3$$

$$\therefore x \in [-3, 3]$$

$$\text{Domain } (f) = [-3, 3]$$

$$\text{When, } x \in [-3, 3], \text{ we have } 0 \leq 9 - x^2 \leq 9$$

$$0 \leq \sqrt{9-x^2} \leq 3 \Rightarrow 0 \leq f(x) \leq 3$$

$$\therefore f(x) \in [0, 3]$$

$$\text{Range } (f) = [0, 3]$$

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1. Find $f + g$, $f - g$, cf ($c \in \mathbb{R}$, $c \neq 0$), fg , $1/f$ and f/g in each of the following:

(i) $f(x) = x^3 + 1$ and $g(x) = x + 1$

(ii) $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{x+1}$

Solution:

(i) $f(x) = x^3 + 1$ and $g(x) = x + 1$

We have $f(x): \mathbb{R} \rightarrow \mathbb{R}$ and $g(x): \mathbb{R} \rightarrow \mathbb{R}$

(a) $f + g$

We know, $(f + g)(x) = f(x) + g(x)$

$$(f + g)(x) = x^3 + 1 + x + 1$$

$$= x^3 + x + 2$$

So, $(f + g)(x): \mathbb{R} \rightarrow \mathbb{R}$

$\therefore f + g: \mathbb{R} \rightarrow \mathbb{R}$ is given by $(f + g)(x) = x^3 + x + 2$

(b) $f - g$

We know, $(f - g)(x) = f(x) - g(x)$

$$(f - g)(x) = x^3 + 1 - (x + 1)$$

$$= x^3 + 1 - x - 1$$

$$= x^3 - x$$

So, $(f - g)(x): \mathbb{R} \rightarrow \mathbb{R}$

$\therefore f - g: \mathbb{R} \rightarrow \mathbb{R}$ is given by $(f - g)(x) = x^3 - x$

(c) cf ($c \in \mathbb{R}$, $c \neq 0$)

We know, $(cf)(x) = c \times f(x)$

$$(cf)(x) = c(x^3 + 1)$$

$$= cx^3 + c$$

So, $(cf)(x) : \mathbb{R} \rightarrow \mathbb{R}$

$\therefore cf: \mathbb{R} \rightarrow \mathbb{R}$ is given by $(cf)(x) = cx^3 + c$

(d) fg

We know, $(fg)(x) = f(x)g(x)$

$$(fg)(x) = (x^3 + 1)(x + 1)$$

$$= (x + 1)(x^2 - x + 1)(x + 1)$$

$$= (x + 1)^2(x^2 - x + 1)$$

So, $(fg)(x) : \mathbb{R} \rightarrow \mathbb{R}$

$\therefore fg: \mathbb{R} \rightarrow \mathbb{R}$ is given by $(fg)(x) = (x + 1)^2(x^2 - x + 1)$

(e) $1/f$

We know, $(1/f)(x) = 1/f(x)$

$$1/f(x) = 1 / (x^3 + 1)$$

Observe that $1/f(x)$ is undefined when $f(x) = 0$ or when $x = -1$.

So, $1/f: \mathbb{R} - \{-1\} \rightarrow \mathbb{R}$ is given by $1/f(x) = 1 / (x^3 + 1)$

(f) f/g

We know, $(f/g)(x) = f(x)/g(x)$

$$(f/g)(x) = (x^3 + 1) / (x + 1)$$

Observe that $(x^3 + 1) / (x + 1)$ is undefined when $g(x) = 0$ or when $x = -1$.

Using $x^3 + 1 = (x + 1)(x^2 - x + 1)$, we have

$$(f/g)(x) = [(x+1)(x^2 - x + 1)/(x+1)]$$

$$= x^2 - x + 1$$

$\therefore f/g: \mathbb{R} - \{-1\} \rightarrow \mathbb{R}$ is given by $(f/g)(x) = x^2 - x + 1$

(ii) $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{x+1}$

We have $f(x): [1, \infty) \rightarrow \mathbb{R}^+$ and $g(x): [-1, \infty) \rightarrow \mathbb{R}^+$ as real square root is defined only for non-negative numbers.

(a) $f + g$

We know, $(f + g)(x) = f(x) + g(x)$

$$(f+g)(x) = \sqrt{x-1} + \sqrt{x+1}$$

Domain of $(f + g) = \text{Domain of } f \cap \text{Domain of } g$

$$\text{Domain of } (f + g) = [1, \infty) \cap [-1, \infty)$$

Domain of $(f + g) = [1, \infty)$

$\therefore f + g: [1, \infty) \rightarrow \mathbb{R}$ is given by $(f+g)(x) = \sqrt{x-1} + \sqrt{x+1}$

(b) $f - g$

We know, $(f - g)(x) = f(x) - g(x)$

$$(f-g)(x) = \sqrt{x-1} - \sqrt{x+1}$$

Domain of $(f - g) = \text{Domain of } f \cap \text{Domain of } g$

$$\text{Domain of } (f - g) = [1, \infty) \cap [-1, \infty)$$

$$\text{Domain of } (f - g) = [1, \infty)$$

$\therefore f - g: [1, \infty) \rightarrow \mathbb{R}$ is given by $(f-g)(x) = \sqrt{x-1} - \sqrt{x+1}$

(c) cf ($c \in \mathbb{R}, c \neq 0$)

We know, $(cf)(x) = c \times f(x)$

$$(cf)(x) = c\sqrt{x-1}$$

Domain of $(cf) = \text{Domain of } f$

$$\text{Domain of } (cf) = [1, \infty)$$

$\therefore cf: [1, \infty) \rightarrow \mathbb{R}$ is given by $(cf)(x) = c\sqrt{x-1}$

(d) fg

We know, $(fg)(x) = f(x)g(x)$

$$(fg)(x) = \sqrt{x-1} \sqrt{x+1}$$

$$= \sqrt{x^2 - 1}$$

Domain of $(fg) = \text{Domain of } f \cap \text{Domain of } g$

$$\text{Domain of } (fg) = [1, \infty) \cap [-1, \infty)$$

$$\text{Domain of } (fg) = [1, \infty)$$

$\therefore fg: [1, \infty) \rightarrow \mathbb{R}$ is given by $(fg)(x) = \sqrt{x^2 - 1}$

(e) $1/f$

We know, $(1/f)(x) = 1/f(x)$

$$(1/f)(x) = 1/\sqrt{x-1}$$

Domain of $(1/f) = \text{Domain of } f$

$$\text{Domain of } (1/f) = [1, \infty)$$

Observe that $1/\sqrt{x-1}$ is also undefined when $x - 1 = 0$ or $x = 1$.

$\therefore 1/f: (1, \infty) \rightarrow \mathbb{R}$ is given by $(1/f)(x) = 1/\sqrt{x-1}$

(f) f/g

We know, $(f/g)(x) = f(x)/g(x)$

$$(f/g)(x) = \sqrt{x-1}/\sqrt{x+1}$$

$$(f/g)(x) = \sqrt{[(x-1)/(x+1)]}$$

Domain of $(f/g) = \text{Domain of } f \cap \text{Domain of } g$

$$\text{Domain of } (f/g) = [1, \infty) \cap [-1, \infty)$$

$$\text{Domain of } (f/g) = [1, \infty)$$

$\therefore f/g: [1, \infty) \rightarrow \mathbb{R}$ is given by $(f/g)(x) = \sqrt{[(x-1)/(x+1)]}$

2. Let $f(x) = 2x + 5$ and $g(x) = x^2 + x$. Describe

(i) $f + g$

(ii) $f - g$

(iii) fg

(iv) f/g

Find the domain in each case.

Solution:

Given:

$$f(x) = 2x + 5 \text{ and } g(x) = x^2 + x$$

Both $f(x)$ and $g(x)$ are defined for all $x \in \mathbb{R}$.

So, domain of $f = \text{domain of } g = \mathbb{R}$

(i) $f + g$

We know, $(f + g)(x) = f(x) + g(x)$

$$(f + g)(x) = 2x + 5 + x^2 + x$$

$$= x^2 + 3x + 5$$

$(f + g)(x)$ is defined for all real numbers x .

$\therefore$ The domain of $(f + g)$ is $\mathbb{R}$

(ii) $f - g$

We know, $(f - g)(x) = f(x) - g(x)$

$$(f - g)(x) = 2x + 5 - (x^2 + x)$$

$$= 2x + 5 - x^2 - x$$

$$= 5 + x - x^2$$

$(f - g)(x)$ is defined for all real numbers x .

$\therefore$ The domain of $(f - g)$ is $\mathbb{R}$

(iii) fg

We know, $(fg)(x) = f(x)g(x)$

$$(fg)(x) = (2x + 5)(x^2 + x)$$

$$= 2x(x^2 + x) + 5(x^2 + x)$$

$$= 2x^3 + 2x^2 + 5x^2 + 5x$$

$$= 2x^3 + 7x^2 + 5x$$

$(fg)(x)$ is defined for all real numbers x .

$\therefore$ The domain of fg is $\mathbb{R}$

(iv) f/g

We know, $(f/g)(x) = f(x)/g(x)$

$$(f/g)(x) = (2x+5)/(x^2+x)$$

$(f/g)(x)$ is defined for all real values of x , except for the case when $x^2 + x = 0$.

$$x^2 + x = 0$$

$$x(x + 1) = 0$$

$$x = 0 \text{ or } x + 1 = 0$$

$$x = 0 \text{ or } -1$$

When $x = 0$ or -1 , $(f/g)(x)$ will be undefined as the division result will be indeterminate.

$\therefore$ The domain of $f/g = \mathbb{R} - \{-1, 0\}$

3. If $f(x)$ be defined on $[-2, 2]$ and is given by

$$f(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x - 1, & 0 < x \leq 2 \end{cases} \text{ and } g(x) = f(|x|) + |f(x)|. \text{ Find } g(x).$$

Solution:

Given:

$$f(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x-1, & 0 < x \leq 2 \end{cases} \text{ and}$$

$$g(x) = f(|x|) + |f(x)|$$

Now we have,

$$f(|x|) = \begin{cases} -1, & -2 \leq |x| \leq 0 \\ |x|-1, & 0 < |x| \leq 2 \end{cases}$$

However, $|x| \geq 0 \Rightarrow f(|x|) = |x| - 1$ when $0 < |x| \leq 2$

We also have,

$$\begin{aligned} |f(x)| &= \begin{cases} |-1|, & -2 \leq x \leq 0 \\ |x-1|, & 0 < x \leq 2 \end{cases} \\ &= \begin{cases} 1, & -2 \leq x \leq 0 \\ |x-1|, & 0 < x \leq 2 \end{cases} \end{aligned}$$

We also know,

$$\begin{aligned} |x-1| &= \begin{cases} -(x-1), & x-1 < 0 \\ x-1, & x-1 \geq 0 \end{cases} \\ &= \begin{cases} -(x-1), & x < 1 \\ x-1, & x \geq 1 \end{cases} \end{aligned}$$

Here, we shall only the range between $[0, 2]$.

$$|x-1| = \begin{cases} -(x-1), & 0 < x < 1 \\ x-1, & 1 \leq x \leq 2 \end{cases}$$

Substituting this value of $|x-1|$ in $|f(x)|$, we get

$$\begin{aligned} |f(x)| &= \begin{cases} 1, & -2 \leq x \leq 0 \\ -(x-1), & 0 < x < 1 \\ x-1, & 1 \leq x \leq 2 \end{cases} \\ &= \begin{cases} 1, & -2 \leq x \leq 0 \\ 1-x, & 0 < x < 1 \\ x-1, & 1 \leq x \leq 2 \end{cases} \end{aligned}$$

Now, we need to find $g(x)$

$$\begin{aligned} g(x) &= f(|x|) + |f(x)| \\ &= |x| - 1 \text{ when } 0 < |x| \leq 2 + \begin{cases} 1, & -2 \leq x \leq 0 \\ 1-x, & 0 < x < 1 \\ x-1, & 1 \leq x \leq 2 \end{cases} \end{aligned}$$

$$\begin{aligned}
 g(x) &= \begin{cases} -x-1, -2 \leq x \leq 0 \\ x-1, 0 < x < 1 \\ x-1, 1 \leq x \leq 2 \end{cases} + \begin{cases} 1, -2 \leq x \leq 0 \\ 1-x, 0 < x < 1 \\ x-1, 1 \leq x \leq 2 \end{cases} \\
 &= \begin{cases} -x-1+1, -2 \leq x \leq 0 \\ x-1+1-x, 0 < x < 1 \\ x-1+x-1, 1 \leq x \leq 2 \end{cases} \\
 &= \begin{cases} -x, -2 \leq x \leq 0 \\ 0, 0 < x < 1 \\ 2(x-1), 1 \leq x \leq 2 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \underline{g(x)} &= f(|x|) + |f(x)| \\
 &= \begin{cases} -x, -2 \leq x \leq 0 \\ 0, 0 < x < 1 \\ 2(x-1), 1 \leq x \leq 2 \end{cases}
 \end{aligned}$$

4. Let f, g be two real functions defined by $f(x) = \sqrt{x+1}$ and $g(x) = \sqrt{9-x^2}$. Then, describe each of the following functions.

- (i) $f + g$
- (ii) $g - f$
- (iii) fg
- (iv) f/g
- (v) g/f
- (vi) $2f - \sqrt{5}g$
- (vii) $f^2 + 7f$
- (viii) $5/g$

Solution:

Given:

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We know the square of a real number is never negative.

So, $f(x)$ takes real values only when $x + 1 \geq 0$

$$x \geq -1, x \in [-1, \infty)$$

Domain of $f = [-1, \infty)$

Similarly, $g(x)$ takes real values only when $9 - x^2 \geq 0$

$$9 \geq x^2$$

$$x^2 \leq 9$$

$$x^2 - 9 \leq 0$$

$$x^2 - 3^2 \leq 0$$

$$(x + 3)(x - 3) \leq 0$$

$$x \geq -3 \text{ and } x \leq 3$$

$$\therefore x \in [-3, 3]$$

$$\text{Domain of } g = [-3, 3]$$

(i) $f + g$

$$\text{We know, } (f + g)(x) = f(x) + g(x)$$

$$(f + g)(x) = \sqrt{x+1} + \sqrt{9-x^2}$$

$$\text{Domain of } f + g = \text{Domain of } f \cap \text{Domain of } g$$

$$= [-1, \infty) \cap [-3, 3]$$

$$= [-1, 3]$$

$$\therefore f + g: [-1, 3] \rightarrow \mathbb{R} \text{ is given by } (f + g)(x) = f(x) + g(x) = \sqrt{x+1} + \sqrt{9-x^2}$$

(ii) $g - f$

$$\text{We know, } (g - f)(x) = g(x) - f(x)$$

$$(g - f)(x) = \sqrt{9-x^2} - \sqrt{x+1}$$

$$\text{Domain of } g - f = \text{Domain of } g \cap \text{Domain of } f$$

$$= [-3, 3] \cap [-1, \infty)$$

$$= [-1, 3]$$

$$\therefore g - f: [-1, 3] \rightarrow \mathbb{R} \text{ is given by } (g - f)(x) = g(x) - f(x) = \sqrt{9-x^2} - \sqrt{x+1}$$

(iii) fg

$$\text{We know, } (fg)(x) = f(x)g(x)$$

$$(fg)(x) = \sqrt{x+1} \sqrt{9-x^2}$$

$$= \sqrt{[(x+1)(9-x^2)]}$$

$$= \sqrt{[x(9-x^2) + (9-x^2)]}$$

$$= \sqrt{(9x-x^3+9-x^2)}$$

$$= \sqrt{(9+9x-x^2-x^3)}$$

$$\text{Domain of } fg = \text{Domain of } f \cap \text{Domain of } g$$

$$= [-1, \infty) \cap [-3, 3]$$

$$= [-1, 3]$$

$$\therefore fg: [-1, 3] \rightarrow \mathbb{R} \text{ is given by } (fg)(x) = f(x)g(x) = \sqrt{x+1} \sqrt{9-x^2} = \sqrt{(9+9x-x^2-x^3)}$$

(iv) f/g

$$\text{We know, } (f/g)(x) = f(x)/g(x)$$

$$(f/g)(x) = \sqrt{x+1} / \sqrt{9-x^2}$$

$$= \sqrt{[(x+1) / (9-x^2)]}$$

$$\text{Domain of } f/g = \text{Domain of } f \cap \text{Domain of } g$$

$$= [-1, \infty) \cap [-3, 3]$$

$$= [-1, 3]$$

However, $(f/g)(x)$ is defined for all real values of $x \in [-1, 3]$, except for the case when $9 - x^2 = 0$ or $x = \pm 3$

When $x = \pm 3$, $(f/g)(x)$ will be undefined as the division result will be indeterminate.

$$\text{Domain of } f/g = [-1, 3] - \{-3, 3\}$$

$$\text{Domain of } f/g = [-1, 3)$$

$$\therefore f/g: [-1, 3) \rightarrow \mathbb{R} \text{ is given by } (f/g)(x) = f(x)/g(x) = \sqrt{x+1} / \sqrt{9-x^2}$$

(v) g/f

$$\text{We know, } (g/f)(x) = g(x)/f(x)$$

$$(g/f)(x) = \sqrt{9-x^2} / \sqrt{x+1}$$

$$= \sqrt{[(9-x^2) / (x+1)]}$$

$$\text{Domain of } g/f = \text{Domain of } f \cap \text{Domain of } g$$

$$= [-1, \infty) \cap [-3, 3]$$

$$= [-1, 3]$$

However, $(g/f)(x)$ is defined for all real values of $x \in [-1, 3]$, except for the case when $x + 1 = 0$ or $x = -1$

When $x = -1$, $(g/f)(x)$ will be undefined as the division result will be indeterminate.

$$\text{Domain of } g/f = [-1, 3] - \{-1\}$$

$$\text{Domain of } g/f = (-1, 3]$$

$$\therefore g/f: (-1, 3] \rightarrow \mathbb{R} \text{ is given by } (g/f)(x) = g(x)/f(x) = \sqrt{9-x^2} / \sqrt{x+1}$$

(vi) $2f - \sqrt{5}g$

$$\text{We know, } (2f - \sqrt{5}g)(x) = 2f(x) - \sqrt{5}g(x)$$

$$(2f - \sqrt{5}g)(x) = 2f(x) - \sqrt{5}g(x)$$

$$= 2\sqrt{x+1} - \sqrt{5}\sqrt{9-x^2}$$

$$= 2\sqrt{x+1} - \sqrt{45-5x^2}$$

$$\text{Domain of } 2f - \sqrt{5}g = \text{Domain of } f \cap \text{Domain of } g$$

$$= [-1, \infty) \cap [-3, 3]$$

$$= [-1, 3]$$

$$\therefore 2f - \sqrt{5}g: [-1, 3] \rightarrow \mathbb{R} \text{ is given by } (2f - \sqrt{5}g)(x) = 2f(x) - \sqrt{5}g(x) = 2\sqrt{x+1} - \sqrt{45-5x^2}$$

(vii) $f^2 + 7f$

$$\text{We know, } (f^2 + 7f)(x) = f^2(x) + (7f)(x)$$

$$(f^2 + 7f)(x) = f(x)f(x) + 7f(x)$$

$$= \sqrt{x+1} \sqrt{x+1} + 7\sqrt{x+1}$$

$$= x + 1 + 7\sqrt{x+1}$$

Domain of $f^2 + 7f$ is same as domain of f .

$$\text{Domain of } f^2 + 7f = [-1, \infty)$$

$$\therefore f^2 + 7f: [-1, \infty) \rightarrow \mathbb{R} \text{ is given by } (f^2 + 7f)(x) = f(x)f(x) + 7f(x) = x + 1 + 7\sqrt{x+1}$$

(viii) $5/g$

$$\text{We know, } (5/g)(x) = 5/g(x)$$

$$(5/g)(x) = 5/\sqrt{9-x^2}$$

$$\text{Domain of } 5/g = \text{Domain of } g = [-3, 3]$$

However, $(5/g)(x)$ is defined for all real values of $x \in [-3, 3]$, except for the case when $9 - x^2 = 0$ or $x = \pm 3$

When $x = \pm 3$, $(5/g)(x)$ will be undefined as the division result will be indeterminate.

$$\text{Domain of } 5/g = [-3, 3] - \{-3, 3\}$$

$$= (-3, 3)$$

$$\therefore 5/g: (-3, 3) \rightarrow \mathbb{R} \text{ is given by } (5/g)(x) = 5/g(x) = 5/\sqrt{9-x^2}$$

5. If $f(x) = \log_e(1 - x)$ and $g(x) = [x]$, then determine each of the following functions:

(i) $f + g$

(ii) fg

(iii) f/g

(iv) g/f

Also, find $(f + g)(-1)$, $(fg)(0)$, $(f/g)(1/2)$ and $(g/f)(1/2)$.

Solution:

Given:

$$f(x) = \log_e(1 - x) \text{ and } g(x) = [x]$$

We know, $f(x)$ takes real values only when $1 - x > 0$

$$1 > x$$

$$x < 1, \therefore x \in (-\infty, 1)$$

$$\text{Domain of } f = (-\infty, 1)$$

Similarly, $g(x)$ is defined for all real numbers x .

$$\text{Domain of } g = [x], x \in \mathbb{R}$$

$$= \mathbb{R}$$

(i) $f + g$

$$\text{We know, } (f + g)(x) = f(x) + g(x)$$

$$(f + g)(x) = \log_e(1 - x) + [x]$$

$$\text{Domain of } f + g = \text{Domain of } f \cap \text{Domain of } g$$

Domain of $f + g = (-\infty, 1) \cap \mathbb{R}$

$$= (-\infty, 1)$$

$\therefore f + g: (-\infty, 1) \rightarrow \mathbb{R}$ is given by $(f + g)(x) = \log_e(1 - x) + [x]$

(ii) fg

We know, $(fg)(x) = f(x)g(x)$

$$(fg)(x) = \log_e(1 - x) \times [x]$$

$$= [x] \log_e(1 - x)$$

Domain of $fg = \text{Domain of } f \cap \text{Domain of } g$

$$= (-\infty, 1) \cap \mathbb{R}$$

$$= (-\infty, 1)$$

$\therefore fg: (-\infty, 1) \rightarrow \mathbb{R}$ is given by $(fg)(x) = [x] \log_e(1 - x)$

(iii) f/g

We know, $(f/g)(x) = f(x)/g(x)$

$$(f/g)(x) = \log_e(1 - x) / [x]$$

Domain of $f/g = \text{Domain of } f \cap \text{Domain of } g$

$$= (-\infty, 1) \cap \mathbb{R}$$

$$= (-\infty, 1)$$

However, $(f/g)(x)$ is defined for all real values of $x \in (-\infty, 1)$, except for the case when $[x] = 0$.

We have, $[x] = 0$ when $0 \leq x < 1$ or $x \in [0, 1)$

When $0 \leq x < 1$, $(f/g)(x)$ will be undefined as the division result will be indeterminate.

Domain of $f/g = (-\infty, 1) - [0, 1)$

$$= (-\infty, 0)$$

$\therefore f/g: (-\infty, 0) \rightarrow \mathbb{R}$ is given by $(f/g)(x) = \log_e(1 - x) / [x]$

(iv) g/f

We know, $(g/f)(x) = g(x)/f(x)$

$$(g/f)(x) = [x] / \log_e(1 - x)$$

However, $(g/f)(x)$ is defined for all real values of $x \in (-\infty, 1)$, except for the case when $\log_e(1 - x) = 0$.

$$\log_e(1 - x) = 0 \Rightarrow 1 - x = 1 \text{ or } x = 0$$

When $x = 0$, $(g/f)(x)$ will be undefined as the division result will be indeterminate.

Domain of $g/f = (-\infty, 1) - \{0\}$

$$= (-\infty, 0) \cup (0, 1)$$

$\therefore g/f: (-\infty, 0) \cup (0, 1) \rightarrow \mathbb{R}$ is given by $(g/f)(x) = [x] / \log_e(1 - x)$

(a) We need to find $(f + g)(-1)$.

We have, $(f + g)(x) = \log_e(1 - x) + [x]$, $x \in (-\infty, 1)$

Substituting $x = -1$ in the above equation, we get

$$(f + g)(-1) = \log_e(1 - (-1)) + [-1]$$

$$= \log_e(1 + 1) + (-1)$$

$$= \log_e 2 - 1$$

$$\therefore (f + g)(-1) = \log_e 2 - 1$$

(b) We need to find $(fg)(0)$.

We have, $(fg)(x) = [x] \log_e(1 - x)$, $x \in (-\infty, 1)$

Substituting $x = 0$ in the above equation, we get

$$(fg)(0) = [0] \log_e(1 - 0)$$

$$= 0 \times \log_e 1$$

$$\therefore (fg)(0) = 0$$

(c) We need to find $(f/g)(1/2)$

We have, $(f/g)(x) = \log_e(1 - x) / [x]$, $x \in (-\infty, 0)$

However, $1/2$ is not in the domain of f/g .

$\therefore (f/g)(1/2)$ does not exist.

(d) We need to find $(g/f)(1/2)$

We have, $(g/f)(x) = [x] / \log_e(1 - x)$, $x \in (-\infty, 0) \cup (0, \infty)$

Substituting $x=1/2$ in the above equation, we get

$$(g/f)(1/2) = [x] / \log_e(1 - x)$$

$$= (1/2) / \log_e(1 - 1/2)$$

$$= 0.5 / \log_e(1/2)$$

$$= 0 / \log_e(1/2)$$

$$= 0$$

$$\therefore (g/f)(1/2) = 0$$