

RD Sharma Solutions for Class 11 Maths Chapter 29 Limits

1. Show that $\lim_{x \rightarrow 0} \frac{x}{|x|}$ does not exist.

Solution:

Firstly let us consider LHS:

$$\lim_{x \rightarrow 0^-} \left(\frac{x}{|x|} \right)$$

So, let $x = 0 - h$, where, $h = 0$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x}{|x|} &= \lim_{h \rightarrow 0} \left(\frac{0 - h}{|0 - h|} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{-h}{h} \right) \\ &= -1 \end{aligned}$$

Now, let us consider RHS:

$$\lim_{x \rightarrow 0^+} \left(\frac{x}{|x|} \right)$$

So, let $x = 0 + h$, where, $h = 0$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x}{|x|} &= \lim_{h \rightarrow 0} \left(\frac{0 + h}{|0 + h|} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{h}{h} \right) \\ &= 1 \end{aligned}$$

Since $LHS \neq RHS$

$\therefore$ Limit does not exist.

2. Find k so that $\lim_{x \rightarrow 2} f(x)$ may exist, where $f(x) = \begin{cases} 2x + 3, & x \leq 2 \\ x + k, & x > 2 \end{cases}$

Solution:

Firstly let us consider LHS:

$$\lim_{x \rightarrow 2^-} f(x)$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (2x + 3)$$

So, let $x = 2 - h$, where $h = 0$

Substituting the value of x , we get

$$\lim_{h \rightarrow 0} [2(2 - h) + 3]$$

$$\Rightarrow 2(2 - 0) + 3 = 7$$

Now let us consider RHS:

$$\lim_{x \rightarrow 2^+} f(x)$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (x + k)$$

So, let $x = 2 + h$, where, $h = 0$

$$\lim_{h \rightarrow 0} (2 + h + k)$$

$$\Rightarrow 2 + 0 + k = 2 + k$$

Since, Limit exists, LHS = RHS

$$7 = 2 + k$$

$$k = 7 - 2$$

$$= 5$$

$\therefore$ Value of k is 5.

3. Show that $\lim_{x \rightarrow 0} \frac{1}{x}$ does not exist.

Solution:

Firstly let us consider LHS:

$$\lim_{x \rightarrow 0^-} \left(\frac{1}{x} \right)$$

So, let $x = 0 - h$, where $h = 0$

$$\begin{aligned} \lim_{x \rightarrow 0^-} \left(\frac{1}{x} \right) &= \lim_{h \rightarrow 0} \left(\frac{1}{0 - h} \right) \\ &= -\infty \end{aligned}$$

Now, let us consider RHS:

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{x} \right)$$

So, let $x = 0 + h$, where $h = 0$

$$\begin{aligned} \lim_{x \rightarrow 0^+} \left(\frac{1}{x} \right) &= \lim_{h \rightarrow 0} \left(\frac{1}{0 + h} \right) \\ &= \infty \end{aligned}$$

Since, $LHS \neq RHS$

$\therefore$ Limit does not exist.

4. Let $f(x)$ be a function defined by $f(x) = \begin{cases} \frac{3x}{|x| + 2x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Show that $\lim_{x \rightarrow 0} f(x)$ does not exist.

Solution:

Firstly let us consider LHS:

$$\lim_{x \rightarrow 0^-} \left[\frac{3x}{|x| + 2x} \right]$$

So, let $x = 0 - h$, where $h = 0$

Substituting the value of x , we get

$$\begin{aligned} \lim_{x \rightarrow 0^-} \left[\frac{3x}{|x| + 2x} \right] &= \lim_{h \rightarrow 0} \left[\frac{3(-h)}{|-h| + 2(-h)} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{-3h}{h - 2h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{-3h}{-h} \right] \\ &= 3 \end{aligned}$$

Now, let us consider RHS:

$$\lim_{x \rightarrow 0^+} \left(\frac{3x}{|x| + 2x} \right)$$

So, let $x = 0 + h$, where $h = 0$

Substituting the value of x , we get

$$\begin{aligned} \lim_{x \rightarrow 0^+} \left(\frac{3x}{|x| + 2x} \right) &= \lim_{h \rightarrow 0} \left(\frac{3h}{|h| + 2h} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{3h}{h + 2h} \right) \\ &= 1 \end{aligned}$$

Since, $LHS \neq RHS$

$\therefore$ Limit does not exist.

5. Let $f(x) = \begin{cases} x + 1, & \text{if } x > 0 \\ x - 1, & \text{if } x < 0 \end{cases}$. Prove that $\lim_{x \rightarrow 0} f(x)$ does not exist.

Solution:

Firstly let us consider LHS:

$$\lim_{x \rightarrow 0^-} f(x)$$

So, let $x = 0 - h$, where $h = 0$

Substituting the value of x , we get

$$\begin{aligned}\lim_{x \rightarrow 0^-} f(x) &= \lim_{h \rightarrow 0} (0 - h - 1) \\ &= -1\end{aligned}$$

Now, let us consider RHS

$$\lim_{x \rightarrow 0^+} f(x)$$

So, let $x = 0 + h$, where $h = 0$

Substituting the value of x , we get

$$\begin{aligned}\lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0} (x + 1) \\ &= \lim_{h \rightarrow 0} (0 + h + 1) \\ &= 1\end{aligned}$$

Since, $LHS \neq RHS$

$\therefore$ Limit does not exist.

EXERCISE 29.2 PAGE NO: 29.18

Evaluate the following limits:

1. $\lim_{x \rightarrow 1} \frac{x^2 + 1}{x + 1}$

Solution:

Given:

$$\lim_{x \rightarrow 1} \frac{x^2 + 1}{x + 1}$$

Let us substitute the value of x directly in the given limit, we get

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{x^2 + 1}{x + 1} &= \frac{1^2 + 1}{1 + 1} \\ &= 2 / 2 \\ &= 1\end{aligned}$$

$\therefore$ The value of the given limit is 1.

$$2. \lim_{x \rightarrow 0} \frac{2x^2 + 3x + 4}{x^2 + 3x + 2}$$

Solution:

Given:

$$\lim_{x \rightarrow 0} \frac{2x^2 + 3x + 4}{x^2 + 3x + 2}$$

Let us substitute the value of x directly in the given limit, we get

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{2x^2 + 3x + 4}{x^2 + 3x + 2} &= \frac{2(0^2) + 3(0) + 4}{0^2 + 3(0) + 2} \\ &= 4 / 2 \\ &= 2 \end{aligned}$$

∴ The value of the given limit is 2.

$$3. \lim_{x \rightarrow 3} \frac{\sqrt{2x+3}}{x+3}$$

Solution:

Given:

$$\lim_{x \rightarrow 3} \frac{\sqrt{2x+3}}{x+3}$$

Let us substitute the value of x directly in the given limit, we get

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{\sqrt{2x+3}}{x+3} &= \frac{\sqrt{2(3)+3}}{3+3} \\ &= \sqrt{9} / 6 \\ &= 3 / 6 \\ &= 1 / 2 \end{aligned}$$

∴ The value of the given limit is 1/2.

$$4. \lim_{x \rightarrow 1} \frac{\sqrt{x+8}}{\sqrt{x}}$$

Solution:

Given:

$$\lim_{x \rightarrow 1} \frac{\sqrt{x+8}}{\sqrt{x}}$$

Let us substitute the value of x directly in the given limit, we get

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\sqrt{x+8}}{\sqrt{x}} &= \frac{\sqrt{1+8}}{1} \\ &= \frac{\sqrt{9}}{1} \\ &= 3 \end{aligned}$$

∴ The value of the given limit is 3.

$$5. \lim_{x \rightarrow a} \frac{\sqrt{x} + \sqrt{a}}{x + a}$$

Solution:

Given:

$$\lim_{x \rightarrow a} \frac{\sqrt{x} + \sqrt{a}}{x + a}$$

Let us substitute the value of x directly in the given limit, we get

$$\begin{aligned} \lim_{x \rightarrow a} \frac{\sqrt{x} + \sqrt{a}}{x + a} &= \frac{\sqrt{a} + \sqrt{a}}{a + a} \\ &= \frac{2\sqrt{a}}{2a} \\ &= \frac{1}{\sqrt{a}} \end{aligned}$$

∴ The value of the given limit is $1/\sqrt{a}$.

EXERCISE 29.3 PAGE NO: 29.23

Evaluate the following limits:

$$1. \lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x + 5}$$

Solution:

Given:
The limit $\lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x + 5}$

By substituting the value of x, we get

$$\begin{aligned} \lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x + 5} &= \frac{2(-5)^2 + 9(-5) - 5}{(-5) + 5} \\ &= \frac{50 - 50}{(-5) + 5} \\ &= \frac{0}{0} \text{ [Since, it is of the form indeterminate]} \end{aligned}$$

By using factorization method:

$$\begin{aligned} \lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x + 5} &= \lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x + 5} \\ &= \lim_{x \rightarrow -5} \frac{2x^2 + 10x - x - 5}{x + 5} \\ &= \lim_{x \rightarrow -5} \frac{2x(x + 5) - (x + 5)}{x + 5} \\ &= \lim_{x \rightarrow -5} \frac{(2x - 1)(x + 5)}{x + 5} \\ &= \lim_{x \rightarrow -5} 2x - 1 \\ &= 2(-5) - 1 \\ &= -11 \end{aligned}$$

∴ The value of the given limit is - 11.

$$2. \lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$$

Solution:

Given:
The limit $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$

By substituting the value of x, we get

$$\begin{aligned}\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} &= \frac{(3)^2 - 4(3) + 3}{(3)^2 - 2(3) - 3} \\&= \frac{12 - 12}{(-9) + 9} \\&= \frac{0}{0} \text{ [Since, it is of the form indeterminate]}\end{aligned}$$

By using factorization method:

$$\begin{aligned}\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} &= \lim_{x \rightarrow 3} \frac{(x^2 - 4x + 3)}{(x^2 - 2x - 3)} \\&= \lim_{x \rightarrow 3} \frac{(x^2 - 3x - x + 3)}{(x^2 - 3x + x - 3)} \\&= \lim_{x \rightarrow 3} \frac{x(x - 3) - 1(x - 3)}{x(x - 3) + 1(x - 3)} \\&= \lim_{x \rightarrow 3} \frac{(x - 3)(x - 1)}{(x - 3)(x + 1)} \\&= \lim_{x \rightarrow 3} \frac{(x - 1)}{(x + 1)} \\&= \frac{(3 - 1)}{(3 + 1)} \\&= 2 / 4 \\&= 1/2\end{aligned}$$

∴ The value of the given limit is $\frac{1}{2}$.

3. $\lim_{x \rightarrow 3} \frac{x^4 - 81}{x^2 - 9}$

Solution:

Given:

The limit $\lim_{x \rightarrow 3} \frac{x^4 - 81}{x^2 - 9}$

By substituting the value of x, we get

$$\begin{aligned}\lim_{x \rightarrow 3} \frac{x^4 - 81}{x^2 - 9} &= \frac{(3)^4 - 81}{(3)^2 - 9} \\&= \frac{81 - 81}{(-9) + 9} \\&= \frac{0}{0} \text{ [Since, it is of the form indeterminate]}\end{aligned}$$

By using factorization method:

$$\begin{aligned}\lim_{x \rightarrow 3} \frac{x^4 - 81}{x^2 - 9} &= \lim_{x \rightarrow 3} \frac{(x^4 - 81)}{(x^2 - 9)} \\&= \lim_{x \rightarrow 3} \frac{(x^4 - 3^4)}{(x^2 - 3^2)} \\&= \lim_{x \rightarrow 3} \frac{((x^2)^2 - (3^2)^2)}{(x^2 - 3^2)} \text{ [Since } a^2 - b^2 = (a + b)(a - b)\text{]}\end{aligned}$$

So,

$$\begin{aligned}&= \lim_{x \rightarrow 3} \frac{(x^2 - 3^2)(x^2 + 3^2)}{(x^2 - 3^2)} \\&= \lim_{x \rightarrow 3} (x^2 + 3^2) \\&= 3^2 + 3^2 \\&= 18\end{aligned}$$

∴ The value of the given limit is 18.

$$4. \lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$$

Solution:

Given:

$$\text{The limit } \lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$$

By substituting the value of x, we get

$$\begin{aligned}\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4} &= \frac{(2)^3 - 8}{(2)^2 - 4} \\ &= \frac{8 - 8}{(4) - 4} \\ &= \frac{0}{0} \text{ [Since, it is of the form indeterminate]}\end{aligned}$$

By using factorization method:

$$\begin{aligned}\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4} &= \lim_{x \rightarrow 2} \frac{(x^3 - 8)}{(x^2 - 4)} \\ &= \lim_{x \rightarrow 2} \frac{(x^3 - 2^3)}{(x^2 - 2^2)} \\ &= \lim_{x \rightarrow 2} \frac{(x - 2)(x^2 + 2^2 + 2x)}{(x + 2)(x - 2)}\end{aligned}$$

[By using the formula, $(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)$ & $(a^2 - b^2) = (a + b)(a - b)$]

$$\begin{aligned}&= \lim_{x \rightarrow 2} \frac{(x^2 + 2^2 + 2x)}{(x + 2)} \\ &= \frac{(2^2 + 2^2 + 2(2))}{(2 + 2)} \\ &= \frac{3.4}{(4)} \\ &= 3\end{aligned}$$

∴ The value of the given limit is 3.

$$\text{5. } \lim_{x \rightarrow -1/2} \frac{8x^3 + 1}{2x + 1}$$

Solution:

Given: $\lim_{x \rightarrow -1/2} \frac{8x^3 + 1}{2x + 1}$

The limit $\lim_{x \rightarrow -1/2} \frac{8x^3 + 1}{2x + 1}$

By substituting the value of x, we get

$$\begin{aligned}\lim_{x \rightarrow -1/2} \frac{8x^3 + 1}{2x + 1} &= \frac{8\left(-\frac{1}{2}\right)^3 + 1}{2\left(-\frac{1}{2}\right) + 1} \\ &= \frac{-1 + 1}{-1 + 1} \\ &= \frac{0}{0} \text{ [Since, it is of the form indeterminate]}\end{aligned}$$

By using factorization method:

$$\begin{aligned}\lim_{x \rightarrow -1/2} \frac{8x^3 + 1}{2x + 1} &= \lim_{x \rightarrow -\frac{1}{2}} \frac{8x^3 + 1}{2x + 1} \\ &= \lim_{x \rightarrow -\frac{1}{2}} \frac{(2x)^3 + (1)^3}{2x + 1}\end{aligned}$$

$$\begin{aligned}[\text{By using the formula, } a^3 + b^3 &= (a + b)(a^2 + b^2 - ab)] \\ &= \lim_{x \rightarrow -\frac{1}{2}} \frac{(2x + 1)((2x)^2 + (1)^2 - 2x)}{2x + 1} \\ &= \lim_{x \rightarrow -\frac{1}{2}} (2x)^2 + (1)^2 - 2x \\ &= (2(\frac{-1}{2}))^2 + (1)^2 - 2(\frac{-1}{2}) \\ &= 1 + 1 + 1 \\ &= 3\end{aligned}$$

∴ The value of the given limit is 3.

EXERCISE 29.4 PAGE NO: 29.28

Evaluate the following limits:

$$1. \lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x}$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x}$

The limit $\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x}$

We need to find the limit of the given equation when $x \Rightarrow 0$

Now let us substitute the value of x as 0, we get an indeterminate form of $0/0$.

Let us rationalizing the given equation, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x} = \lim_{x \rightarrow 0} \frac{(\sqrt{1+x+x^2} - 1)(\sqrt{1+x+x^2} + 1)}{x(\sqrt{1+x+x^2} + 1)}$$

[By using the formula: $(a+b)(a-b) = a^2 - b^2$]

$$= \lim_{x \rightarrow 0} \frac{1+x+x^2 - 1}{x(\sqrt{1+x+x^2} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{x(1+x)}{x(\sqrt{1+x+x^2} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{(1+x)}{(\sqrt{1+x+x^2} + 1)}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of x as 0, we get

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x} &= \frac{1}{1+1} \\ &= \frac{1}{2} \end{aligned}$$

$\therefore$ The value of the given limit is $\frac{1}{2}$.

$$2. \lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}}$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}}$

The limit $\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}}$

We need to find the limit of the given equation when $x \Rightarrow 0$

Now let us substitute the value of x as 0, we get an indeterminate form of $0/0$.

Let us rationalizing the given equation, we get

$$\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} = \lim_{x \rightarrow 0} \frac{2x}{(\sqrt{a+x} - \sqrt{a-x})} \frac{(\sqrt{a+x} + \sqrt{a-x})}{(\sqrt{a+x} + \sqrt{a-x})}$$

[By using the formula: $(a+b)(a-b) = a^2 - b^2$]

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{2x(\sqrt{a+x} + \sqrt{a-x})}{a+x - a+x} \\ &= \lim_{x \rightarrow 0} \frac{2x(\sqrt{a+x} + \sqrt{a-x})}{2x} \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{a+x} + \sqrt{a-x})}{1} \end{aligned}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of x as 0, we get

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} &= \sqrt{a} + \sqrt{a} \\ &= 2\sqrt{a} \end{aligned}$$

$\therefore$ The value of the given limit is $2\sqrt{a}$

$$3. \lim_{x \rightarrow 0} \frac{\sqrt{a^2 + x^2} - a}{x^2}$$

Solution:

$$\text{Given: } \lim_{x \rightarrow 0} \frac{\sqrt{a^2 + x^2} - a}{x^2}$$

The limit

We need to find the limit of the given equation when $x \Rightarrow 0$

Now let us substitute the value of x as 0, we get an indeterminate form of $0/0$.

Let us rationalizing the given equation, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{a^2 + x^2} - a}{x^2} = \lim_{x \rightarrow 0} \frac{(\sqrt{a^2 + x^2} - a)(\sqrt{a^2 + x^2} + a)}{x^2(\sqrt{a^2 + x^2} + a)}$$

[By using the formula: $(a+b)(a-b) = a^2 - b^2$]

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{(a^2 + x^2 - a^2)}{x^2(\sqrt{a^2 + x^2} + a)} \\ &= \lim_{x \rightarrow 0} \frac{x^2}{x^2(\sqrt{a^2 + x^2} + a)} \\ &= \lim_{x \rightarrow 0} \frac{1}{(\sqrt{a^2 + x^2} + a)} \end{aligned}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of x as 0, we get

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sqrt{a^2+x^2} - a}{x^2} &= \frac{1}{a+a} \\ &= \frac{1}{2a}\end{aligned}$$

∴ The value of the given limit is $\frac{1}{2a}$.

4. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x}$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x}$

The limit $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x}$

We need to find the limit of the given equation when $x \Rightarrow 0$

Now let us substitute the value of x as 0, we get an indeterminate form of 0/0.

Let us rationalizing the given equation, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x} = \lim_{x \rightarrow 0} \frac{(\sqrt{1+x} - \sqrt{1-x}) (\sqrt{1+x} + \sqrt{1-x})}{2x (\sqrt{1+x} + \sqrt{1-x})}$$

[By using the formula: $(a+b)(a-b) = a^2 - b^2$]

$$= \lim_{x \rightarrow 0} \frac{1+x - 1+x}{2x (\sqrt{1+x} + \sqrt{1-x})}$$

$$= \lim_{x \rightarrow 0} \frac{2x}{2x (\sqrt{1+x} + \sqrt{1-x})}$$

$$= \lim_{x \rightarrow 0} \frac{1}{(\sqrt{1+x} + \sqrt{1-x})}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of x as 0, we get

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x} &= \frac{1}{1+1} \\ &= \frac{1}{2}\end{aligned}$$

∴ The value of the given limit is $\frac{1}{2}$.

$$5. \lim_{x \rightarrow 2} \frac{\sqrt{3-x} - 1}{2-x}$$

Solution:

Given: $\lim_{x \rightarrow 2} \frac{\sqrt{3-x} - 1}{2-x}$

The limit $\lim_{x \rightarrow 2} \frac{\sqrt{3-x} - 1}{2-x}$

We need to find the limit of the given equation when $x \Rightarrow 0$

Now let us substitute the value of x as 0, we get an indeterminate form of $0/0$.

Let us rationalizing the given equation, we get

$$\lim_{x \rightarrow 2} \frac{\sqrt{3-x} - 1}{2-x} = \lim_{x \rightarrow 2} \frac{(\sqrt{3-x} - 1)(\sqrt{3-x} + 1)}{(2-x)(\sqrt{3-x} + 1)}$$

[By using the formula: $(a+b)(a-b) = a^2 - b^2$]

$$= \lim_{x \rightarrow 2} \frac{(3-x-1)}{(2-x)(\sqrt{3-x} + 1)}$$

$$= \lim_{x \rightarrow 2} \frac{(2-x)}{(2-x)(\sqrt{3-x} + 1)}$$

$$= \lim_{x \rightarrow 2} \frac{1}{(\sqrt{3-x} + 1)}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of x as 0, we get

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{\sqrt{3-x} - 1}{2-x} &= \frac{1}{1+1} \\ &= \frac{1}{2} \end{aligned}$$

$\therefore$ The value of the given limit is $\frac{1}{2}$.

EXERCISE 29.5 PAGE NO: 29.33

Evaluate the following limits:

$$1. \lim_{x \rightarrow a} \frac{(x+2)^{5/2} - (a+2)^{5/2}}{x-a}$$

Solution:

Given: $\lim_{x \rightarrow a} \frac{(x+2)^{5/2} - (a+2)^{5/2}}{x-a}$

The limit $\lim_{x \rightarrow a} \frac{(x+2)^{5/2} - (a+2)^{5/2}}{x-a}$

When $x = a$, the expression $\lim_{x \rightarrow a} \frac{(x+2)^{5/2} - (a+2)^{5/2}}{x-a}$ assumes the form $(0/0)$.

So let, $Z = \lim_{x \rightarrow a} \frac{(x+2)^{5/2} - (a+2)^{5/2}}{x-a}$

By using the formula: $\lim_{x \rightarrow a} \frac{(x)^n - (a)^n}{x-a} = na^{n-1}$

Since, Z is not of the form as described above.

Let us simplify, we get

$$Z = \lim_{x \rightarrow a} \frac{(x+2)^{5/2} - (a+2)^{5/2}}{x-a}$$

$$Z = \lim_{x \rightarrow a} \frac{(x+2)^{5/2} - (a+2)^{5/2}}{x+2-(a+2)}$$

Let $x+2 = y$ and $a+2 = k$

As $x \rightarrow a$; $y \rightarrow k$

So,

$$Z = \lim_{y \rightarrow k} \frac{(y)^{5/2} - (k)^{5/2}}{y-k}$$

By using the formula: $\lim_{x \rightarrow a} \frac{(x)^n - (a)^n}{x-a} = na^{n-1}$

$$Z = \frac{5}{2} k^{\frac{5}{2}-1}$$

$$= \frac{5}{2} k^{\frac{3}{2}}$$

$$= \frac{5}{2} (a+2)^{\frac{3}{2}}$$

$$\therefore \lim_{x \rightarrow a} \frac{(x+2)^{5/2} - (a+2)^{5/2}}{x-a} = \frac{5}{2} (a+2)^{\frac{3}{2}}$$

$$2. \lim_{x \rightarrow a} \frac{(x+2)^{3/2} - (a+2)^{3/2}}{x-a}$$

Solution:

Given: $\lim_{x \rightarrow a} \frac{(x+2)^{3/2} - (a+2)^{3/2}}{x-a}$

The limit $\lim_{x \rightarrow a} \frac{(x+2)^{3/2} - (a+2)^{3/2}}{x-a}$

When $x = a$, the expression $\lim_{x \rightarrow a} \frac{(x+2)^{3/2} - (a+2)^{3/2}}{x-a}$ assumes the form $(0/0)$.

So let, $Z = \lim_{x \rightarrow a} \frac{(x+2)^{3/2} - (a+2)^{3/2}}{x-a}$

By using the formula: $\lim_{x \rightarrow a} \frac{(x)^n - (a)^n}{x-a} = na^{n-1}$

Since, Z is not of the form as described above.

Let us simplify, we get

$$Z = \lim_{x \rightarrow a} \frac{(x+2)^{3/2} - (a+2)^{3/2}}{x-a}$$

$$Z = \lim_{x \rightarrow a} \frac{(x+2)^{3/2} - (a+2)^{3/2}}{x+2 - (a+2)}$$

Let $x+2 = y$ and $a+2 = k$

As $x \rightarrow a$; $y \rightarrow k$

$$Z = \lim_{y \rightarrow k} \frac{(y)^{3/2} - (k)^{3/2}}{y-k}$$

By using the formula: $\lim_{x \rightarrow a} \frac{(x)^n - (a)^n}{x-a} = na^{n-1}$

$$Z = \frac{3}{2} k^{\frac{3}{2}-1}$$

$$= \frac{3}{2} k^{\frac{1}{2}}$$

$$= \frac{3}{2} (a+2)^{\frac{1}{2}}$$

$$\therefore \lim_{x \rightarrow a} \frac{(x+2)^{3/2} - (a+2)^{3/2}}{x-a} = \frac{3}{2} \sqrt{a+2}$$

$$3. \lim_{x \rightarrow a} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$$

Solution:

Given: $\lim_{x \rightarrow a} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$

The limit $\lim_{x \rightarrow a} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$

When $x = a$, the expression $\lim_{x \rightarrow a} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$ assumes the form $(0/0)$.

So let, $Z = \lim_{x \rightarrow a} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$

$$Z = \frac{(1+a)^6 - 1}{(1+a)^2 - 1} = \frac{\{(1+a)^2\}^3 - 1}{(1+a)^2 - 1}$$

[This can be further simplified using the formula: $a^3 - 1 = (a-1)(a^2 + a + 1)$]

$$Z = \frac{\{(1+a)^2 - 1\} \{(1+a)^4 + (1+a)^2 + 1\}}{(1+a)^2 - 1}$$

$$= (1+a)^4 + (1+a)^2 + 1$$

$$= 1 + 1 + 1$$

$$= 3$$

$$\therefore \lim_{x \rightarrow a} \frac{(1+x)^6 - 1}{(1+x)^2 - 1} = 3$$

$$4. \lim_{x \rightarrow a} \frac{x^{2/7} - a^{2/7}}{x - a}$$

Solution:

Given: $\lim_{x \rightarrow a} \frac{x^{2/7} - a^{2/7}}{x - a}$

The limit $\lim_{x \rightarrow a} \frac{x^{2/7} - a^{2/7}}{x - a}$

When $x = a$, the expression $\lim_{x \rightarrow a} \frac{x^{2/7} - a^{2/7}}{x - a}$ assumes the form $(0/0)$.

So let, $Z = \lim_{x \rightarrow a} \frac{x^{2/7} - a^{2/7}}{x - a}$

By using the formula: $\lim_{x \rightarrow a} \frac{(x)^n - (a)^n}{x - a} = na^{n-1}$

Since, Z is not of the form as described above.

Let us simplify, we get

$$Z = \lim_{x \rightarrow a} \frac{x^{2/7} - a^{2/7}}{x - a}$$

By using the formula: $\lim_{x \rightarrow a} \frac{(x)^n - (a)^n}{x - a} = na^{n-1}$

$$Z = \frac{2}{7} a^{\frac{2}{7}-1}$$

$$= \frac{2}{7} a^{-\frac{5}{7}}$$

$$\therefore \lim_{x \rightarrow a} \frac{x^{2/7} - a^{2/7}}{x - a} = \frac{2}{7} a^{-\frac{5}{7}}$$

$$5. \lim_{x \rightarrow a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$$

Solution:

Given: $\lim_{x \rightarrow a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$

The limit $\lim_{x \rightarrow a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$

When $x = a$, the expression $\lim_{x \rightarrow a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$ assumes the form $(0/0)$.

So let, $Z = \lim_{x \rightarrow a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$

By using the formula: $\lim_{x \rightarrow a} \frac{(x)^n - (a)^n}{x - a} = na^{n-1}$

Since, Z is not of the form as described above.

Let us simplify, we get

$$Z = \lim_{x \rightarrow a} \frac{\frac{5}{x^7} - \frac{5}{a^7}}{\frac{2}{x^7} - \frac{2}{a^7}}$$

Let us divide the numerator and denominator by $(x - a)$, we get

$$Z = \lim_{x \rightarrow a} \frac{\frac{5}{x^7} - \frac{5}{a^7}}{\frac{2}{x^7} - \frac{2}{a^7}} \cdot \frac{x - a}{x - a}$$

By using algebra of limits, we have

$$Z = \lim_{x \rightarrow a} \frac{\frac{5}{x^7} - \frac{5}{a^7}}{\frac{2}{x^7} - \frac{2}{a^7}} \cdot \frac{x - a}{x - a}$$

So now again, by using the formula: $\lim_{x \rightarrow a} \frac{(x)^n - (a)^n}{x - a} = na^{n-1}$

$$Z = \frac{\frac{5}{a^7} - \frac{5}{a^7}}{\frac{2}{a^7} - \frac{2}{a^7}}$$

$$= \frac{5a^{-\frac{2}{7}}}{2a^{-\frac{5}{7}}}$$

$$= \frac{5}{2} a^{\frac{3}{7}}$$

$$\therefore \lim_{x \rightarrow a} \frac{\frac{5}{x^7} - \frac{5}{a^7}}{\frac{2}{x^7} - \frac{2}{a^7}} = \frac{5}{2} a^{\frac{3}{7}}$$

EXERCISE 29.6 PAGE NO: 29.38

Evaluate the following limits:

$$1. \lim_{x \rightarrow \infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)}$$

Solution:

Given: $\lim_{x \rightarrow \infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)}$

The limit $\lim_{x \rightarrow \infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)}$

Let us simplify the expression, we get

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)} &= \lim_{x \rightarrow \infty} \frac{(12x^2 - 10x + 2)}{(x^2 + 9x - 8)} \\ &= \lim_{x \rightarrow \infty} \left(\frac{12 - \frac{10}{x} + \frac{2}{x^2}}{1 + \frac{9}{x} - \frac{8}{x^2}} \right) \end{aligned}$$

When substituting the value of x as $x \rightarrow \infty$ and $\frac{1}{x} \rightarrow 0$ then,

$$\begin{aligned} &= \frac{12 - 0 + 0}{1} \\ &= 12 \end{aligned}$$

$$\therefore \lim_{x \rightarrow \infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)} = 12$$

$$2. \lim_{x \rightarrow \infty} \frac{3x^3 - 4x^2 + 6x - 1}{2x^3 + x^2 - 5x + 7}$$

Solution:

Given: $\lim_{x \rightarrow \infty} \frac{3x^3 - 4x^2 + 6x - 1}{2x^3 + x^2 - 5x + 7}$

The limit $\lim_{x \rightarrow \infty} \frac{3x^3 - 4x^2 + 6x - 1}{2x^3 + x^2 - 5x + 7}$

Let us simplify the expression, we get

$$\lim_{x \rightarrow \infty} \frac{3x^3 - 4x^2 + 6x - 1}{2x^3 + x^2 - 5x + 7} = \lim_{x \rightarrow \infty} \frac{3 - \frac{4}{x} + \frac{6}{x^2} - \frac{1}{x^3}}{2 + \frac{1}{x} - \frac{5}{x^2} + \frac{7}{x^3}}$$

When substituting the value of x as $x \rightarrow \infty$ and $\frac{1}{x} \rightarrow 0$ then,

$$\begin{aligned} &= \frac{3 - 0 + 0 - 0}{2 + 0 - 0 + 0} \end{aligned}$$

$$= 3/2$$

$$\therefore \lim_{x \rightarrow \infty} \frac{3x^3 - 4x^2 + 6x - 1}{2x^3 + x^2 - 5x + 7} = \frac{3}{2}$$

$$3. \lim_{x \rightarrow \infty} \frac{5x^3 - 6}{\sqrt{9 + 4x^6}}$$

Solution:

Given: $\lim_{x \rightarrow \infty} \frac{5x^3 - 6}{\sqrt{9 + 4x^6}}$

The limit

Let us simplify the expression, we get

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{5x^3 - 6}{\sqrt{9 + 4x^6}} &= \lim_{x \rightarrow \infty} \frac{5 - \frac{6}{x^3}}{\sqrt{\frac{9}{x^6} + \frac{4x^6}{x^6}}} \\ &= \lim_{x \rightarrow \infty} \frac{\left(5 - \frac{6}{x^3}\right)}{\sqrt{\frac{9}{x^6} + 4}} \end{aligned}$$

When substituting the value of x as $x \rightarrow \infty$ and $\frac{1}{x} \rightarrow 0$ then,

$$\begin{aligned} &= \frac{5}{\sqrt{4}} \\ &= 5/2 \end{aligned}$$

$$\therefore \lim_{x \rightarrow \infty} \frac{5x^3 - 6}{\sqrt{9 + 4x^6}} = \frac{5}{2}$$

$$4. \lim_{x \rightarrow \infty} \sqrt{x^2 + cx} - x$$

Solution:

Given: $\lim_{x \rightarrow \infty} \sqrt{x^2 + cx} - x$

The limit

Let us simplify the expression by rationalizing the numerator, we get

$$\begin{aligned} \lim_{x \rightarrow \infty} \sqrt{x^2 + cx} - x &= \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + cx} - x \right) \cdot \frac{\sqrt{x^2 + cx} + x}{\sqrt{x^2 + cx} + x} \\ &= \lim_{x \rightarrow \infty} \frac{(x^2 + cx - x^2)}{\sqrt{x^2 + cx} + x} \\ &= \lim_{x \rightarrow \infty} \frac{cx}{\sqrt{x^2 + cx} + x} \end{aligned}$$

By taking 'x' as common from both numerator and denominator, we get

$$= \lim_{x \rightarrow \infty} \frac{c}{\sqrt{1 + \frac{c}{x} + 1}}$$

When substituting the value of x as $x \rightarrow \infty$ and $\frac{1}{x} \rightarrow 0$ then,

$$= \frac{c}{1 + 1}$$

$$= \frac{c}{2}$$

$$\therefore \lim_{x \rightarrow \infty} \sqrt{x^2 + cx} - x = \frac{c}{2}$$

$$5. \lim_{x \rightarrow \infty} \sqrt{x+1} - \sqrt{x}$$

Solution:

Given: $\lim_{x \rightarrow \infty} \sqrt{x+1} - \sqrt{x}$

The limit $x \rightarrow \infty$

Let us simplify the expression by rationalizing the numerator, we get

On rationalizing the numerator we get,

$$\lim_{x \rightarrow \infty} \sqrt{x+1} - \sqrt{x} = \lim_{x \rightarrow \infty} \sqrt{x+1} - \sqrt{x} \cdot \frac{\sqrt{x+1} + \sqrt{x}}{(\sqrt{x+1} + \sqrt{x})}$$

$$= \lim_{x \rightarrow \infty} \frac{(x+1-x)}{\sqrt{x+1} + \sqrt{x}}$$

$$= \lim_{x \rightarrow \infty} \left(\frac{1}{\sqrt{x+1} + \sqrt{x}} \right)$$

When substituting the value of x as $x \rightarrow \infty$ and $\frac{1}{x} \rightarrow 0$ then,

$$= \frac{1}{\infty}$$

$$= 0$$

$$\therefore \lim_{x \rightarrow \infty} \sqrt{x+1} - \sqrt{x} = 0$$

EXERCISE 29.7 PAGE NO: 29.49

Evaluate the following limits:

$$1. \lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$

The limit $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$

Let us consider the limit:

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{5x} = \frac{1}{5} \lim_{x \rightarrow 0} \frac{\sin 3x}{x}$$

Now let us multiply and divide the expression by 3, we get

$$\begin{aligned} &= \frac{1}{5} \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \times 3 \\ &= \frac{3}{5} \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \end{aligned}$$

Now, put $3x = y$

$$= \frac{3}{5} \lim_{y \rightarrow 0} \frac{\sin y}{y} \quad \left[\text{We know that, } \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right]$$

So,

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin 3x}{5x} &= \frac{3}{5} \lim_{y \rightarrow 0} \frac{\sin y}{y} \\ &= \frac{3}{5} \times 1 \\ &= \frac{3}{5} \end{aligned}$$

$\therefore$ The value of $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x} = \frac{3}{5}$

$$2. \lim_{x \rightarrow 0} \frac{\sin x}{x}$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

The limit $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

We know, $1^\circ = \frac{\pi}{180}$ radians

So,

$$x^\circ = \frac{\pi x}{180} \text{ radians}$$

Let us consider the limit,

$$\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x} = \lim_{x \rightarrow 0} \frac{\sin \frac{\pi x}{180}}{x}$$

Now let us multiply and divide the expression by $\frac{\pi}{180}$, we get

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{\sin \frac{\pi x}{180} \times \frac{\pi}{180}}{x \times \frac{\pi}{180}} \\ &= \frac{\pi}{180} \lim_{x \rightarrow 0} \frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}} \end{aligned}$$

Now, put $\frac{\pi x}{180} = y$

$$= \frac{\pi}{180} \lim_{y \rightarrow 0} \frac{\sin y}{y} \quad \left[\text{We know that, } \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right]$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin x^\circ}{x} &= \frac{\pi}{180} \lim_{y \rightarrow 0} \frac{\sin y}{y} \\ &= \frac{\pi}{180} \times 1 \\ &= \frac{\pi}{180} \end{aligned}$$

$\therefore$ The value of $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x} = \frac{\pi}{180}$

$$3. \quad \lim_{x \rightarrow 0} \frac{x^2}{\sin x^2}$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{x^2}{\sin x^2}$

The limit $\lim_{x \rightarrow 0} \frac{x^2}{\sin x^2}$, we get

$$\lim_{x \rightarrow 0} \frac{x^2}{\sin x^2} = \lim_{x \rightarrow 0} \frac{1}{\frac{\sin x^2}{x^2}}$$

Now, put $x^2 = y$

$$\lim_{x \rightarrow 0} \frac{1}{\frac{\sin x^2}{x^2}} = \frac{1}{\lim_{y \rightarrow 0} \frac{\sin y}{y}} \quad \left[\text{We know that, } \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right]$$

$$= \frac{1}{1}$$

$$= 1$$

∴ The value of $\lim_{x \rightarrow 0} \frac{x^2}{\sin x^2} = 1$

4. $\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x}$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x}$

The limit

Let us consider the limit

$$\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x} = \frac{1}{3} \lim_{x \rightarrow 0} \frac{\sin x \cos x}{x}$$

$$= \frac{1}{3} \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \cos x$$

We know,

$$\lim_{x \rightarrow 0} A(x) \cdot B(x) = \lim_{x \rightarrow 0} A(x) \times \lim_{x \rightarrow 0} B(x)$$

So,

$$= \frac{1}{3} \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \lim_{x \rightarrow 0} \cos x \quad \left[\text{We know that, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x} = \frac{1}{3} \times 1 \times \cos 0$$

$$= \frac{1}{3} \times 1 \times 1 \quad [\text{Since, } \cos 0 = 1]$$

$$= \frac{1}{3}$$

∴ The value of $\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x} = \frac{1}{3}$

$$5. \lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x}$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x}$

The limit $\lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x}$

We know that, $\sin 3x = 3 \sin x - 4 \sin^3 x$

So,

$$\lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x} = \lim_{x \rightarrow 0} \frac{\sin 3x}{x}$$

Now multiply and divide the expression by 3, we get

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin 3x}{x} &= \lim_{x \rightarrow 0} \frac{\sin 3x \times 3}{3x} \\ &= 3 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \end{aligned}$$

Now, put $3x = y$

$$= 3 \lim_{y \rightarrow 0} \frac{\sin y}{y} \quad \left[\text{We know that, } \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right]$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x} &= 3 \lim_{y \rightarrow 0} \frac{\sin y}{y} \\ &= 3 \times 1 \\ &= 3 \end{aligned}$$

$$\therefore \text{The value of } \lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x} = 3$$

EXERCISE 29.8 PAGE NO: 29.62

Evaluate the following limits:

1. $\lim_{x \rightarrow \pi/2} \left(\frac{\pi}{2} - x \right) \tan x$

Solution:

Given: $\lim_{x \rightarrow \pi/2} \left(\frac{\pi}{2} - x \right) \tan x$

The limit Let us assume, $y = \frac{\pi}{2} - x$

So,

$x \rightarrow \frac{\pi}{2}, y \rightarrow 0$

$$\begin{aligned} \lim_{x \rightarrow \pi/2} \left(\frac{\pi}{2} - x \right) \tan x &= \lim_{y \rightarrow 0} y \tan \left(\frac{\pi}{2} - y \right) \\ &= \lim_{y \rightarrow 0} y \frac{\sin \left(\frac{\pi}{2} - y \right)}{\cos \left(\frac{\pi}{2} - y \right)} \quad [\text{We know that, } \tan = \sin/\cos] \\ &= \lim_{y \rightarrow 0} y \frac{\cos y}{\sin y} \end{aligned}$$

Upon simplification, we get

$$= \lim_{y \rightarrow 0} \cos y - \lim_{y \rightarrow 0} \frac{y}{\sin y}$$

Substituting the value of $y = 0$, then

$$\begin{aligned} &= \cos 0 - \frac{0}{\sin 0} \\ &= 1 - 0 \\ &= 1 \end{aligned}$$

$\therefore$ The value of $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\pi}{2} - x \right) \tan x = 1$

2. $\lim_{x \rightarrow \pi/2} \frac{\sin 2x}{\cos x}$

Solution:

Given: $\lim_{x \rightarrow \pi/2} \frac{\sin 2x}{\cos x}$

The limit

We know, $\sin 2x = 2\sin x \cdot \cos x$

So,

$$\lim_{x \rightarrow \pi/2} \frac{\sin 2x}{\cos x} = \lim_{x \rightarrow \pi/2} \frac{2 \sin x \cos x}{\cos x}$$

Upon simplification, we get

$$= \lim_{x \rightarrow \pi/2} 2 \sin x$$

Substitute the value of x, we get

$$\begin{aligned} &= 2 \sin \frac{\pi}{2} \\ &= 2 \times 1 \\ &= 2 \end{aligned}$$

$\therefore$ The value of $\lim_{x \rightarrow \pi/2} \frac{\sin 2x}{\cos x} = 2$

$$3. \lim_{x \rightarrow \pi/2} \frac{\cos^2 x}{1 - \sin x}$$

Solution:

Given: $\lim_{x \rightarrow \pi/2} \frac{\cos^2 x}{1 - \sin x}$

The limit

We know that, $\cos^2 x = 1 - \sin^2 x$

So, by substituting this value we get,

$$\lim_{x \rightarrow \pi/2} \frac{\cos^2 x}{1 - \sin x} = \lim_{x \rightarrow \pi/2} \frac{1 - \sin^2 x}{1 - \sin x}$$

Upon expansion,

$$= \lim_{x \rightarrow \pi/2} \frac{(1 - \sin x)(1 + \sin x)}{1 - \sin x}$$

When simplified, we get

$$= \lim_{x \rightarrow \pi/2} 1 + \sin x$$

Now, substitute the value of x, we get

$$\begin{aligned} &= 1 + \sin \frac{\pi}{2} \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

$\therefore$ The value of $\lim_{x \rightarrow \pi/2} \frac{\cos^2 x}{1 - \sin x} = 2$

4. $\lim_{x \rightarrow \pi/3} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2}(\pi/3 - x)}$

Solution:

Given: $\lim_{x \rightarrow \pi/3} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2}(\pi/3 - x)}$

The limit $\lim_{x \rightarrow \pi/3} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2}(\pi/3 - x)}$

We know that, $1 - \cos 2x = 2\sin^2 x$

So,

$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2}(\frac{\pi}{3} - x)} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{2 \sin^2 3x}}{\sqrt{2}(\frac{\pi}{3} - x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{2} \sin 3x}{\sqrt{2}(\frac{\pi}{3} - x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin 3x}{(\frac{\pi}{3} - x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{3}} \frac{3 \sin 3x}{\pi - 3x}$$

We know that, $\sin x = \sin(\pi - x)$

So,

$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2}(\frac{\pi}{3} - x)} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{3 \sin(\pi - 3x)}{\pi - 3x} \quad \left[\text{We know that, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$= 3$$

$\therefore$ The value of $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2}(\frac{\pi}{3} - x)} = 3$

5. $\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a}$

Solution:

Given: $\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a}$

The limit $\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a}$

We know that,

$$\left[\cos A - \cos B = 2 \sin \left(\frac{A - B}{2} \right) \sin \left(\frac{A + B}{2} \right) \right]$$

By substituting in the formula, we get

$$\begin{aligned}\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a} &= \lim_{x \rightarrow a} \frac{\left(-2 \sin\left(\frac{x+a}{2}\right) \sin\left(\frac{x-a}{2}\right)\right)}{x - a} \\ &= -2 \lim_{x \rightarrow a} \sin\left(\frac{x+a}{2}\right) \lim_{x \rightarrow a} \sin\left(\frac{x-a}{2}\right) \frac{1}{x-a}\end{aligned}$$

Upon simplification, we get

$$\begin{aligned}&= -2 \sin\left(\frac{a+a}{2}\right) \left(\lim_{x \rightarrow a} \sin\left(\frac{x-a}{2}\right) \times \frac{1}{x-a}\right) \\ &= -2 \sin a \times 1 \times \frac{1}{2} \\ &= -\sin a\end{aligned}$$

$\therefore$ The value of $\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a} = -\sin a$

EXERCISE 29.9 PAGE NO: 29.65

Evaluate the following limits:

1. $\lim_{x \rightarrow \pi} \frac{1 + \cos x}{\tan^2 x}$

Solution:

Given: $\lim_{x \rightarrow \pi} \frac{1 + \cos x}{\tan^2 x}$

The limit $\lim_{x \rightarrow \pi} \frac{1 + \cos x}{\tan^2 x}$

When $x = \pi$, the expression $\lim_{x \rightarrow \pi} \frac{1 + \cos x}{\tan^2 x}$ assumes the form $(0/0)$.

So, let us multiply the expression by $\cos^2 x$

$$\begin{aligned}\lim_{x \rightarrow \pi} \frac{1 + \cos x}{\tan^2 x} &= \lim_{x \rightarrow \pi} \left[\frac{(1 + \cos x)}{\sin^2 x} \times \cos^2 x \right] \\ &= \lim_{x \rightarrow \pi} \left[\frac{(1 + \cos x)}{1 - \cos^2 x} \times \cos^2 x \right]\end{aligned}$$

Upon expansion, we get

$$\begin{aligned}&= \lim_{x \rightarrow \pi} \left[\frac{(1 + \cos x)}{(1 - \cos x)(1 + \cos x)} \times \cos^2 x \right] \\ &= \lim_{x \rightarrow \pi} \left[\frac{\cos^2 x}{(1 - \cos x)} \right]\end{aligned}$$

Now, substitute the value of x, we get

$$= \frac{\cos^2 \pi}{1 - \cos \pi}$$

$$= \frac{(-1)^2}{1 - (-1)}$$

$$= \frac{1}{2}$$

∴ The value of $\lim_{x \rightarrow \pi} \frac{1 + \cos x}{\tan^2 x} = \frac{1}{2}$

2. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$

Solution:

Given: $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$

The limit $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$

When $x = \pi/4$, the expression $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$ assumes the form (0/0).

So,

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1} = \lim_{x \rightarrow \frac{\pi}{4}} \left[\frac{1 + \cot^2 x - 2}{\cot x - 1} \right] \quad [\text{Since, } \operatorname{cosec}^2 x = 1 + \cot^2 x]$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \left[\frac{\cot^2 x - 1}{\cot x - 1} \right]$$

Upon expansion, we get

$$= \lim_{x \rightarrow \frac{\pi}{4}} \left[\frac{(\cot x - 1)(\cot x + 1)}{(\cot x - 1)} \right]$$

Now, substitute the value of x, we get

$$= \cot \frac{\pi}{4} + 1$$

$$= 2$$

∴ The value of $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1} = 2$

$$3. \lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$$

Solution: $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$

Given:

The limit $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$

When $x = \pi/6$, the expression $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$ assumes the form $(0/0)$.

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2} = \lim_{x \rightarrow \frac{\pi}{6}} \left[\frac{\operatorname{cosec}^2 x - 1 - 3}{\operatorname{cosec} x - 2} \right] \quad [\text{Since, } \cot^2 x = \operatorname{cosec}^2 x - 1]$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \left[\frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2} \right]$$

Upon expansion, we get

$$= \lim_{x \rightarrow \frac{\pi}{6}} \left[\frac{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 2)}{(\operatorname{cosec} x - 2)} \right]$$

Now, substitute the value of x , we get

$$\begin{aligned} &= \operatorname{cosec} \frac{\pi}{6} + 2 \\ &= 2 + 2 \\ &= 4 \end{aligned}$$

$\therefore$ The value of $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2} = 4$

$$4. \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x}$$

Solution:

Given: $\lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x}$

The limit $\lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x}$

When $x = \pi/4$, the expression $\lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x}$ assumes the form $(0/0)$.

So,

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x} &= \lim_{x \rightarrow \frac{\pi}{4}} \left[\frac{2 - (1 + \cot^2 x)}{1 - \cot x} \right] \text{ [Since, } \operatorname{cosec}^2 x = 1 + \cot^2 x \text{]} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \left[\frac{1 - \cot^2 x}{1 - \cot x} \right] \end{aligned}$$

Upon expansion, we get

$$= \lim_{x \rightarrow \frac{\pi}{4}} \left[\frac{(1 - \cot x)(1 + \cot x)}{(1 - \cot x)} \right]$$

Now, substitute the value of x , we get

$$\begin{aligned} &= 1 + \cot\left(\frac{\pi}{4}\right) \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

$$\therefore \text{The value of } \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x} = 2$$

5. $\lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$

Solution:

Given: $\lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$

The limit $\lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$

When $x = \pi$, the expression $\lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$ assumes the form $(0/0)$.

So, let us rationalize the numerator, we get

$$\lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} = \lim_{x \rightarrow \pi} \left[\frac{(\sqrt{2 + \cos x} - 1) \times (\sqrt{2 + \cos x} + 1)}{(\pi - x)^2 (\sqrt{2 + \cos x} + 1)} \right]$$

Let us simplify the above expression, we get

$$\begin{aligned} &= \lim_{x \rightarrow \pi} \left[\frac{2 + \cos x - 1}{(\pi - x)^2 (\sqrt{2 + \cos x} + 1)} \right] \\ &= \lim_{x \rightarrow \pi} \left[\frac{1 + \cos x}{(\pi - x)^2 [\sqrt{2 + \cos x} + 1]} \right] \end{aligned}$$

Now, let $x = \pi - h$

When $x = \pi$, then $h = 0$

So,

$$\begin{aligned} &= \lim_{h \rightarrow 0} \left[\frac{1 + \cos(\pi - h)}{[\pi - (\pi - h)]^2 [\sqrt{2 + \cos(\pi - h)} + 1]} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{1 - \cos h}{h^2 [\sqrt{2 - \cos h} + 1]} \right] \{ \because \cos(\pi - \theta) = -\cos \theta \} \end{aligned}$$

Let us simplify further,

$$= \lim_{h \rightarrow 0} \left[\frac{2 \sin^2\left(\frac{h}{2}\right)}{4 \times \frac{h^2}{4} [\sqrt{2 - \cos h} + 1]} \right]$$

$$= \frac{1}{2} \lim_{h \rightarrow 0} \left[\left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \right)^2 \times \frac{1}{[\sqrt{2 - \cos h} + 1]} \right]$$

Now, substitute the value of h, we get

$$= \frac{1}{2} \times 1 \times \frac{1}{(\sqrt{2 - \cos 0} + 1)}$$

$$= \frac{1}{2} \times \frac{1}{(\sqrt{1} + 1)}$$

$$= \frac{1}{2 \times 2}$$

$$= \frac{1}{4}$$

$$\therefore \text{The value of } \lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} = \frac{1}{4}$$

EXERCISE 29.10 PAGE NO: 29.71

Evaluate the following limits:

$$1. \lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4 + x} - 2}$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4 + x} - 2}$

The limit $\lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4 + x} - 2}$ assumes the form $(0/0)$.

So, $\lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4 + x} - 2}$

Now, multiply both numerator and denominator by $\sqrt{4 + x} + 2$ so that we can remove the indeterminate form.

$$\begin{aligned} Z &= \lim_{x \rightarrow 0} \frac{(5^x - 1)\sqrt{4 + x} + 2}{(\sqrt{4 + x} - 2)(\sqrt{4 + x} + 2)} \\ &= \lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4 + x} - 2} \times \frac{\sqrt{4 + x} + 2}{\sqrt{4 + x} + 2} \end{aligned}$$

{By using $a^2 - b^2 = (a + b)(a - b)$ }

$$Z = \lim_{x \rightarrow 0} \frac{(5^x - 1)\sqrt{4+x} + 2}{4+x-4}$$

$$= \lim_{x \rightarrow 0} \frac{(5^x - 1)\sqrt{4+x} + 2}{x}$$

By using basic algebra of limits, we get

$$Z = \lim_{x \rightarrow 0} \frac{(5^x - 1)}{x} \times \lim_{x \rightarrow 0} \sqrt{4+x} + 2 = \{\sqrt{4+0} + 2\} \lim_{x \rightarrow 0} \frac{(5^x - 1)}{x}$$

$$= 4 \lim_{x \rightarrow 0} \frac{(5^x - 1)}{x} \quad [\text{By using the formula: } \lim_{x \rightarrow 0} \frac{(a^x - 1)}{x} = \log a]$$

$$Z = 4 \log 5$$

$$\therefore \text{The value of } \lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4+x} - 2} = 4 \log 5$$

$$2. \lim_{x \rightarrow 0} \frac{\log(1+x)}{3^x - 1}$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{\log(1+x)}{3^x - 1}$

The limit $\lim_{x \rightarrow 0} \frac{\log(1+x)}{3^x - 1}$

When $x = 0$, the expression $\lim_{x \rightarrow 0} \frac{\log(1+x)}{3^x - 1}$ assumes the form $(0/0)$.

So,

$$\text{As } Z = \lim_{x \rightarrow 0} \frac{\log(1+x)}{3^x - 1}$$

Let us divide numerator and denominator by x , we get

$$Z = \lim_{x \rightarrow 0} \frac{\frac{\log(1+x)}{x}}{\frac{3^x - 1}{x}} = \frac{\lim_{x \rightarrow 0} \frac{\log(1+x)}{x}}{\lim_{x \rightarrow 0} \frac{3^x - 1}{x}} \quad \{\text{by using basic limit algebra}\}$$

$$[\text{By using the formula: } \lim_{x \rightarrow 0} \frac{(a^x - 1)}{x} = \log a]$$

$$= \frac{1}{\log 3}$$

$$\therefore \text{The value of } \lim_{x \rightarrow 0} \frac{\log(1+x)}{3^x - 1} = \frac{1}{\log 3}$$

$$3. \lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2}$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2}$

The limit $\lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2}$

When $x = 0$, the expression $\lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2}$ assumes the form $(0/0)$.

So,

As $Z = \lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2}$

$$= \lim_{x \rightarrow 0} \frac{a^{-x}(a^{2x} - 2a^x + 1)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(a^{2x} - 2a^x + 1)}{a^x x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(a^x - 1)^2}{a^x x^2} \quad \{\text{By using } (a + b)^2 = a^2 + b^2 + 2ab\}$$

Let us use algebra of limit, we get

$$Z = \lim_{x \rightarrow 0} \left(\frac{a^x - 1}{x} \right)^2 \times \lim_{x \rightarrow 0} \frac{1}{a^x}$$

[By using the formula: $\lim_{x \rightarrow 0} \frac{(a^x - 1)}{x} = \log a$]

$$Z = (\log a)^2 \frac{1}{a^0} = (\log a)^2$$

$$\therefore \text{The value of } \lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2} = (\log a)^2$$

$$4. \lim_{x \rightarrow 0} \frac{a^{mx} - 1}{b^{nx} - 1}, n \neq 0$$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{a^{mx} - 1}{b^{nx} - 1}, n \neq 0$

The limit $\lim_{x \rightarrow 0} \frac{a^{mx} - 1}{b^{nx} - 1}, n \neq 0$

When $x = 0$, the expression $\lim_{x \rightarrow 0} \frac{a^{mx} - 1}{b^{nx} - 1}, n \neq 0$ assumes the form $(0/0)$.

So, let us include mx and nx as follows:

$$\begin{aligned} Z &= \lim_{x \rightarrow 0} \frac{a^{mx} - 1}{b^{nx} - 1} = \lim_{x \rightarrow 0} \frac{a^{\frac{a^{mx} - 1}{a^{mx} - 1} \times mx}}{b^{\frac{b^{nx} - 1}{b^{nx} - 1} \times nx}} \\ &= \frac{m}{n} \lim_{x \rightarrow 0} \frac{\frac{a^{mx} - 1}{a^{mx} - 1}}{\frac{b^{nx} - 1}{b^{nx} - 1}} \end{aligned}$$

By using algebra of limits, we get

$$Z = \frac{m}{n} \frac{\lim_{x \rightarrow 0} \frac{a^{mx} - 1}{mx}}{\lim_{x \rightarrow 0} \frac{b^{nx} - 1}{nx}}$$

[By using the formula: $\lim_{x \rightarrow 0} \frac{(a^x - 1)}{x} = \log a$]

$$Z = \frac{m}{n} \frac{\log a}{\log b}, n \neq 0$$

$$\therefore \text{The value of } \lim_{x \rightarrow 0} \frac{a^{mx} - 1}{b^{nx} - 1} = \frac{m}{n} \frac{\log a}{\log b}, n \neq 0$$

5. $\lim_{x \rightarrow 0} \frac{a^x + b^x - 2}{x}$

Solution:

Given: $\lim_{x \rightarrow 0} \frac{a^x + b^x - 2}{x}$

The limit $\lim_{x \rightarrow 0} \frac{a^x + b^x - 2}{x}$

When $x = 0$, the expression $\lim_{x \rightarrow 0} \frac{a^x + b^x - 2}{x}$ assumes the form $(0/0)$.

So,

As $Z = \lim_{x \rightarrow 0} \frac{a^x + b^x - 2}{x}$
 $= \lim_{x \rightarrow 0} \frac{a^x - 1 + b^x - 1}{x}$

By using algebra of limits, we get

$$Z = \lim_{x \rightarrow 0} \frac{a^x - 1}{x} + \lim_{x \rightarrow 0} \frac{b^x - 1}{x}$$

[By using the formula: $\lim_{x \rightarrow 0} \frac{(a^x - 1)}{x} = \log a$]

$$Z = \log a + \log b = \log ab$$

$\therefore$ The value of $\lim_{x \rightarrow 0} \frac{a^x + b^x - 2}{x} = \log ab$

EXERCISE 29.11 PAGE NO: 29.71

Evaluate the following limits:

1. $\lim_{x \rightarrow \pi} \left(1 - \frac{x}{\pi}\right)^\pi$

Solution:

Given: $\lim_{x \rightarrow \pi} \left(1 - \frac{x}{\pi}\right)^\pi$

Let us substitute the value of $x = \pi$ directly, we get

$$Z = \lim_{x \rightarrow \pi} \left(1 - \frac{x}{\pi}\right)^\pi = \left(1 - \frac{\pi}{\pi}\right)^\pi = (1 - 1)^\pi = 0^\pi = 0$$

Since, it is not of indeterminate form.

$Z = 0$
 $\therefore$ The value of $\lim_{x \rightarrow \pi} \left(1 - \frac{x}{\pi}\right)^\pi = 0$

$$2. \lim_{x \rightarrow 0^+} \left\{ 1 + \tan \sqrt{x} \right\}^{1/2x}$$

Solution:

Given: $\lim_{x \rightarrow 0^+} \left\{ 1 + \tan \sqrt{x} \right\}^{1/2x}$

The limit $x \rightarrow 0^+$

Let us use the theorem given below

If $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ such that $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ exists, then $\lim_{x \rightarrow a} [1 + f(x)]^{\frac{1}{g(x)}} = e^{\lim_{x \rightarrow a} \frac{f(x)}{g(x)}}$.

So here,

$$f(x) = \tan^2 \sqrt{x}$$

$$g(x) = 2x$$

Then,

$$\begin{aligned} \lim_{x \rightarrow 0^+} \left\{ 1 + \tan \sqrt{x} \right\}^{1/2x} &= e^{\lim_{x \rightarrow 0^+} \left(\frac{\tan^2 \sqrt{x}}{2x} \right)} \\ &= e^{\lim_{x \rightarrow 0^+} \left(\frac{\tan \sqrt{x}}{\sqrt{x}} \right) \times \left(\frac{\tan \sqrt{x}}{\sqrt{x}} \right) \times \frac{1}{2}} \\ &= e^{1 \times 1 \times \frac{1}{2}} \\ &= \sqrt{e} \end{aligned}$$

$$\therefore \text{The value of } \lim_{x \rightarrow 0^+} \left\{ 1 + \tan \sqrt{x} \right\}^{1/2x} = \sqrt{e}$$

3. $\lim_{x \rightarrow 0} (\cos x)^{1/\sin x}$

Solution:

Given: $\lim_{x \rightarrow 0} (\cos x)^{1/\sin x}$

The limit $x \rightarrow 0$

Let us use the theorem given below

If $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ such that $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ exists, then $\lim_{x \rightarrow a} [1 + f(x)]^{\frac{1}{g(x)}} = e^{\lim_{x \rightarrow a} \frac{f(x)}{g(x)}}$.

So here,

$f(x) = \cos x - 1$

$g(x) = \sin x$

Then,

$$\begin{aligned} \lim_{x \rightarrow 0} (\cos x)^{1/\sin x} &= e^{\lim_{x \rightarrow 0} \left(\frac{\cos x - 1}{\sin x} \right)} \\ &= e^{\lim_{x \rightarrow 0} \left(\frac{-2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} \right)} \\ &= e^{\lim_{x \rightarrow 0} \left(-\tan \frac{x}{2} \right)} \\ &= e^0 \\ &= 1 \end{aligned}$$

$\therefore$ The value of $\lim_{x \rightarrow 0} (\cos x)^{1/\sin x} = 1$

4. $\lim_{x \rightarrow 0} (\cos x + \sin x)^{1/x}$

Solution:

Given: $\lim_{x \rightarrow 0} (\cos x + \sin x)^{1/x}$

The limit $x \rightarrow 0$

Let us add and subtract '1' to the given expression, we get

$\lim_{x \rightarrow 0} [1 + \cos x + \sin x - 1]^{\frac{1}{x}}$

Let us use the theorem given below

If $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ such that $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ exists, then $\lim_{x \rightarrow a} [1 + f(x)]^{\frac{1}{g(x)}} = e^{\lim_{x \rightarrow a} \frac{f(x)}{g(x)}}$.

So here,

$f(x) = \cos x + \sin x - 1$

$g(x) = x$

Then,

$$\lim_{x \rightarrow 0} (\cos x + \sin x)^{1/x} = e^{\lim_{x \rightarrow 0} \left(\frac{\cos x + \sin x - 1}{x} \right)}$$

Upon computing, we get

$$\begin{aligned} &= e^{\lim_{x \rightarrow 0} \left[\frac{\sin x}{x} - \frac{(1 - \cos x)}{x} \right]} \\ &= e^{\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} - \frac{2 \sin^2 \frac{x}{2}}{x} \right)} \\ &= e^{\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} - \frac{2 \sin\left(\frac{x}{2}\right) \times \sin\left(\frac{x}{2}\right)}{2 \times \frac{x}{2}} \right)} \end{aligned}$$

Now, substitute the value of x, we get

$$\begin{aligned} &= e^{1-0} \\ &= e^1 \\ &= e \end{aligned}$$

$$\therefore \text{The value of } \lim_{x \rightarrow 0} (\cos x + \sin x)^{1/x} = e$$

5. $\lim_{x \rightarrow 0} (\cos x + a \sin bx)^{1/x}$

Solution:

Given: $\lim_{x \rightarrow 0} (\cos x + a \sin bx)^{1/x}$

The limit $\lim_{x \rightarrow 0}$

Let us add and subtract '1' to the given expression, we get

$$\lim_{x \rightarrow 0} [1 + \cos x + a \sin bx - 1]^{\frac{1}{x}}$$

Let us use the theorem given below

If $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ such that $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ exists, then $\lim_{x \rightarrow a} [1 + f(x)]^{\frac{1}{g(x)}} = e^{\lim_{x \rightarrow a} \frac{f(x)}{g(x)}}$.

So here,

$$f(x) = \cos x + a \sin bx - 1$$

$$g(x) = x$$

Then,

$$\lim_{x \rightarrow 0} (\cos x + a \sin bx)^{1/x} = e^{\lim_{x \rightarrow 0} \left[\frac{\cos x + a \sin bx - 1}{x} \right]}$$

Let us compute now, we get

$$\begin{aligned}
 &= e^{\lim_{x \rightarrow 0} \left[\frac{b \times a \sin bx}{bx} - \frac{(1 - \cos x)}{x} \right]} \\
 &= e^{\lim_{x \rightarrow 0} \left(\frac{ab \sin bx}{bx} - \frac{2 \sin^2 \frac{x}{2}}{x} \right)}
 \end{aligned}$$

Now, substitute the value of x, we get
 $= e^{ab}$

$\therefore$ The value of $\lim_{x \rightarrow 0} (\cos x + a \sin bx)^{1/x} = e^{ab}$