

HC VERMA Solutions for Class 11 Physics Chapter 2 - Physics and Mathematics

Question 1

A vector $\vec{A}$ makes an angle of 20° and $\vec{B}$ makes an angle of 110° with the X-axis. The magnitudes of these vectors are 3 m and 4 m respectively. Find the resultant.

Solution 1

The angle between $\vec{A}$ and $\vec{B}$ from the x-axis are 20° and 110° respectively.

Their magnitudes are 3 units and 4 units respectively.

Thus the angle between $\vec{A}$ and $\vec{B}$ is $= 110 - 20 = 90^\circ$

Now, $R^2 = A^2 + B^2 + 2AB\cos\theta$

$$= 3^2 + 4^2 + 2 \cdot 3 \cdot 4 \cos(90)$$

$$= 5^2$$

$$\text{Or, } R = 5$$

Let ϕ is the angle between R and A,

$$\frac{B\sin\theta}{A+B\cos\theta} = \frac{4}{3}, \text{ or } \phi = 53^\circ.$$

The resultant makes an angle of $(53+20)^\circ = 73^\circ$ with the x axis.

Question 2

Let $\vec{A}$ and $\vec{B}$ be the two vectors of magnitude 10 unit each. If they are inclined to the X-axis at angles 30° and 60° respectively, find the resultant.

Solution 2

A and B are inclined at angles of 30 degrees and 60 degrees with respect to the x axis

Angle between them $= (60 - 30) = 90$ degrees

Given that $|\vec{A}| = |\vec{B}| = 10$ units, we get

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$= 10^2 + 10^2 + 2 \cdot 10 \cdot 10 \cos(30)$$

$$R = 20\cos 15^\circ$$

$$\text{Or, } R = 19.3 \text{ units}$$

$$\text{And } \tan \phi = \frac{B\sin\theta}{A+B\cos\theta} = \frac{1}{2+\sqrt{3}}, \text{ or } \phi = 15^\circ.$$

Therefore, this resultant makes an angle of $(15+30) = 45$ degrees with the x axis

Question 3

Add vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ each having magnitude of 100 unit and inclined to the X-axis at angles 45° , 135° and 315° respectively.

Solution 3

Vectors A, B and C are oriented at 45° , 135° and 315° respectively.

$$|\vec{A}| = |\vec{B}| = |\vec{C}| = 100 \text{ units}$$

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, and $\vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$, and we can write that,

$$A_x = C_x = 100\cos(45^\circ) = 100/\sqrt{2}, \text{ by considering their components}$$

$$B_x = -100/\sqrt{2}$$

$$\text{Now } A_y = 100\sin(45^\circ) = 100/\sqrt{2},$$

$$B_y = 100 \sin(135^\circ) = 100/\sqrt{2}$$

$$\text{Similarly, } C_y = -100/\sqrt{2}$$

$$\text{Net x component} = 100/\sqrt{2} + 100/\sqrt{2} - 100/\sqrt{2} = 100/\sqrt{2}$$

$$\text{Net y component} = 100/\sqrt{2} + 100/\sqrt{2} - 100/\sqrt{2} = 100/\sqrt{2}$$

$$R^2 = x^2 + y^2 = 100^2$$

$$R = 100 \text{ and } \tan \phi = (100/\sqrt{2}) / (100/\sqrt{2}) = 1, \text{ and } \phi = 45^\circ$$

Question 4

Let $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} + 4\hat{j}$. (a) Find the magnitudes of (a) $\vec{a}$, (b) $\vec{b}$, (c) $\vec{a} + \vec{b}$ and (d) $\vec{a} - \vec{b}$.

Solution 4

$$a = 4i + 3j, b = 3i + 4j$$

$$|a| = |b| = \sqrt{(3^2 + 4^2)} = 5$$

$$a + b = 7i + 7j \text{ and } a - b = i - j$$

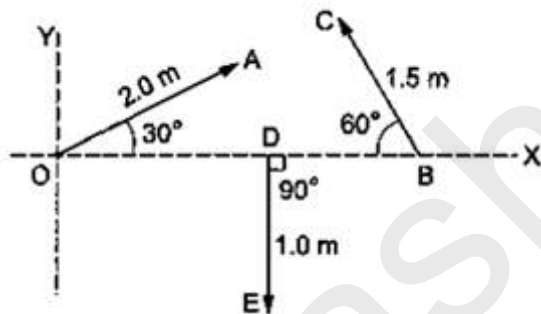
$$|a + b|$$

$$= \sqrt{(7^2 + 7^2)} = 7\sqrt{2} \text{ and } |a - b| = \sqrt{(1^2 + 1^2)} = \sqrt{2}$$

Question 5

Refer to the figure. (a) Find the magnitude, (b) x and y

components and (c) the angle with the X-axis of the resultant of $\vec{OA}, \vec{BC}$ and $\vec{DE}$.



Solution 5

$$\text{x component of } OA = 2 \cos 30^\circ = \sqrt{3}$$

$$\text{x component of } BC = 1.5 \cos 120^\circ = -0.75$$

$$\text{x component of } DE = 1 \cos 270^\circ = 0$$

$$\text{y component of } OA = 2 \sin 30^\circ = 1$$

$$\text{component of } BC = 1.5 \sin 120^\circ = 1.3$$

$$\text{component of } DE = 1 \sin 270^\circ = -1$$

$$R_x = \text{x component of resultant} = 3 - 0.75 + 0 = 0.98 \text{ m}$$

$$R_y = \text{resultant y component} = 1 + 1.3 - 1 = 1.3 \text{ m}$$

$$\text{So, } R = (R_x^2 + R_y^2)^{1/2} = 1.6 \text{ m}$$

$$\text{It makes an angle } \theta \text{ with positive x-axis then } \tan \theta = R_y/R_x = 1.32 \quad \theta = \tan^{-1}(1.32)$$

Question 6

Two vectors have magnitudes 3 unit and 4 unit respectively. What should be the angle between them if the magnitude of the resultant is (a) 1 unit, (b) 5 unit and (c) 7 unit.

Solution 6

$$|a| = 3 \text{ and } |b| = 4$$

Let Θ be the angle between them.

Then, using the relation $R^2 = A^2 + B^2 + 2AB\cos\Theta$,

a)

We get for $R = 1$,

$$1 = 9 + 16 + 24\cos\Theta$$

$$\text{Or, } \Theta = 180^\circ$$

b)

For, $R = 5$, we have

$$25 = 9 + 16 + 24\cos\Theta$$

$$\text{Or, } \cos\Theta = 0;$$

$$\Theta = 90^\circ$$

c) For $R = 7$,

$$49 = 9 + 16 + 24\cos\Theta,$$

$$\text{Or } \cos\Theta = 1,$$

$$\text{And } \Theta = 0^\circ$$

Question 7

A spy report about a suspected car reads as follows. "The car moved 2.00 km towards east, made a perpendicular left turn, ran for 500 m, made a perpendicular right turn, ran for 4.00 km and stopped." Find the displacement of the car.

Solution 7

$$AB = 2i + 0.5j + 4i = 6i + 0.5j$$

As the car went forward, took a left and then a right.

$$\text{So, } AB = (6^2 + 0.5^2)^{1/2} = 6.02 \text{ km}$$

$$\text{And } \phi = \tan^{-1}(BE/AE) = \tan^{-1}(0.5/6) = \tan^{-1}(1/12)$$

Question 8

A carrom board (4 ft $\times$ 4 ft square) has the queen at the center. The queen, hit by the striker moves to the front edge, rebounds and goes in the hole behind the striking line. Find the magnitude of displacement of the queen (a) from the center to the front edge, (b) from the front edge to the hole and (c) from the center to the hole.

Solution 8

In $\triangle ABC$, $\tan l = x/2$ and in $\triangle DCF$, $\tan l = (2 - x)/4$, So, $(x/2) = (2 - x)/4$. Solving,

$$4 - 2x = 4x$$

$$\text{Or, } 6x = 4$$

$$\text{Or, } x = 2/3 \text{ ft}$$

$$\text{a) In } \triangle ABC, AC = (AB^2 + BC^2)^{1/2} = 2\sqrt{10/3} \text{ ft}$$

$$\text{b) In } \triangle CFD, DF = 1 - (2/3) = 4/3 \text{ ft}$$

$$\text{So, } CF = 4 \text{ ft. Now, } CD = (CF^2 + FD^2)^{1/2} = 4\sqrt{10/3} \text{ ft}$$

$$\text{c) In } \triangle ADE, AE = (AE^2 + ED^2)^{1/2} = 2\sqrt{2} \text{ ft.}$$

Question 9

A mosquito net over a 7 ft $\times$ 4 ft bed is 3 ft high. The net has a hole at one corner of the bed through which a mosquito enters the bed. It flies and sits at the diagonally opposite upper corner of the net. (a) Find the magnitude of the displacement of the mosquito. (b) Taking the hole as the origin, the length of the bed as the X-axis, its width as the Y-axis, and vertically up as the Z-axis, write the components of the displacement vector.

Solution 9

The displacement vector is given by $r = 7i + 4j + 3k$

Magnitude of displacement of the mosquitoes = $(7^2 + 4^2 + 3^2)^{1/2} = 74\text{ft}$

The components of displacement along x axis, y axis and z axis are 7 units, 4 units and 3 units respectively.

Question 10

Suppose $\vec{a}$ is a vector of magnitude 4.5 unit due north. What is the vector (a) $3\vec{a}$, (b) $-4\vec{a}$?

Solution 10

$a = 4.5 \mathbf{n}$, where $\mathbf{n}$ is unit vector in north direction

a) $3a = 4.5 \times 3\mathbf{n} = 13.5$ in north direction

b) $-4a = 4.5 \times -4\mathbf{n} = 18$ in south direction

Question 11

Two vectors have magnitudes 2 m and 3 m. The angle between them is 60° . Find (a) the scalar product of the two vectors, (b) the magnitude of their vector product?

Solution 11

We have $a = 2\text{m}$, $b = 3\text{m}$.

$\theta = 60^\circ$ is the angle between the two vectors

Scalar product between the two vectors = $a \cdot b =$

$$2 \times 3 \times \cos(60^\circ) = 3\text{m}^2$$

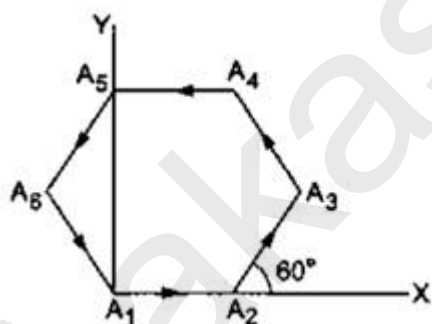
Vector product between the two vectors = $a \times b =$

$$2 \times 3 \times \sin(60^\circ) = 3\sqrt{3}\text{m}^2$$

Question 12

Let $A_1A_2A_3A_4A_5A_6A_1$ be a regular hexagon. Write the x-components of the vectors represented by the six sides taken in order. Use the fact that the resultant of these six vectors is zero, to prove that $\cos 0 + \cos \pi/3 + \cos 2\pi/3 + \cos 4\pi/3 + \cos 5\pi/3 = 0$

Use the known cosine values to verify the result.



Solution 12

From polygon law of vector addition, the resultant of the six vectors can be affirmed to be zero. Here their magnitudes are the same.

That is, $A = B = C = D = E = F$.

$$R_x = A \cos 0 + A \cos \pi/3 + A \cos 2\pi/3 + A \cos 3\pi/3 + A \cos 4\pi/3 + A \cos 5\pi/3 = 0$$

[As resultant is zero, x component of resultant is also 0]

Now taking A common and putting $R_x = 0$,

$$\cos 0 + \cos \pi/3 + \cos 2\pi/3 + \cos 3\pi/3 + \cos 4\pi/3 + \cos 5\pi/3 = 0$$

Question 13

Let $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{j} + 5\vec{k}$. Find the angle between them.

Solution 13

$a = 2i + 3j + 4k$ and $b = 3i + 4j + 5k$

Let angle between them is Θ

Then $a \cdot b = 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 = 38$

$$|a| = \sqrt{(2^2 + 3^2 + 4^2)} = \sqrt{29}$$

$$|b| = \sqrt{(3^2 + 4^2 + 5^2)} = \sqrt{50}$$

$$\text{Now } \cos \Theta = a \cdot b / |a||b| = 38 / \sqrt{1450}$$

Question 14

Prove that $\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$.

Solution 14

$(\vec{A} \times \vec{B}) = AB \sin \Theta \hat{u}$, where $\hat{u}$ is a unit vector perpendicular to both A and B.

Now, $\vec{A} \cdot (\vec{A} \times \vec{B})$ is basically a dot product between two vectors which are perpendicular to each other.

Then $\cos 90^\circ = 0$, and thus

$$\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$$

Question 15

If $\vec{A} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{B} = 4\vec{i} + 3\vec{j} + 2\vec{k}$, find $\vec{A} \times \vec{B}$.

Solution 15

$A = 2i + 3j + 4k$, $B = 4i + 3j + 2k$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 4 \\ 4 & 3 & 2 \end{vmatrix} = -6\vec{i} + 12\vec{j} - 6\vec{k}$$

Question 16

If $\vec{A}, \vec{B}, \vec{C}$ are mutually perpendicular, show that $\vec{C} \times (\vec{A} \times \vec{B}) = 0$. Is the converse true?

Solution 16

A, B and C are mutually perpendicular vectors. Now, if we take cross product between any two vectors, the resultant vector will be in parallel to the third vector, as there are only three axis perpendicular to each other.

So if we consider $(\vec{A} \times \vec{B})$, then it is parallel to C, and so angle between the resultant vector and C is 0° , and $\sin(0^\circ) = 0$. So, $\vec{C} \times (\vec{A} \times \vec{B}) = 0$

Question 17

A particle moves on a given straight line with a constant v. At a certain time it is at a point P on its straight line path. O is a fixed point. Show that $\vec{OP} \times \vec{v}$ is independent of the position P.

Solution 17

The particle moves from PP' in a straight line with a constant speed v.

From the figure, we see that $OP \times v = (OP)v \sin \theta \hat{u}$, where $\hat{u}$ is a unit vector perpendicular to v and OP. Now,

We know, $OQ = OP \sin \theta = OP' \sin \theta'$

So, the position of the particle may vary, but the magnitude and direction of $OP \times v$ will be constant. $OP \times v$ is independent of P.

Question 18

The force on a charged particle due to electric and magnetic fields is given by $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$.

Suppose $\vec{E}$ is along the X-axis and $\vec{B}$ along the Y-axis. In what direction and with what minimum speed v should a positively charged particle be sent so that the net force on it is zero?

Solution 18

$$F = q(E + v \times B)$$

Now, for net force to be 0, we must have $E = -(v \times B)$

So, the direction of E must be opposite to that of $(v \times B)$, so v must be in Z axis and its magnitude is $E/(B \sin \theta)$. For v to be minimum, $\theta = 90^\circ$, so $v = E/B$.

Question 19

Give an example for which $\vec{A} \cdot \vec{B} = \vec{C} \cdot \vec{B}$ but $\vec{A} \neq \vec{C}$.

Solution 19

Let us assume that B is along Y axis, and A is along positive x axis and C is along negative X axis. Now, $A \cdot B = B \cdot C = 0$. But $A \neq C$

Question 20

Draw a graph from the following data. Draw tangents at $x=2, 4, 6$ and 8 . Find the slopes of these

tangents. Verify that the curve is drawn is $y=2x^2$ and the slope of tangent is $\tan \theta = \frac{dy}{dx} = 4x$.

x	1	2	3	4	5	6	7	8	9	10
y	2	8	18	32	50	72	98	128	162	200

Solution 20

To find the slope at any point, we draw a tangent and we extend it to meet the X axis. Then we can find θ as shown in the figure.

We can use another way, which is by differentiation. We write,

$$\frac{dy}{dx} = \frac{d}{dx}(2x^2) = 4x$$

Question 21

A curve is represented by $y = \sin x$. If x is changed from $\frac{\pi}{3}$ to $\frac{\pi}{3} + \frac{\pi}{100}$, find approximately the change in y .

Solution 21

$$y = \sin(x)$$

$$\text{Let } y_1 = \sin(\pi/3) \text{ and } y_2 = \sin(\pi/3 + \pi/100)$$

$$\text{Change in } y = y_2 - y_1 = \sin(\pi/3 + \pi/100) - \sin(\pi/3)$$

$$= \sin(\pi/3 + (\pi/3 + \pi/100 - \pi/3)) - \sin(\pi/3)$$

$$= 0.0157$$

Question 22

The electric current in a charging R-C circuit is given by $i = i_0 e^{-t/RC}$ where i_0 , R and C are constant parameters of the circuit and t is time. Find the rate of change of current at (a) $t=0$, (b) $t=RC$, (c) $t=10RC$.

Solution 22

We have, $i = i_0 e^{-t/RC}$

$$\text{Rate of change of current} = \frac{di}{dt} = \frac{d}{dt} \left(i_0 e^{-\frac{t}{RC}} \right) = -\frac{i_0}{RC} \times e^{-\frac{t}{RC}}$$

a) When $t = 0$, $di/dt = -i_0/RC$

b) When $t = RC$, $di/dt = -i_0/RCe$

c) When $t = 10RC$, $di/dt = -\frac{i_0}{RCe^{10}}$

Question 23

The electric current in a discharging R-C circuit is given by $i = i_0 e^{-t/RC}$ where i_0 , R and C are constant parameters of the circuit and t is time. Let $i_0 = 2.00$ A, $R = 6.00 \times 10^5 \Omega$ and $C = 0.500 \mu F$. (a) Find the current at $t = 0.3$ s. (b) Find the rate of change of current at $t = 0.3$ s. (c) Find approximately the current at $t = 0.31$ s.

Solution 23

$$i = i_0 e^{-t/RC}$$

Here, $i_0 = 2$ A, $R = 6 \times 10^5 \Omega$, $C = 5 \times 10^{-6} F = 5 \times 10^{-7} F$

a) $i|_{t=0.3} = 2 \times e^{-(0.3/0.3)} = 2/e$ Amperes.

b) $di/dt|_{t=0.3} = -2e^{-(0.3/0.3)}/0.3 = -20/3e$

c) $i|_{t=0.31} = 5.8/3e$ Amperes

Question 24

Find the area bounded under the curve $y = 3x^2 + 6x + 7$ and the X-axis with the co-ordinates at $x = 5$ and $x = 10$.

Solution 24

$$y = 3x^2 + 6x + 7$$

Area bounded under the curve within $x = 5$ and $x = 10$ is calculated by the method of integration.

$$\text{Area} = \int_{x=5}^{x=10} y dx = \int_5^{10} (3x^2 + 6x + 7) dx = \left[3 \frac{x^3}{3} + 6 \frac{x^2}{2} + 7x \right]_5^{10} = 1135 \text{ sq. units}$$

Question 25

Find the area enclosed by the curve $y = \sin x$ and the X-axis between $x = 0$ and $x = \pi$.

Solution 25

$$y = \sin(x)$$

Area under the curve from $x = 0$ to $x = \pi$ is calculated by the method of integration.

$$\text{Area} = \int_{x=0}^{x=\pi} y dx = \int_0^{\pi} \sin x dx = -[\cos \pi - \cos 0] = 2$$

Question 26

Find the area bounded by the curve, $y = e^{-x}$, the X-axis and the Y-axis.

Solution 26

The equation of the curve is $y = e^{-x}$. And when $x = 0$, then $y = e^{-0} = 1$. Now, $y = 0$ when $x = \infty$. Using these boundaries, we get, $\text{Area} = \int_0^{\infty} e^{-x} dx = 1$.

Question 27

A rod of length L is placed along the X-axis between $x = 0$ and $x = L$. The linear density (mass/length) ρ of the rod varies with the distance x from the origin as $\rho = a + bx$. (a) Find the SI units of a and b . (b) Find the mass of the rod in terms of a , b and L .

Solution 27

It is given that linear density (mass per unit length) $\rho = a + bx$, where x is distance from origin.

a) S.I. unit of $a = \text{kg/m}$
S.I. unit of $b = \text{kg/m}^2$

b) To find the mass of the rod, we have to take a small segment dx placed at a distance x from the origin.

or, $dm = \rho dx$,

$$\text{or, } m = \int_0^L \rho dx = \int_0^L (a + bx) dx = [ax + bx^2/2]_0^L = aL + 0.5bL^2$$

Question 28

The momentum p of a particle changes with time t according to the relation $\frac{dp}{dt} = (10N) + (2N/s)t$. If the momentum is zero at $t=0$, what will be the momentum be at $t=10s$?

Solution 28

It is given that $\frac{dp}{dt} = (10N) + \left(\frac{2N}{s}\right)t$

Now, $p=0$ at $t = 0$. So,

$$p = \int_0^{10} (10dt + 2tdt) = 10(10-0) + (100-0) = 200N/s = 200 \text{ kg m/s}$$

Question 29

The changes in a function y and the independent variable x are related as $\frac{dy}{dx} = x^2$. Find y as a function of x .

Solution 29

It is given that $\frac{dy}{dx} = x^2$

We can write $dy = x^2 dx$

$$\text{or, } y = \int x^2 dx = \frac{x^3}{3} + c$$

Question 30

Write the number significant digits in (a) 1001, (b) 100.1, (c) 100.10, (d) 0.001001.

Solution 30

The number of significant digits are as follows:

- a) 4
- b) 4
- c) 5
- d) 4

Question 31

A meter scale is graduated at every millimeter. How many significant digits will be there in a length measurement with this scale?

Solution 31

It is stated that the meter scale is graduated at every millimeter. Now, we know $1m = 1000mm$. So minimum measurement can be of 1 digit value(say, 3mm, 5mm), then there can be up to two digits and three digits(say, 76mm, 305 mm) and the maximum value can be 1000mm. So, there can be 1, 2, 3 or 4 significant digits.

Question 32

Round the following numbers to 2 significant digits. (a) 3472, (b) 84.16, (c) 2.55 and (d) 28.5.

Solution 32

(a) Here after 4, the number is 7, which is greater than 5. So, the next two digits are neglected and the 4 becomes 5. Hence, rounded value = 3500

(b) Here after 4, the number is 1, which is less than 5. So, the next two digits are neglected and the 4 doesn't change. Hence, rounded value = 84

(c) Here after 5, the number is 5. So, the next two digits are neglected and the 5 is increased by 1. Hence, rounded value 2.6

(d) Here, after 8 the number is 5, and hence 8 is increased by 1. So the rounded value is 29

Question 33

The length and the radius of a cylinder measured with a slide calipers are found to be 4.54 cm and 1.75 cm respectively. Calculate the volume of the cylinder.

Solution 33

Length $L = 4.54\text{cm}$, radius $r = 1.75\text{cm}$

$$\text{Volume} = \pi r^2 L = \pi \times (4.54)^2 \times (1.75)^2 = 43.6577\text{cm}^3.$$

Now, it has to be rounded off to 3 significant digits, so we see that after 6, the value is 5. So 6 becomes 7, and thus the rounded off value is given by $V = 43.7\text{ cm}^3$.

Question 34

The thickness of a plate is measured to be 2.17 mm, 2.17 mm and 2.18 mm at three different places. Find the average thickness of the plate from this data.

Solution 34

Average thickness

$$= \frac{2.17 + 2.17 + 2.18}{3} = 2.1733\text{mm}.$$

Now we have to round it off to three significant figures. So, after 7, the digit is 3. Hence, Average thickness = 2.17mm.

Question 35

The length of the string of a simple pendulum is measured with a meter scale to be 90.0 cm. The radius of the bob plus the length of the hook is calculated to be 2.13 cm using measurements with a side calipers. What is the effective length of the pendulum? (The effective length is defined as the distance between the point of suspension and the center of the bob.)

Solution 35

$$\text{Effective Length} = (90.0 + 2.13)\text{ cm}$$

In the measurement of the 90.0cm, there are only two significant digits. Keeping this in mind, we have to consider only two significant digits during addition. So,

$$\text{Effective Length} = (90.0 + 2.1)\text{cm} = 92.1\text{cm}$$