

HC VERMA Solutions for Class 11 Physics Chapter 18 Geometrical Optics

Question 1

A concave mirror having a radius of curvature 40cm is placed in front of an illuminated point source at a distance of 30cm from it. Find the location of the images.

Solution 1

By using mirror equation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R}$$

Hence,

$$\frac{1}{v} = \frac{2}{R} - \frac{1}{u}$$

$$\frac{1}{v} = \frac{2}{-40} - \frac{1}{-30}$$

$$v = -60\text{cm}$$

Question 2

A concave mirror forms an image of 20cm high object on a screen placed 5.0cm away from the mirror. The height of the image is 50cm. Find the focal length of the mirror and the distance between the mirror and the object.

Solution 2

We know that,

$$\frac{hi}{ho} = \frac{-v}{u}$$

$$\frac{-50}{20} = \frac{500}{u}$$

$$u = -200\text{cm}$$

Now by using mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{-5} + \frac{1}{-2}$$

$$f = -1.44\text{m}$$

Question 3

A concave mirror has a focal length of 20cm. Find the position or position of an object for which the image size is double of the object size.

Solution 3

There are two possible cases:

(i) $u = -u$, $v = 2u$, $f = -20\text{cm}$

By using mirror formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-u} + \frac{1}{2u} = \frac{1}{-20}$$

$u = 10\text{cm}$

(ii) $u = -u$, $v = -2u$, $f = -20\text{cm}$

By using mirror formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-u} + \frac{1}{-2u} = \frac{1}{-20}$$

$u = 30\text{cm}$

Question 4

A 1cm object is placed perpendicular to the principal axis of a convex mirror of focal length 7.5cm. Find its distance from the mirror if the image formed is 0.6 cm in size.

Solution 4

Using mirror equation,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{0.6u} + \frac{1}{u} = \frac{1}{7.5}$$

As,

$$\left\{m = \frac{-v}{u}\right\}$$

$$\left\{0.6 = \frac{-v}{u}\right\}$$

$u = 5\text{cm}$

Question 5

A candle flame 1.6cm high is imaged in a hall bearing of diameter 0.4cm. If the ball bearing is 20cm away from the flame, find the location and the height of the image.

Solution 5

Focal length

$$f = \frac{R}{2} = \frac{0.2}{2} = 0.1 \text{ cm}$$

Now by using mirror formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{-20} = \frac{1}{0.1}$$

$$v = 0.1 \text{ cm}$$

and magnification,

$$m = \frac{h_i}{h_o} = \frac{-v}{u}$$

$$h_i = \frac{-v}{u} \times h_o$$

$$= \frac{-0.1}{-20} \times 1.6$$

$$h_i = 0.008 \text{ cm}$$

Question 6

A 3cm tall object is placed at a distance of 7.5cm from a convex mirror of focal length 6cm. Find the location, size and nature of the image.

Solution 6

By using mirror formula ,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{6} + \frac{1}{v} = \frac{1}{-7.5}$$

$$v = \frac{10}{3} \text{ cm}$$

And,

$$m = \frac{-v}{u}$$

$$= \frac{-10}{3} \times \frac{1}{7.5}$$

$$m = \frac{-100}{75 \times 3}$$

Also,

$$m = \frac{h_i}{h_o}$$

So,

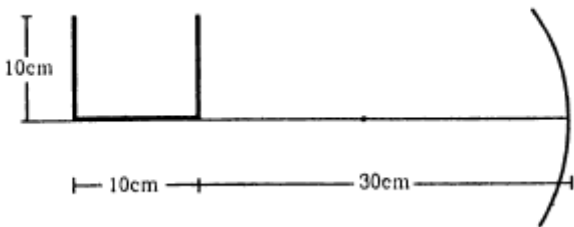
$$\frac{h_i}{h_o} = \frac{-100}{75 \times 3}$$

$$h_i = \frac{-100}{75 \times 3} \times h_o = \frac{-100}{75 \times 3} \times 3$$

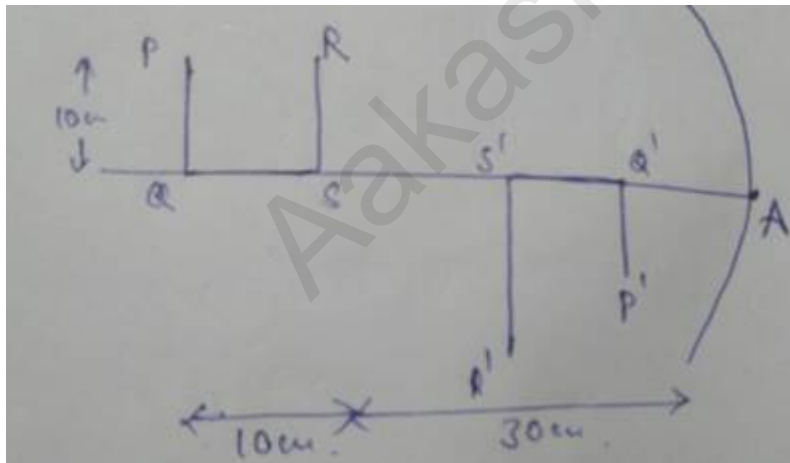
$$h_i = 1.33 \text{ cm}$$

Question 7

A U-shaped wire is placed before a concave mirror having radius of curvature 20cm as shown in figure. Find the total length of the image.



Solution 7



For PQ,
 $U = AQ = 40 \text{ cm}$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-40} - \frac{1}{-10} = \frac{1}{v}$$

$$v = -13.3 \text{ cm}$$

Also,

$$m = \frac{hi}{ho} = \frac{-v}{u}$$

$$hi = \frac{-v}{u} \times ho = \frac{13.3}{-40} \times 10$$

$$hi = -3.3 \text{ cm}$$

For RS

$$U = -30 \text{ cm}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-30} - \frac{1}{-10} = \frac{1}{v}$$

$$v = -15 \text{ cm}$$

Also,

$$m = \frac{hi'}{ho} = \frac{-v}{u}$$

$$hi' = \frac{-v}{u} \times ho = \frac{15}{30} \times 10$$

$$hi = 5 \text{ cm}$$

Now,

$$Q'S' = AS' - AQ'$$

$$= 15 - 13.3$$

$$= 1.7 \text{ cm}$$

Thus, total image length is,

$$3.3 + 5 + 1.7 = 10 \text{ cm}$$

Question 8

A man uses a concave mirror for shaving. He keeps his face at a distance of 25 cm from the mirror and gets an image which is 1.4 times enlarged. Find the focal length of the mirror.

Solution 8

As we know that,

$$m = \frac{hi}{ho} = \frac{-v}{u}$$

$$1.4 = \frac{-v}{-25}$$

$$v = 35\text{cm}$$

Now,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{35} + \frac{1}{-25} = \frac{1}{f}$$

$$f = -87.5\text{cm}$$

Chapter 18 - Geometrical Optics Exercise 413

Question 9

Find the diameter of the images of the moon formed by a spherical concave mirror of focal length 7.6m. The diameter of the moon is 3450km and the distance between the earth and the moon is 3.8×10^5 km.

Solution 9

By using mirror formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{3.8 \times 10^5 \times 10^3} + \frac{1}{v} = \frac{1}{-7.6}$$

$$v = -7.6\text{m}$$

Now,

$$m = \frac{-v}{u} = \frac{-(-7.6)}{-3.8 \times 10^8} = \frac{hi}{ho}$$

$$hi = \frac{-7.6}{3.8 \times 10^8} \times 3450 \times 10^3$$

$$hi = 0.069\text{m}$$

Question 10

A particle goes in a circle of radius 2.0cm. A concave mirror of focal length 20cm is placed with its principal axis passing through the centre of the circle and perpendicular to its plane. The distance between the pole of the mirror and the center of the circle is 30cm. Calculate the radius of the circle formed by the image.

Solution 10

By using mirror formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-30} + \frac{1}{v} = \frac{1}{-20}$$

$$v = -60 \text{ cm}$$

Also,

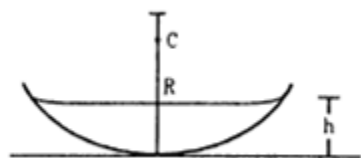
$$m = \frac{-v}{u} = \frac{Ri}{Ro}$$

$$\frac{-60}{-30} \times 2 = Ri$$

$$Ri = 4 \text{ cm}$$

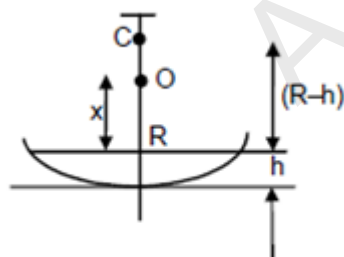
Question 11

A concave mirror of radius R is kept on a horizontal table. Water (refractive index = μ) is poured into it up to a height h . Where an object should be placed so that its image is formed on itself?



Solution 11

Let x be height of object above water.



We know,

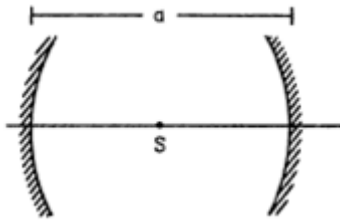
$$\frac{\text{Real depth}}{\text{Apparent depth}} = \frac{1}{\mu}$$

$$\frac{x}{R-h} = \frac{1}{\mu}$$

$$x = \frac{R-h}{\mu}$$

Question 12

A point source S is placed midway between two converging mirror having equal focal length f as shown in fig. Find the values of d for which only one image is formed.



Solution 12

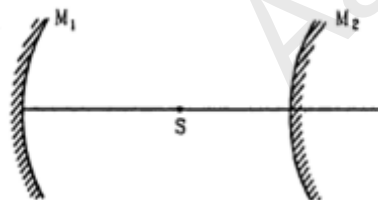
Both mirror produces one image in following two cases:

(A) When Source is at distance $=2f$ or at center of curvature then image is produce at $v=2f+2f=4f$

(B) When source is at distance f then rays become parallel after reflecting through one mirror, then it is reflected through other mirror and only one image is formed
 $V=f + f=2f$

Question 13

A converging mirror M_1 , a point source S and a diverging mirror M_2 are arranged as shown in figure. The source is placed at a distance of 30cm from M_1 . The focal length of each of the mirror is 20cm. Consider only the images formed by a maximum of two reflections. It is found that one image is formed on the source itself. (a) Find the distance between the two mirrors. (b) Find the location of the images formed by the single reflection from M_2 .



Solution 13

By using mirror formula, for reflection,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-30} + \frac{1}{v} = \frac{1}{-20}$$

$$v = -60\text{cm}$$

Now, for 2nd reflection,

$$u = 60 - (30 + x) = 30 - x$$

Using Mirror equation,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{30 - x} + \frac{1}{-x} = \frac{1}{20}$$

$$x^2 + 10x - 600 = 0$$

Solving equation we get,

$$x = 20\text{cm or } x = -30\text{cm}$$

Therefore,

$$\text{Total distance is } 20 + 30 = 50\text{cm}$$

Question 14

A light ray falling at an angle of 45° with the surface of a clean slab of ice of thickness 1m is refracted into it at an angle of 30° . Calculate the time taken by the light rays to cross the slab. Speed of light in vacuum $= 3 \times 10^8 \text{ m/s}$.

Solution 14

We know that,

$$\frac{\sin i}{\sin r} = \frac{c}{v}$$

$$\frac{\sin 45^\circ}{\sin 30^\circ} = \frac{3 \times 10^8}{v}$$

$$v = \frac{3 \times 10^8}{\sqrt{2}} \text{ m/s}$$

Distance

$$x = \frac{1\text{m}}{\cos 30^\circ} = \frac{2}{\sqrt{3}}$$

And time taken,

$$t = \frac{2}{\sqrt{3}} \div \frac{3 \times 10^8}{\sqrt{2}}$$

Total time taken

$$t = 5.4 \times 10^{-8} \text{ s}$$

Question 15

A pole of length 1m stands half dipped in a swimming pool with water level 50.0cm higher than the bed. The refractive index of water is 1.33 and sunlight is coming at an angle of 45° with the vertical. Find the length of the shadow of the pole on the bed.

Solution 15

Length of Shadow = $0.5 + 0.5 \tan r$

Also,

$$\frac{\sin 45^\circ}{\sin r} = 1.33$$

$$\sin r = 0.53$$

$$\text{and } \cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - (0.53)^2} \\ = 0.85$$

$$\tan r = \frac{\sin r}{\cos r} = \frac{0.53}{0.85} = 0.623$$

$$\text{Length of shadow} = 0.5(1 + \tan r) \\ = 0.5(1 + 0.623) \\ = 81.5 \text{ cm}$$

Question 16

A small piece of wood is floating on the surface of a 2.5 m deep lake. Where does the shadow from on the bottom when the sun is just setting? Refractive index of water = $\frac{4}{3}$.

Solution 16

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

$$\sin r = \frac{3}{4}$$

$$r = 49^\circ$$

According to figure,

$$\tan r = \frac{x}{2.5}$$

$$x = 2.8 \text{ m}$$

Question 17

An object P is focused by a microscope M. A glass slab of thickness 2.1 cm is introduced between P and M. If the refractive index of the slab is 1.5, by what distance should the microscope be shifted to focus the object again?

Solution 17

Shift,

$$\Delta t = \left(1 - \frac{1}{\mu}\right) d$$

$$= \left(1 - \frac{1}{1.5}\right) \times 2.1$$

$$\Delta t = 0.7 \text{ cm}$$

Question 18

A vessel contains water up to a height of 20cm and above it oil up to another 20cm. The refractive index of the water and the oil are 1.33 and 1.30 resp. Find the apparent depth of the vessel when viewed from above.

Solution 18

Shift due to water,

$$\Delta t_w = \left(1 - \frac{1}{\mu}\right) d$$

$$= \left(1 - \frac{1}{1.33}\right) 20$$

$$\Delta t_w = 5 \text{ cm}$$

Shift due to oil,

$$\Delta t_o = \left(1 - \frac{1}{\mu}\right) d$$

$$= \left(1 - \frac{1}{1.3}\right) 20$$

$$\Delta t_o = 4.6 \text{ cm}$$

Adding Shift of water and oil,

$$\Delta t = \Delta t_w + \Delta t_o$$

$$= 5 + 4.6$$

$$= 9.6 \text{ cm}$$

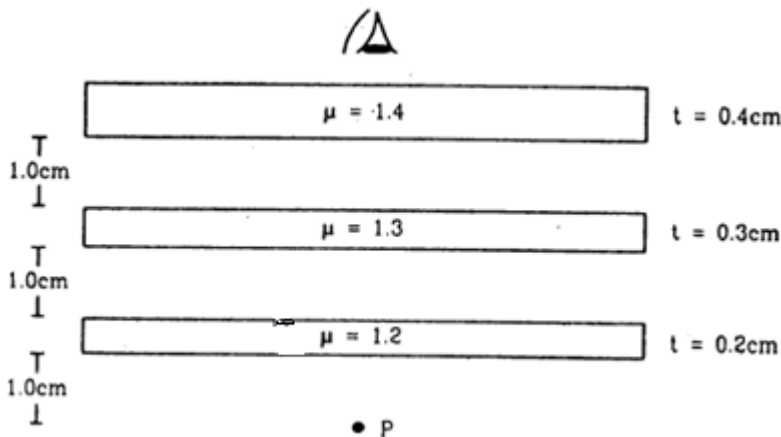
$$\text{Apparent depth} = 40 - \Delta t$$

$$= 40 - 9.6$$

$$= 30.4 \text{ cm}$$

Question 19

Locate the image of the point P as seen by the eye in the fig



Solution 19

Shift is given as:

$$\Delta t = \left(1 - \frac{1}{1.2}\right)(0.2) + \left(1 - \frac{1}{1.3}\right)(0.3) + \left(1 - \frac{1}{1.4}\right)(0.4)$$

$$\Delta t = 0.2 \text{ cm}$$

Shift of 0.2 cm above P point takes place due to 3 sheets.

Question 20

k transparent slabs are arranged one over another. The refractive index of the slabs are $\mu_1, \mu_2, \mu_3, \dots, \mu_k$ and the thickness are $t_1, t_2, t_3, \dots, t_k$. An object is seen through this combination with nearly perpendicular light. Find the equivalent refractive index of the system which will allow the image to be formed at the same place.

Solution 20

Total shift due to 'k' no. of slabs is:

$$\Delta t = \left(1 - \frac{1}{\mu_1}\right)t_1 + \left(1 - \frac{1}{\mu_2}\right)t_2 + \dots + \left(1 - \frac{1}{\mu_k}\right)t_k \quad \dots \dots \dots (1)$$

Where $t_1, t_2, t_3, \dots, t_k$ are thickness and $\mu_1, \mu_2, \mu_3, \dots, \mu_k$ are refractive index

$$\Delta t = \left(1 - \frac{1}{\mu}\right)(t_1 + t_2 + t_3 \dots + t_k) \quad \dots \dots \dots (2)$$

Comparing equation(1) and (2)

$$\begin{aligned} & \left(1 - \frac{1}{\mu}\right)(t_1 + t_2 + t_3 \dots + t_k) \\ &= \left(1 - \frac{1}{\mu_1}\right)t_1 + \left(1 - \frac{1}{\mu_2}\right)t_2 + \dots + ()t_k \end{aligned}$$

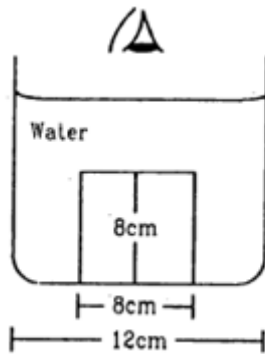
$$\left(1 - \frac{1}{\mu}\right)(t_1 + t_2 + t_3 + \dots + t_k)$$

$$= (t_1 + t_2 + t_3 + \dots + t_k) - \left(\frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} + \dots + \frac{t_k}{\mu_k}\right)$$

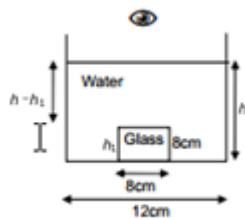
$$\mu = \frac{\sum_{i=1}^k t_i}{\sum_{i=1}^k \left(\frac{t_i}{\mu_i}\right)}$$

Question 21

A cylindrical vessel of diameter 12cm contains 800π cm of water. A cylindrical glass piece of diameter 8.0cm and height 8cm is placed in the vessel. If the bottom of the vessel under the glass piece is seen by the paraxial rays fig. locate its image. The index of refractive of glass is 1.50 and that of water is 1.33.



Solution 21



When glass piece is inserted inside water then volume of cylindrical water column becomes,

$$\pi r^2 h = 800\pi + \pi r^2 h_1$$

$$6^2 \times h = 800 + 4^2 \times 8$$

$$h = 25.7\text{cm}$$

Now, shift due to glass block is,

$$\Delta t_1 = \left(1 - \frac{1}{\mu_g}\right) t_g$$

$$= \left(1 - \frac{1}{3/2}\right) 8$$

$$\Delta t_1 = 2.26 \text{ cm}$$

Now, shift due to water is,

$$\Delta t_2 = \left(1 - \frac{1}{\mu_w}\right) t_w$$

$$= \left(1 - \frac{1}{\frac{4}{3}}\right) (25.7 - 8)$$

$$\Delta t_1 = 4.44 \text{ cm}$$

Net shift

$$\Delta t = \Delta t_1 + \Delta t_2$$

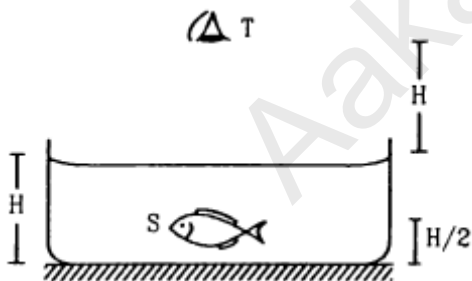
$$= 2.26 + 4.44$$

$$\Delta t = 7.1 \text{ cm}$$

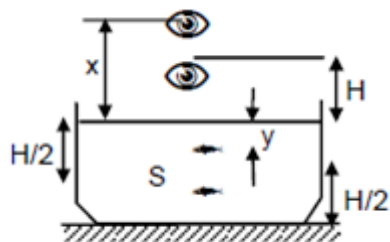
Chapter 18 - Geometrical Optics Exercise 414

Question 22

Consider the situation in figure. The bottom of the pot is a reflecting plane mirror, S is a small fish and T is a human eye. Refractive index of the water is μ (a) At what distance from itself will the fish see the image of the eye? (b) At what distance from itself will the eye see the image of the fish.



Solution 22



(A) We know ,

$$\frac{H}{x} = \frac{1}{\mu}$$

$$x = \mu H$$

Distance =

$$\frac{H}{2} + \mu H = H\left(\mu + \frac{1}{2}\right)$$

Image through mirror

$$= \frac{H}{2} + H + x$$

$$= H\left(\mu + \frac{3}{2}\right)$$

(B) We know that,

$$\frac{H}{\frac{y}{2}} = \mu$$

$$y = \frac{H}{2\mu}$$

Direct image= H+y

$$= H\left(1 + \frac{1}{2\mu}\right)$$

Real depth

$$= H + \frac{H}{2} = \frac{3H}{2}$$

Apparent depth

$$= \frac{3H}{2\mu}$$

$$\text{Distance of image} = H + \frac{3H}{2\mu} = H \left(1 + \frac{3}{2\mu} \right)$$

Question 23

A small object is placed at the center of the bottom of a cylindrical vessel of radius 3cm and height 4cm filled completely with water. Consider the ray leaving the vessel through a corner. Suppose this ray and the ray along the axis of the vessel are used to trace the image. Find the apparent depth under the assumption taken. Refractive index of water is 1.33.

Solution 23

By above diagram,

$$\frac{x}{3} = \cot i \cot r \quad (1)$$

Also,

$$\frac{\sin i}{\sin r} = \frac{1}{1.33}$$

$$\sin r = 1.33 \times \sin i$$

$$\sin r = 4/5$$

$$\text{Then, } \cot r = 3/4 \quad (2)$$

Comparing equation (1) and (2),

$$\frac{x}{3} = \frac{3}{4}$$

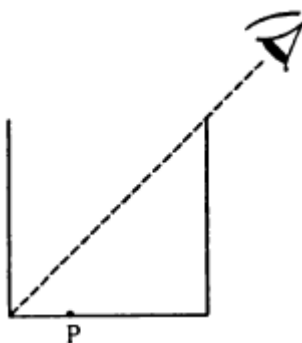
$$x = 2.25 \text{ cm}$$

Ration

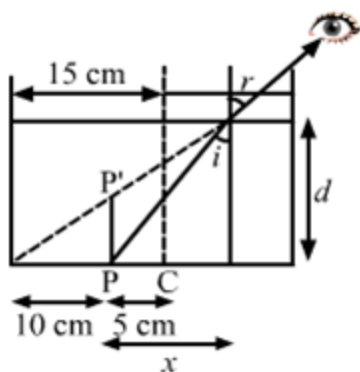
$$\frac{\text{Real depth}}{\text{Apparent depth}} = \frac{4}{2.25} = 1.78$$

Question 24

A cylindrical vessel, whose diameter and height both are equal to 30 cm is placed on a horizontal surface and a small particle in it at a distance of 5cm from the center. An eye is placed at a position such that the edge of the bottom is just visible (fig) The particle P is in the plane of drawing. Up to what minimum height should water be poured in the vessel to make the particle P visible?



Solution 24



We know,

$$\frac{\sin i}{\sin r} = \frac{3}{4}$$

$$\sin i = \frac{1}{\sqrt{2}} \times \frac{3}{4}$$

$$\text{As } \{ \sin 45^\circ = \frac{1}{\sqrt{2}} \}$$

Let x = distance between P and X
Now

$$\tan 45^\circ = \frac{x + 10}{d}$$

$$d = x + 10$$

Also,

$$\tan i = \frac{x}{d}$$

$$\left\{ \sin i = \frac{1}{\sqrt{2}} \times \frac{3}{4} \right\}$$

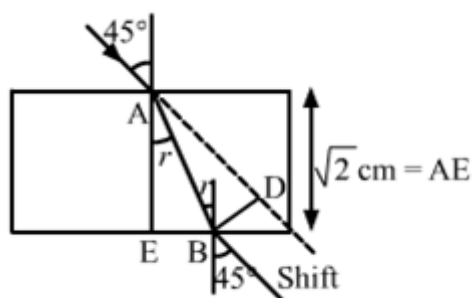
$$\frac{3}{\sqrt{23}} = \frac{d - 10}{d} = 26.7 \text{ cm}$$

Question 25

A light ray is incident at an angle of 45° with the normal to a $\sqrt{2}$ cm thick plate (refractive index = 2). Find the shift in the path of the light as it emerges out from the plate.

Solution 25

According to fig,



$$\frac{\sin i}{\sin r} = \frac{2}{1}$$

$$\sin r = \frac{1}{2\sqrt{2}}$$

$$\text{And } r = 21^\circ$$

$$\theta = 45^\circ - 21^\circ$$

$$= 24^\circ$$

$$\text{Shift} = AB \sin \theta$$

$$\frac{AE}{\cos 21^\circ} \times 0.406$$

$$= 0.62 \text{ cm}$$

Question 26

An optical fibre ($\mu=1.72$) is surrounded by a glass coating ($\mu=1.50$) > find the critical angle for total internal reflection at the fiber glass interface.

Solution 26

We know,

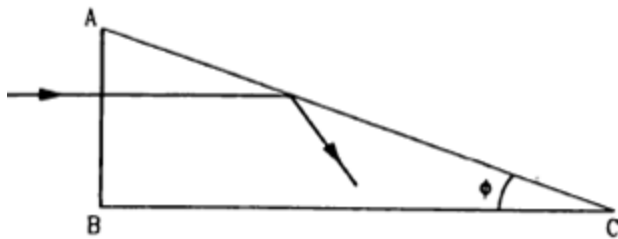
$$\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2}$$

$$\frac{\sin c}{\sin 90} = \frac{1.5}{1.72}$$

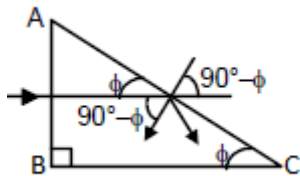
$$c = \sin^{-1}\left(\frac{75}{86}\right)$$

Question 27

A light ray is incident normally on the face AB of a right angled prism ABC ($\mu=1.50$) as shown in fig. What is the largest angle ϕ for which the light ray is totally reflected at the surface AC?



Solution 27



We know,

$$\frac{\sin c}{\sin 90} = \frac{1}{x} = \frac{1}{1.5}$$

$$c = \sin^{-1}\left(\frac{2}{3}\right)$$

And

$$90^\circ - \phi > c$$

$$90^\circ - c > \phi$$

$$\phi < \cos^{-1}\frac{2}{3}$$

Hence, $\cos^{-1}\frac{2}{3}$ is the largest possible angle.

Question 28

Find the maximum angle of refractive index when a light ray is refracted from glass ($\mu=1.50$) to air.

Solution 28

When light ray travels from glass to air then total internal reflection takes place. In this refracted angle is more than 90° for reflection to occur. Thus, maximum angle is 90° .

Question 29

Light is incident from glass ($\mu=1.50$) to air. Sketch the variation of the angle of deviation δ with the angle of incident i for $0 < i < 90^\circ$.

Solution 29

We know,

$$\frac{\sin i}{\sin r} = \frac{\sin c}{\sin r} = \frac{n_1}{n_2}$$

$$\frac{\sin i}{\sin 90^\circ} = \frac{1}{1.5}$$

$$c = \sin^{-1}\left(\frac{1}{1.5}\right)$$

$$c = 40^\circ 48'$$

Angle increases from 0° to $40^\circ 48'$ then to 45° angle decreases.

Question 30

Light is incident from glass ($\mu=1.50$) to water ($\mu=1.33$) Find the range of the angle of deviation for which there are two angles of incidence.

Solution 30

(i) CASE I: When light passes through normal ,

$$i = 0^\circ$$

$$r = 0^\circ$$

Angle of deviation $d = 0^\circ$

(ii) CASE II: When light is incident at critical angle

$$\frac{n_w}{n_g} = \frac{\sin c}{\sin r} \Rightarrow \frac{\sin c}{\sin 90^\circ} = \frac{8}{9}$$

$$\sin i = \frac{8}{9}$$

$$C = 62.73^\circ$$

Angle of deviation

$$= 90^\circ - c$$

$$= 90^\circ - 62.73^\circ$$

$$= 27.27^\circ$$

Range 0 to $\cos^{-1}(8/9)$

Question 31

Light falls from glass ($\mu=1.50$) to air. Find the angle of incidence for which the angle of deviation of 90° .

Solution 31

We know that,

$$c = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{1.5}\right)$$

$$c = 41.8^\circ$$

$$\text{Maximum Deviation} = 90^\circ - 41.8^\circ \\ = 47.2^\circ$$

Thus, total internal reflection will take place.

In reflection deviation is given as:

$$D = 180^\circ - 2i$$

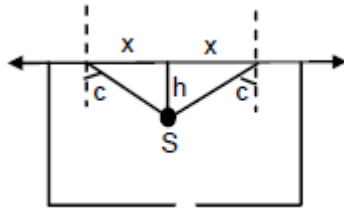
$$i = 45^\circ$$

Question 32

A point source is placed at a depth h below the surface of water (refractive index $= \mu$). (a) Show that light escape through a circular area on the water surface with its center directly above the point source. (b) Find the angle subtended by a radius of the area on the source.

Solution 32

(a) As per figure,



$$\tan A = \frac{x}{h}$$

Also,

$$\frac{x}{h} = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$$

$$\frac{\frac{1}{\mu}}{\sqrt{1 - \frac{1}{\mu^2}}}$$

$$\left\{ \begin{array}{l} \sin A = \frac{1}{\mu} \end{array} \right.$$

$$x = \frac{h}{\sqrt{\mu^2 - 1}}$$

(b) Angle $A = \sin^{-1}(1/\mu)$

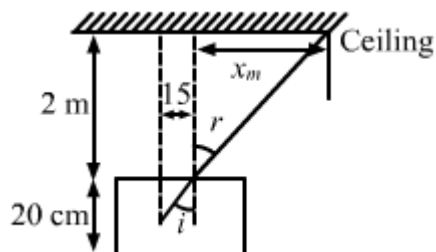
Question 33

A container contains water up to a height of 20cm and there is a point source at the center of the bottom of the container. A rubber ring of radius r floats centrally on the water surface. (a) Find the

radius of the shadow of the ring formed on the ceiling if $r=15\text{cm}$ (b) Find the maximum value of r which the shadow of the ring is formed on the ceiling refractive index of water= $4/3$.

Solution 33

$$\sin i = \frac{15}{25} = \frac{3}{5}$$



And also,

$$\frac{\sin i}{\sin r} = \frac{1}{\mu} = \frac{3}{4}$$

$$\sin r = \sin i \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{5}$$

$$\sin r = \frac{4}{5}$$

$$\sin r = \frac{\tan r}{\sqrt{1 + \tan^2 r}}$$

$$\frac{x}{\sqrt{4 + x^2}} = \frac{4}{5}$$

$$x = 2.667\text{m}$$

$$\text{Total radius} = 2.667 + 0.15 = 2.817\text{m}$$

$$\frac{\sin i \sin c}{\sin r \sin r} = \frac{R}{\sqrt{20^2 + R^2}}$$

$$\frac{3}{4} = \frac{R}{\sqrt{400 + R^2}}$$

$$R = 22.67\text{cm}$$

Question 34

Find the angle of minimum deviation for an equilateral prism made of a material of refractive index 1.732 What is the angle of incidence for this deviation?

Solution 34

Angle of minimum deviation is given as :

$$\mu = \frac{\sin\left(\frac{A + \delta m}{2}\right)}{\sin\frac{A}{2}}$$

$$1.732 = \frac{\sin\left(\frac{60 + \delta m}{2}\right)}{\sin\frac{60}{2}}$$

$$\delta m = 60^\circ$$

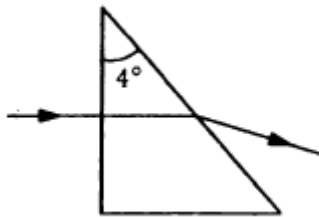
As we know,

$$\delta m = i + i' - 60^\circ$$

$$i = 60^\circ$$

Question 35

Find the angle of deviation suffered by the light ray shown in fig The refractive index $\mu=1.5$ for the prism material.



Solution 35

We know that,

$$\mu = \frac{\sin\left(\frac{A + \delta m}{2}\right)}{\sin\frac{A}{2}}$$

As angle is small,

$$\mu = \frac{A + \delta m}{2}$$

$$1.5 = \frac{4^\circ + \delta m}{4^\circ}$$

$$\delta m = 2^\circ$$

Question 36

A light ray, going through a prism with the angle of prism 60° , is found to deviate by 30° . What limit on the refractive index can be put from these data?

Solution 36

We know that,

$$\mu = \frac{\sin\left(\frac{A + \delta m}{2}\right)}{\sin\frac{A}{2}}$$

$$\mu = \frac{\sin\left(\frac{60 + \delta m}{2}\right)}{\sin\frac{60}{2}}$$

$$\mu = 2\sin\left(\frac{60 + \delta m}{2}\right)$$

As ,

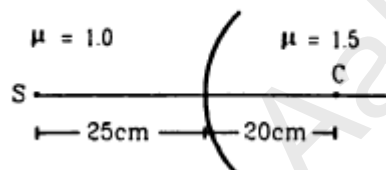
$$\delta = 30^\circ$$

$$\delta m \leq 30^\circ$$

$$\mu \leq \sqrt{2}$$

Question 37

Locate the image by refractive index in the situation shown in figure



Solution 37

We know,

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{-25} = \frac{1.5 - 1}{20}$$

$$v = -100\text{cm}$$

Question 38

A spherical surface of radius 30cm separates two transparent media A and B with refractive index 1.33 and 1.48 respectively. The medium A is on the convex side of the surface. Where should a point object be placed in medium A so that the paraxial rays become parallel after refractive at the surface?

Solution 38

We know,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

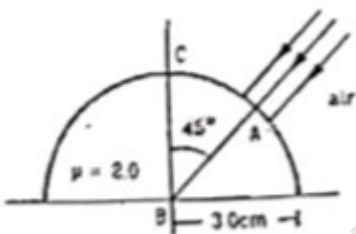
$$\frac{1.48}{\infty} - \frac{1.33}{u} = \frac{1.48 - 1.33}{30}$$

$$u = -266\text{cm}$$

Chapter 18 - Geometrical Optics Exercise 415

Question 39

Figure shows a transparent hemisphere of radius 3.0cm made of a material of refractive index 2.0 (a) A narrow beam of parallel rays is incident on the hemisphere as shown in the figure. Are the rays totally reflected at the plane surface? (b) Find the image formed by the refractive index at the first surface. (c) Find the image formed by the reflection or by the refraction at the plane surface. (d) Trace qualitatively the final rays as they come out the hemisphere.



Solution 39

We know that,

$$c = \sin^{-1} \left(\frac{1}{\mu_2} \right)$$

And

$$\mu_2 = 2$$

Therefore,

$$c = 30^\circ$$

$$i > c$$

Therefore, rays reflected internally.

We know,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{2}{v} - \frac{(-1)}{\infty} = \frac{2-1}{3}$$

$$v = 6\text{cm}$$

Question 40

A small object is embedded in a glass sphere $\mu=1.5$ of radius 5.0cm at a distance 1.5cm left to the center. Locate the image of the object as seen by an observer standing (a) to the left of the sphere and (b) to the right of the sphere.

Solution 40

(a) Image distance from left $= -(5-3.5)$
 $= -3.5\text{cm}$

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} + \frac{1.5}{3.5} = \frac{-(5-3.5)}{5}$$

$$v = -3\text{cm}$$

Image will be formed at 2cm left from center.

(b) Image distance from right $u = -(5+1.5)$
 $= -6.5\text{cm}$

We know,

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} + \frac{1.5}{6.5} = \frac{1-1.5}{-5}$$

$$v = -7.65\text{cm}$$

Image will be formed at 2.65 cm towards left from center

Question 41

A biconvex thick lens is constructed with glass ($\mu=1.50$) Each of the surface has a radius of 10cm and the thickness at the middle is 5cm. Locate the image of an object placed far away from the lens.

Solution 41

When rays refract through A then,

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{0}{v} - 0 = \frac{1.5}{10}$$

$$v = 30\text{cm}$$

Now for second refraction,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} - \frac{1.5}{25} = \frac{1 - 1.5}{-10}$$

$$v = 9.1 \text{ cm}$$

Question 42

A narrow pencil of parallel light is incident normally on a solid transparent sphere of radius r . What should be the refractive index if the pencil is to be focused (a) at the surface of the sphere, (b) at the center of the sphere?

Solution 42

Focused at the surface

We know,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{\mu_1}{2r} - \frac{1}{\infty} = \frac{\mu_2 - 1}{r}$$

$$\mu_2 = 2$$

Focused at the center

We know,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{\mu_2}{r} - \frac{1}{\infty} = \frac{\mu_2 - 1}{r}$$

$$\mu_2 = \mu_2 - 1$$

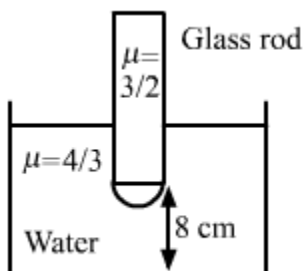
This is not possible. Thus, image cannot be focused at the center of the sphere.

Question 43

One end of a cylindrical glass rod ($\mu=1.5$) of radius 1.0cm is rounded in the shape of a hemisphere. The rod is immersed in water ($\mu=4/3$) and an object is placed in the water along the axis of the rod at a distance of 8.0cm from the rounded edge. Locate the image of the object.

Solution 43

We know,



$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$

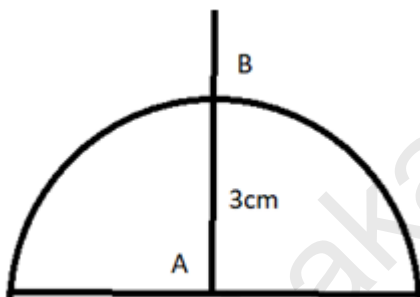
$$\frac{3}{2v} - \frac{4}{3 \times 8} = \frac{3/2 - 4/3}{1}$$

$$v = \infty$$

Question 44

A paperweight in the form of a hemisphere of radius 3.0cm is used to hold down a printed page. An observer looks at the page vertically through the paperweight. At what height above the page will the printed letters near the center appear to the observer?

Solution 44



When image is refracted by A

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{0} = \frac{\mu_2 - \mu_1}{\infty}$$

$$v = 0$$

Now when image is refracted at B,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

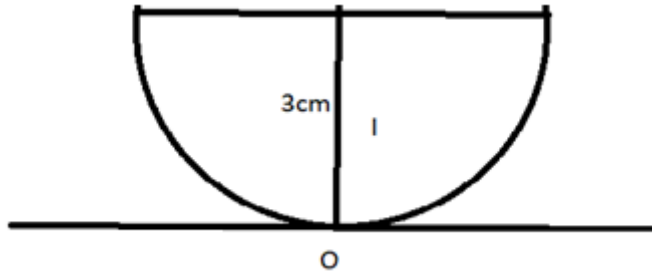
$$\frac{1}{v} - \frac{3}{2 \times 3} = \frac{1 - 3/2}{-3}$$

$$v = -3\text{cm}$$

Question 45

Solve the problem if the paperweight is inverted at its place so that the spherical surface touches the paper.

Solution 45



Shift in image is given as:

$$\Delta s = t \left(1 - \frac{1}{\mu} \right)$$

$$\Delta s = 3 \left(1 - \frac{1}{1.5} \right)$$

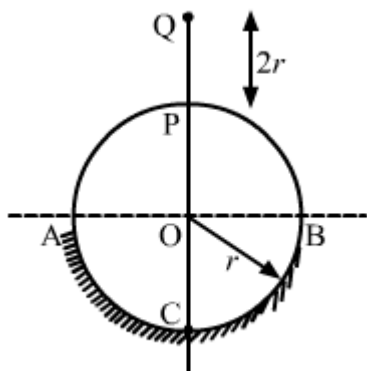
$$\Delta s = 1\text{cm}$$

Question 46

A hemispherical portion of the radius of a solid glass ($\mu=1.5$) of radius r is silvered to make the inner side reflecting. An object is placed in the axis of the hemisphere at a distance $3r$ from the center of the sphere. The light from the object is refracted of the sphere. The light from the object is refracted at the unsilvered part, and then reflected from the object is refracted from the silvered part and again refracted at the unsilvered part. Locate the final image formed.

Solution 46

When refraction takes place from ACB then,



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{-2r} = \frac{1.5 - 1}{r}$$

$$v = \infty$$

Now, refraction takes of reflected image above so $v=f=r/2$ and $u=-3r/2$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} - \frac{1.5}{-3r/2} = \frac{1 - 1.5}{-r}$$

Hence,

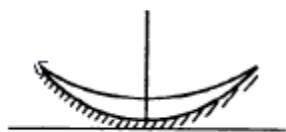
$$v = -2r$$

Therefore, Image is formed at E.

Question 47

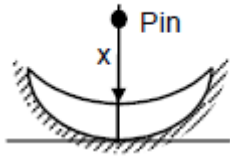
The convex surface of a thin concave convex lens of glass of refractive index 1.5 has a radius of curvature 20cm. The concave surface has a radius of curvature 60cm.

The convex side is silvered side and placed on a horizontal surface as shown in figure (a). Where should a pin be placed on the axis so that its image is formed at the same place? (b) If the concave part is filled with water (refractive index = 4/3) Find the distance through which the pin should be moved so that the image of the pin again coincides with the pin.



Solution 47

For 1st refraction,



We know

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{-x} = \frac{1.5 - 1}{-60}$$

$$v = \frac{-180x}{120 + x}$$

By using mirror formula,

$$\frac{1}{v} + \frac{1}{\frac{-180x}{120 + x}} = \frac{1}{-10}$$

$$v = \frac{-180x}{120 - 17x}$$

Now, for 2nd refraction,

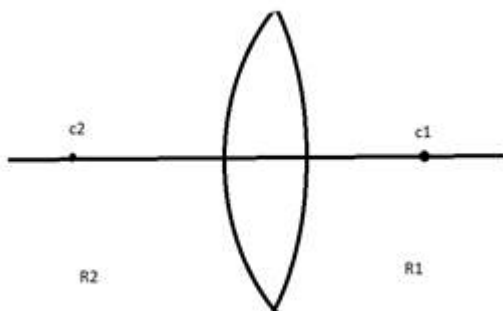
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{-x} - \frac{1.5}{\frac{-180x}{120 - 17x}} = \frac{-0.5}{-60}$$

$$x = 15\text{cm}$$

Question 48

A double convex lens has focal length 25cm. The radius of curvature of one of the surface is double of the other. Find the radii, if the refractive index of the material of the lens is 1.5.



Solution 48

We know,

$$\frac{1}{f} = (\mu - 1) \left[\frac{1}{R1} - \frac{1}{R2} \right]$$

$$\frac{1}{25} = (15 - 1) \left[\frac{1}{R} - \frac{1}{2R} \right]$$

$$R = 18.75cm$$

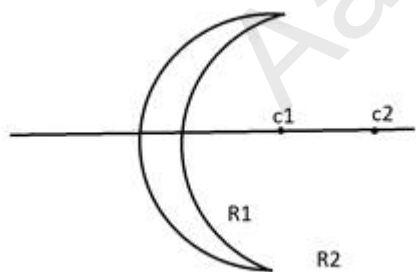
$$R = R1 = 18.75cm$$

$$R1 = R * 2 = 2 * 18.75 = 37.5cm$$

Question 49

The radii of curvature of a lens are 20cm and 30cm. The material of the lens has a refractive index 1.6. Find the focal length of the lens (a) if it is placed in air, and (b) If it is placed in water (refractive index = 1.33).

μ_w lens refractive index = μ_g



Solution 49

Air :

$$\frac{1}{f} = (\mu_g - 1) \left[\frac{1}{R1} - \frac{1}{R2} \right]$$

$$\frac{1}{f} = (1.6 - 1) \left[\frac{1}{20} - \frac{1}{30} \right]$$

$$f = 100 \text{ cm}$$

Air :

$$\frac{1}{f} = (\mu_w - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\frac{1}{f} = (1.6/1.33 - 1) \left[\frac{1}{20} - \frac{1}{30} \right]$$

$$f = 300 \text{ cm}$$

Question 50

Lenses are constructed by a material of refractive index 1.50. The magnitude of the radii of curvature are 20cm and 30cm. Find the focal length of the possible lenses with the above specification.

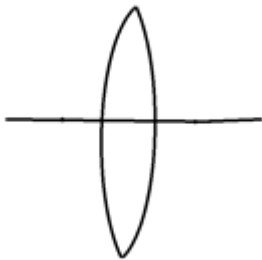
Solution 50

There are four types of lenses:

We know that,

$$\frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

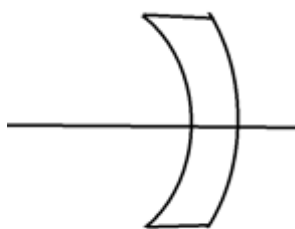
(1) If $R_1 = +ve$ and $R_2 = -ve$



$$\frac{1}{f} = (1.5 - 1) \left[\frac{1}{20} - \frac{1}{-30} \right]$$

$$f = 24 \text{ cm}$$

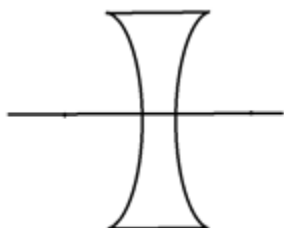
(2) If $R_1 = -ve$ and $R_2 = -ve$



$$\frac{1}{f} = (15 - 1) \left[\frac{1}{-20} - \frac{1}{-30} \right]$$

$$f = -120 \text{ cm}$$

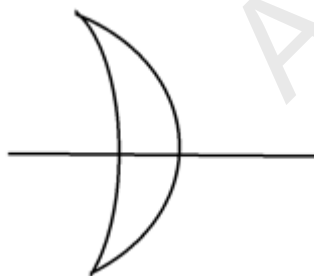
(3) If $R_1 = -ve$ and $R_2 = +ve$



$$\frac{1}{f} = (15 - 1) \left[\frac{1}{-20} - \frac{1}{30} \right]$$

$$f = -24 \text{ cm}$$

(4) If $R_1 = +ve$ and $R_2 = +ve$



$$\frac{1}{f} = (15 - 1) \left[\frac{1}{20} - \frac{1}{30} \right]$$

$$f = 120 \text{ cm}$$

Question 51

A thin lens made of a material of refractive index μ_2 has a medium of refractive index μ_1 on one side and a medium of refractive index μ_3 on the other side. The lens is biconvex and the two radii of curvature have equal magnitude R . A beam of light travelling parallel to the principal axis is incident on the lens. Where will the image be formed if the beam is incident from (a) the medium μ_1 (b) from the medium μ_3 ?

Solution 51

Beam is incident on lens, so,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{-\infty} = \frac{\mu_2 - \mu_1}{R}$$

$$v = \frac{\mu_2 R}{\mu_2 - \mu_1}$$

Now beam is refracted second time,

$$\frac{\mu_3}{v} - \frac{\mu_2}{u} = \frac{\mu_3 - \mu_2}{R}$$

$$v = \frac{-\mu_3 R}{\mu_3 - 2\mu_2 + \mu_1}$$

Therefore, Image is formed at $\frac{\mu_3 R}{\mu_3 - 2\mu_2 + \mu_1}$

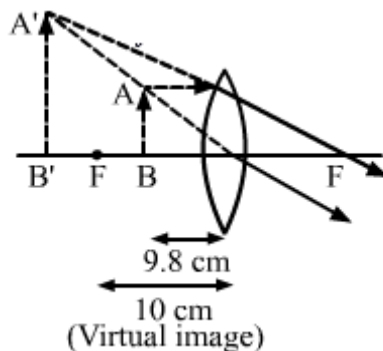
Image formed when rays are incident from medium with refractive index μ_3 is:

$$\frac{\mu_1 R}{2\mu_2 - \mu_1 - \mu_3}$$

Question 52

A convex lens has a focal length of 10cm. Find the location and nature of the image if a point object is placed on the principal axis at a distance of (a) 9.8cm, (b) 10.2cm from the lens.

Solution 52



(a) We know,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-9.8} = \frac{1}{10}$$

$$v = -490\text{cm}$$

And magnification $m = v/u = -490/-9.8 = 50\text{cm}$

As $m > 1$ Then, Image is virtual and erect.

(b) We know,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-10.2} = \frac{1}{10}$$

$$v = 510\text{cm}$$

And magnification $m = v/u = 510/-9.8 = -52.04\text{cm}$

As $m < 1$. Then, Image is real and inverted.

Question 53

A slide projector has to project a 35mm slide (35mm × 23mm) on a 2m × 2m screen at a distance of 10 m from the lens. What should be the focal length of the lens in the project?

Solution 53

In case of projector,

$$m = \frac{v}{u}$$

$$200 = \frac{v}{u}$$

$$u = 17.5\text{cm}$$

We know,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

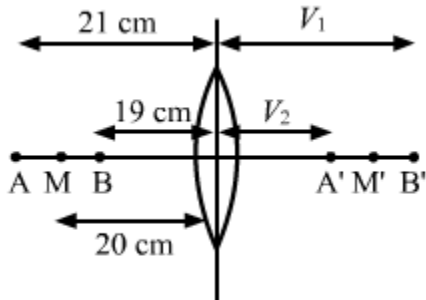
$$\frac{1}{100} - \frac{1}{-17.5} = \frac{1}{f}$$

$$f = 17.19\text{cm}$$

Question 54

A particle executes a simple harmonic motion of amplitude 1.0cm along the principal axis of a convex lens of focal length 12cm. The mean position of oscillation is at 20cm. from the lens. Find the amplitude of oscillation of the image of the particle.

Solution 54



$$\frac{1}{v_B} - \frac{1}{u_1} = \frac{1}{f}$$

$$\frac{1}{v_B} - \frac{1}{-19} = \frac{1}{12}$$

$$v_B = 32.57 \text{ cm}$$

When object is at A, then

$$\frac{1}{v_A} - \frac{1}{u_2} = \frac{1}{f}$$

$$\frac{1}{v_A} - \frac{1}{21} = \frac{1}{12}$$

$$v_A = 28 \text{ cm}$$

Now, Amplitude of vibration is given as:

$$\frac{v_A - v_B}{2}$$

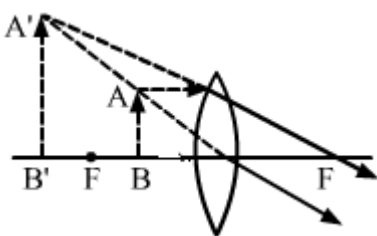
$$= \frac{32.57 - 28}{2}$$

$$\text{Amplitude} = 2.28 \text{ cm}$$

Question 55

An extended object is placed at a distance of 5.0cm from a convex lens of focal length 8cm (a) draw a ray diagram (to the scale) to locate the image and from this, measure the distance of the image from the lens. (b) Find the position of the image from the lens formula and see how close the drawing is to the correct result.

Solution 55



We know,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-5} = \frac{1}{8}$$

$$v = -13.3 \text{ cm}$$

As image formed on left side.

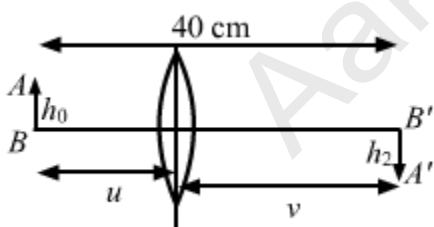
Therefore, Virtual image

Question 56

A pin of length 2cm is placed perpendicular to the principal axis of a converging lens. An inverted image of size 1cm is formed at a distance of 40cm from the pin. Find the focal length of the lens and its distance from the pin.

Solution 56

$$m = \frac{-v}{u} = \frac{hi}{ho}$$



$$\frac{-v}{u} = \frac{1}{2}$$

$$u = -2v \quad (1)$$

And

$$-u + v = 40 \quad (2)$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{2v} = \frac{1}{f}$$

$$\frac{2v}{3} = f \quad (3)$$

From equation (1) and (2)

$$-u + v = 40$$

$$2v + v = 40$$

$$v = 40/3$$

And from eq.(3)

$$f = \frac{2v}{3} = \frac{2 \times 40/3}{3} = 8.89 \text{ cm}$$

From eq (1) and (3)

$$u = 26.7 \text{ cm}$$

Chapter 18 - Geometrical Optics Exercise 416

Question 57

A convex lens produces a double size real image when an object is placed at a distance of 18cm from it. Where should the object be placed to produce a triple size real image?

Solution 57

We know,

$$m = \frac{v}{u}$$

$$-2 = \frac{v}{u}$$

$$-2 \times -18 = v$$

$$36 = v$$

Now,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{36} - \frac{1}{-18} = \frac{1}{f}$$

$$f = 12\text{cm}$$

When the image is tripled then,
We know,

$$m = \frac{v}{u}$$

$$-3 = \frac{v}{u}$$

$$-3u = v$$

Now,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

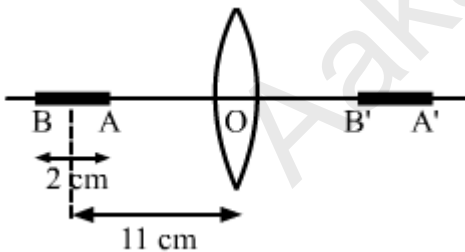
$$\frac{1}{-3u} - \frac{1}{u} = \frac{1}{12}$$

$$u = -16\text{cm}$$

Question 58

A pin of length 2cm lies along the principal axis of a converging lens, the center being at a distance of 11cm from the lens. The focal length of the lens is 6cm. Find the size of the image.

Solution 58



We know that,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

For OB'

$$\frac{1}{v} - \frac{1}{-10} = \frac{1}{6}$$

$$v = 15\text{cm}$$

For OA'

$$\frac{1}{v} - \frac{1}{-12} = \frac{1}{6}$$

$$v = 12\text{cm}$$

Now,

$$A'B' = OB' - OA'$$

$$= 15 - 12$$

$$A'B' = 3\text{cm}$$

Question 59

The diameter of the sun is $1.4 \times 10^9\text{m}$ and its distance from the earth is $1.5 \times 10^{11}\text{m}$. Find the radius of the image of the sun formed by a lens of focal length 20cm.

Solution 59

We know,

$$m = \frac{v}{u} = \frac{hi}{ho}$$

$$\frac{20 \times 10^{-2}}{1.5 \times 10^{11}} = \frac{hi}{1.4 \times 10^9}$$

$$hi = 1.8\text{mm}$$

$$r = \frac{hi}{2} = \frac{1.8}{2}$$

$$r = 0.9\text{mm}$$

Question 60

A 5.0 diopter lens forms a virtual image which is 4 times the object placed perpendicular on the principal axis of the lens. Find the distance of the object from the lens.

Solution 60

As given,

$$4 = \frac{v}{u}$$

$$v = 4u \quad (1)$$

And

$$f = \frac{1}{p} = \frac{1}{5} = 20\text{cm} \quad (2)$$

Now,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{4u} - \frac{1}{u} = \frac{1}{20}$$

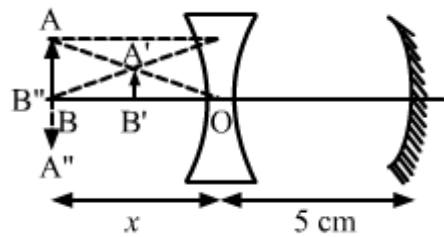
$$u = -15\text{cm}$$

Object is at 15cm from the lens.

Question 61

A diverging lens of focal length 20cm and a converging mirror of focal length 10cm are placed coaxially at a separation of 5cm. Where should an object be placed so that a real image is formed at the object itself?

Solution 61



We know,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-x} = \frac{1}{-20}$$

$$v = \frac{-20x}{20 + x}$$

Therefore, For concave mirror,

$$u = \frac{-20x}{20 + x} + 5$$

$$u = -\left[\frac{25x + 100}{x + 20}\right]$$

Now, Using mirror equation for concave mirror,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-\left[\frac{25x+100}{x+20}\right]} + \frac{1}{v} = \frac{1}{-10}$$

$$v = \left[\frac{50(x+4)}{3x-20}\right]$$

Now, in case of second refraction,

$$u = -\left(\left[-\frac{50(x+4)}{3x-20}\right] + 5\right)$$

And

$$v = x$$

Therefore,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-20} = \frac{1}{x} = \frac{1}{-\left(\frac{50(x+4)}{3x-20}\right)}$$

$$x = 60\text{cm}$$

Question 62

A converging lens of focal length 12cm and a diverging mirror of focal length 7.5cm are placed 5cm apart with their principal axis coinciding. Where should an object be placed so that its image falls on itself?

Solution 62

Let x be the object distance.

First using lens formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{-12} = \frac{1}{v} + \frac{1}{-x}$$

$$v = \frac{-12x}{12+x}$$

Now,

$$u = \frac{-12x}{12+x} - 5 = -\left(\frac{17x+60}{12+x}\right)$$

Now,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{-7.5} = \frac{1}{v} + \frac{1}{-\left(\frac{17x+60}{12+x}\right)}$$

$$u = \frac{50(x+4)}{3x-20}$$

Now again it will refract through lens,

$$u = -\left[5 - \frac{50(x+4)}{3x-20}\right]$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

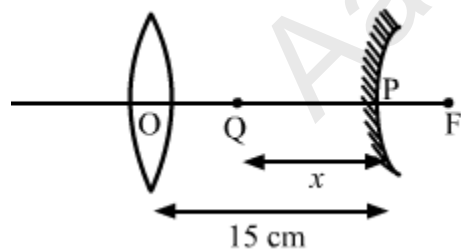
$$\frac{1}{-20} = \frac{1}{x} + \frac{1}{-\left[5 - \frac{50(x+4)}{3x-20}\right]}$$

$$x = 30\text{cm}$$

Question 63

A converging lens and a diverging mirror are placed at a separation of 15cm. The focal length of the lens is 25cm and that of the mirror is 40cm. Where should a point source be placed between the lens and the mirror so that the light, after getting reflected by the mirror and then getting transmitted by the lens, comes out parallel to the principal axis?

Solution 63



We know,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{40} = \frac{1}{10} + \frac{1}{-x}$$

$$x = \frac{40}{3}$$

Therefore,
Object distance = $15 - 40/3$
= 1.67 cm

Question 64

A converging lens focal length 15 cm and a converging mirror of focal length 10 cm are placed 50 cm apart with common principal axis. A point source is placed in between the lens and the mirror at a distance of 40 cm from the lens. Find the location of the images formed.

Solution 64

(I) Image formed due to direct transmission through lens.

We know,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{15} = \frac{1}{v} + \frac{1}{-40}$$

$$v = 24 \text{ cm}$$

(II) Image due to reflection of rays by mirror and then transmission through lens.

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

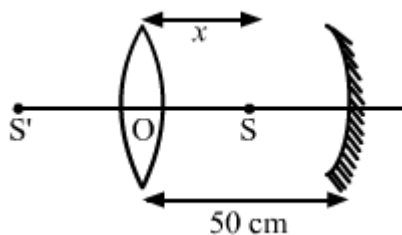
$$\frac{1}{15} = \frac{1}{v} - \frac{1}{\infty}$$

$$v = 15 \text{ cm}$$

Question 65

Consider the situation describe in the previous problem. Where should a point source be placed on the principal axis so that the two image form at the same place?

Solution 65



In case of lens,

$$\frac{1}{f1} = \frac{1}{v1} - \frac{1}{u1}$$

$$\frac{1}{15} = \frac{1}{v1} - \frac{1}{-x}$$

$$v1 = \frac{15x}{x - 15}$$

In case of mirror,

$$\frac{1}{f2} = \frac{1}{v2} - \frac{1}{u2}$$

$$\frac{1}{-10} = \frac{1}{v2} - \frac{1}{-(50 - x)}$$

$$v1 = \frac{10(50 - x)}{x - 40}$$

Now, distance between mirror and lens is,

$$v1 - v2 = 50$$

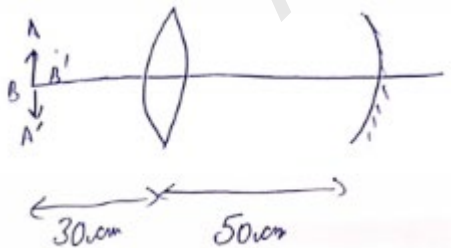
$$\frac{15x}{x - 15} - \frac{10(50 - x)}{x - 40} = 50$$

$$x = 30\text{cm}$$

Question 66

A converging lens of focal length 15cm and a converging mirror of focal length 10cm are placed 50cm apart. If a pin of length 2cm is placed 30cm from the lens farther away from the mirror, where will the final image form and what will be the size of the final image?

Solution 66



We know,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{15} = \frac{1}{v} - \frac{1}{-30}$$

$$v = 30\text{cm}$$

Image will be real and inverted and it will be at a distance of 20cm from mirror and $f=10\text{cm}$.
Hence,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{-10} = \frac{1}{v} + \frac{1}{-20}$$

$$v = -20\text{cm}$$

Now again rays will pass through lens, So

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{15} = \frac{1}{v} - \frac{1}{-30}$$

$$v = -30\text{cm}$$

Therefore, Image is formed at the place of object. It real and inverted and of same size as object.

Question 67

A point object is placed on the principal axis of a convex lens ($f=15\text{cm}$) at a distance of 30cm from it. A glass Plate ($\mu=1.50$) of thickness 1cm is placed on the other side of the lens perpendicular to the axis, Locate the image of the point object.

Solution 67

We know,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{15} = \frac{1}{v} - \frac{1}{-30}$$

Now, shift of the image due to slab is:

$$\Delta t = t \left(1 - \frac{1}{\mu g} \right)$$

$$= 1 - \frac{1}{1.5}$$

$$\Delta t = 0.33\text{cm}$$

Image is formed at $30+0.33=30.33\text{cm}$.

Question 68

A convex lens of focal length 20cm and a concave lens of focal length 10cm are placed 10cm apart with their principal axis coinciding. A beam of light travelling parallel to the principal axis and having a beam diameter 5mm, is incident on the combination. Show that the emergent beam is parallel to the incident one. Find the beam diameter of the emergent beam.

Solution 68

For convex lens,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{\infty}$$

$$v = 20\text{cm}$$

Therefore, For concave lens,

$$u = 10\text{cm}$$

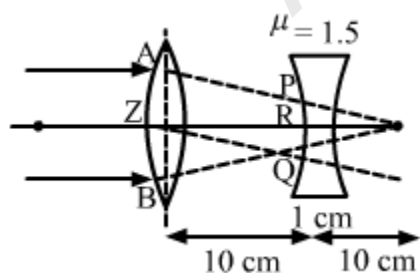
And

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{-10} = \frac{1}{v} - \frac{1}{10}$$

$$v = \infty$$

Thus, when rays pass through concave lens become parallel
According to fig.



$$\frac{AO}{OI} = \frac{PR}{RI}$$

$$\frac{0.5}{20} = \frac{PR}{10}$$

$$PR = 2.5\text{mm}$$

Question 69

A diverging lens of focal length 20 cm and a converging lens of focal length 30 cm are placed 15cm apart with their principal axes coinciding. Where should an object be placed on the principal axis so that its image is formed at infinity?

Solution 69

Net focal length is:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$= \frac{1}{30} + \frac{1}{-20} - \frac{15}{30 \times -20}$$

$$F = 120\text{cm}$$

Equivalent lens is converging lens.

Now,

$$d_1 = \frac{d \times F}{f_1} = \frac{15 \times 120}{30}$$

$$d_1 = 60\text{cm}$$

$$d_2 = \frac{d \times F}{f_2} = \frac{15 \times 120}{20}$$

$$d_2 = 90\text{cm}$$

Distance of object from diverging lens is:

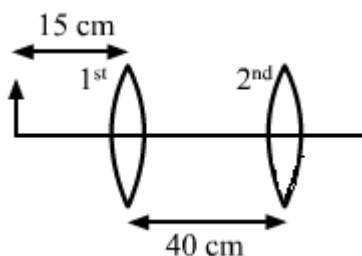
$$\text{For } d_1 \quad 120 - 60 = 60\text{cm}$$

$$\text{For } d_2 \quad 120 + 90 = 210\text{cm}$$

Question 70

A 5 mm high pin is placed at a distance of 15cm from a convex lens focal length 10cm. A second lens of focal length 5cm is placed 40cm from the first lens and 55cm from the pin. Find (a) the position of the final image, (b) its nature and (c) its size.

Solution 70



For lens I
We know,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{-10} = \frac{1}{v} - \frac{1}{-15}$$

$$v = 30\text{cm}$$

Magnification

$$m = \frac{v}{u} = \frac{hi}{ho}$$

$$\frac{30}{-15} = \frac{hi}{5}$$

$$hi = -10\text{m}$$

Image is inverted

Size of image is 10mm.

For lens II

We know,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{5} = \frac{1}{v} - \frac{1}{-10}$$

$$v = 10\text{cm}$$

Magnification

$$m = \frac{v}{u} = \frac{hi}{ho}$$

$$\frac{10}{10} = \frac{hi}{-10}$$

$$h_i = 10\text{mm}$$

Image is real and erect.

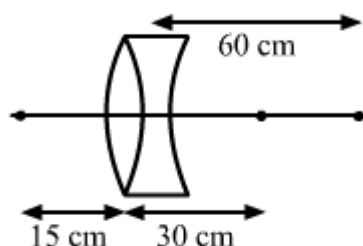
Size of image is 10mm.

Question 71

A point object is placed at a distance of 15cm from a convex lens. The image is formed on the other side at a distance of 30cm from the lens. When a concave lens is placed in contact with the convex lens, the image shifts away further by 30cm. Calculate the focal lengths of the two lenses.

Solution 71

We know,



$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$

$$= \frac{1}{30} - \frac{1}{-15}$$

$$f_1 = 10\text{cm}$$

$$\text{Also } v = 30 + 30 = 60\text{cm}$$

$$v_1 = 30\text{cm}$$

Now,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}$$

Hence

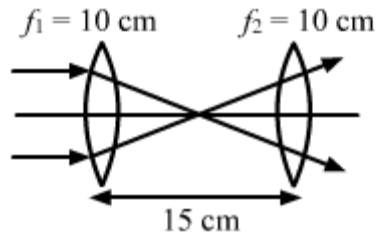
$$f = 10\text{ cm (convex lens)}$$

$$F = 60\text{ cm (concave lens)}$$

Question 72

Two convex lenses, each of focal length 10cm, are placed at a separation of 15cm with their principal axes coinciding (a) show that a light beam coming parallel to the principal axis diverges as it comes out of the lens system. (b) Find the location of the virtual image formed by the lens system of an object placed far away. (c) Find the focal length of the equivalent lens. (note that the sign of the focal length is positive although the lens system actually diverges a parallel beam incident on it.)

Solution 72



Beam will diverge after refracting from both lenses.

1st lens:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-\infty}$$

$$v = 10\text{cm}$$

2nd lens:

$$\frac{1}{f} = \frac{1}{v'} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{v'} - \frac{1}{-(15 - 10)}$$

$$v' = 10\text{cm}$$

Therefore, Image is formed at distance of 5cm for 1st convex lens.

(c) Equivalent focal length is given as :

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$= \frac{1}{10} + \frac{1}{10} - \frac{15}{10 * 10}$$

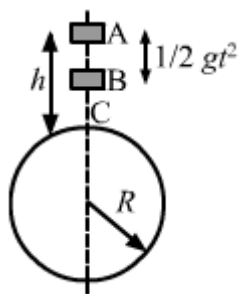
$$F = 20\text{cm}$$

Question 73

A ball is kept at a height h above the surface of a heavy transparent sphere made of a material of refractive index μ . The radius of the sphere is R . At $t=0$, the ball is dropped to fall normally on the

sphere. Find the speed of the image formed as a function of time for $t < \sqrt{\frac{2h}{g}}$. Consider only the image by a single refraction.

Solution 73



$$AB = \frac{1}{2}gt^2$$

$$\text{And } BC = h - \frac{1}{2}gt^2$$

$$\text{And also } u = -\left(h - \frac{1}{2}gt^2\right)$$

Now we know,

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$$

$$\frac{\mu}{v} - \frac{1}{-\left(h - \frac{gt^2}{2}\right)} = \frac{\mu - 1}{R}$$

$$v = \frac{\mu R \left(h - \frac{gt^2}{2}\right)}{(\mu - 1) \left(h - \frac{gt^2}{2}\right) - R}$$

And velocity

$$v = \frac{dv}{dt} = \frac{d}{dt} \left[\frac{\mu R (h - \frac{gt^2}{2})}{(\mu - 1) \left(h - \frac{gt^2}{2}\right) - R} \right]$$

$$v = \frac{\mu R^2 gt}{(\mu - 1) \left(h - \frac{gt^2}{2}\right) - R}$$

Question 74

A particle is moving at a constant speed V from a large distance towards a concave mirror of radius R along its principal axis. Find the speed of the image formed by the mirror as a function of the distance x of the particle from the mirror.

Solution 74

We know,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-x} + \frac{1}{v} = \frac{1}{-R/2}$$

$$v = \frac{xR}{R - 2x}$$

And velocity $V = dv/dt$

$$= \frac{d}{dt} \left[\frac{xR}{R - 2x} \right]$$

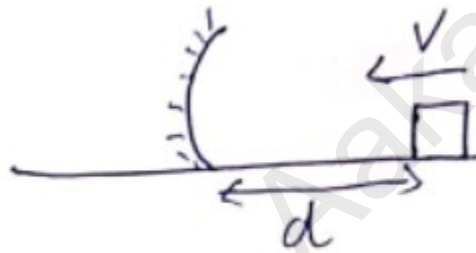
$$v = \frac{R[vR - 2xV + 2xV]}{(R - 2x)(R - 2x)}$$

Question 75

A small block of mass m and a concave mirror of radius R fitted with a stand lie on a smooth horizontal table with a separation d between them. The mirror together with its stand has a mass m . The block is pushed at $t=0$ towards the mirror so that it starts moving towards the mirror at a constant speed V and collides with it. The collision is perfectly elastic. Find the velocity of the image (a) at a time $t < d/V$, (b) at a time $t > d/V$.

Solution 75

$t < d/V$



as we know,

$$vi = \frac{v \times R^2}{2 \times \text{distance} - R^2}$$

$$vi = \frac{v \times R^2}{2(d - vt) - R^2}$$

$t > d/v$

as we know that,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-R/2} = \frac{1}{v} + \frac{1}{(d - vt)}$$

$$v = \frac{-R(d - vt)}{R - 2(d - vt)}$$

$$V = \frac{dv}{dt} = \frac{d}{dt} \left[\frac{-R(d - vt)}{R - 2(d - vt)} \right]$$

After solving we get,

$$vi = \frac{-vR^2}{(R + 2y)(R + 2y)}$$

Absolute velocity is given as

$$v \left[1 - \frac{R^2}{(R + 2y)^2} \right]$$

$$v \left[1 - \frac{R^2}{2(vt - d) - (R)^2} \right]$$

$$V = v \left[1 - \frac{R^2}{2(vt - d) - (R)^2} \right]$$

Question 76

A gun of mass M fires a bullet of mass m with a horizontal speed V. The gun is fitted with a concave mirror of focal length f facing towards the receding bullet. Find the speed of separation of the bullet and the image just after the gun was fired.

Solution 76

Recoil velocity is given as,

$$v = \frac{mv}{M}$$

At time 't' position = $vt + \frac{mv}{M}t$

$$\left(1 + \frac{m}{M}\right)vt$$

And $u = kv$

Also,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{-f} = \frac{1}{v} - \frac{1}{kVt}$$

$$v = \frac{kVtf}{f - kVt}$$

Velocity is given as:

$$V' = \frac{dv}{dt} = \frac{d}{dt} \left[\frac{kVtf}{f - kVt} \right]$$

$$V' = \frac{kVt^2}{(f - kVt)^2}$$

Thus velocity of separation is given as:

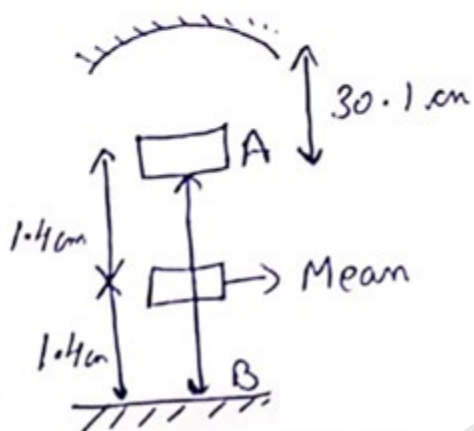
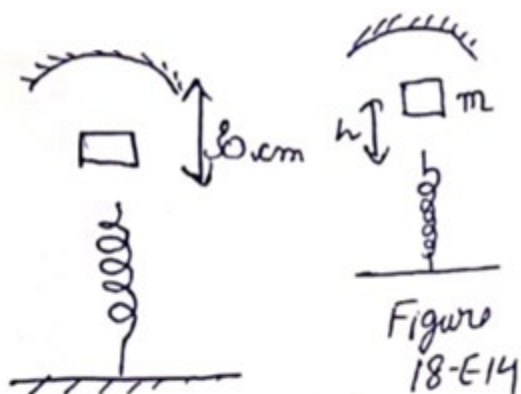
$$V'' = v + \frac{mv}{M} + \frac{kVt^2}{(f - kVt)^2}$$

If $t=0$

$$V'' = v + \frac{mv}{M} + kv$$

Question 77

A mass $m=50\text{g}$ is dropped on a vertical spring of spring constant 500N/m from a height $h=10\text{cm}$ as shown in fig. The mass sticks to the spring and executes simple harmonic oscillation after that. A concave mirror of focal length 12cm facing the mass is fixed with its principal axis coinciding with the line of motion of the mass, its pole being at a distance of 30cm from the free end of the spring. Find the length in which the image of the mass oscillates.



Solution 77

In case of equilibrium,

$$mg = kx$$

$$x = mg/k = 0.1 \text{ cm}$$

Thus, mean position = $30 + 0.1 = 30.1 \text{ cm}$

Max. Compression = 8

And work done = $KE_2 - KE_1$

$$mg(h + \delta) - \frac{k\delta^2}{2} = 0 - 0$$

$$\delta = 0.015 \text{ m} = 1.5 \text{ cm}$$

From fig2

Position of B = 31.3 cm

Amplitude = $31.5 - 30.1 = 1.4$

Position of A = $30.1 - 1.4 = 28.7 \text{ cm}$

For A,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-31.5} + \frac{1}{v} = \frac{1}{-12}$$

$$v = -19.38 \text{ cm}$$

For B,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

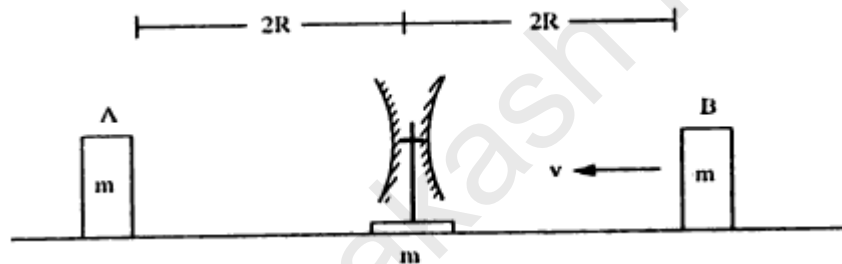
$$\frac{1}{-28.7} + \frac{1}{v} = \frac{1}{-12}$$

$$v = -20.62 \text{ cm}$$

Therefore, Vibration length = $20.62 - 19.38 = 1.24 \text{ cm}$

Question 78

Two concave mirrors of equal radii of curvature R are fixed on a stand facing opposite directions. The whole system has a mass m and is kept on a frictionless horizontal table (fig.)

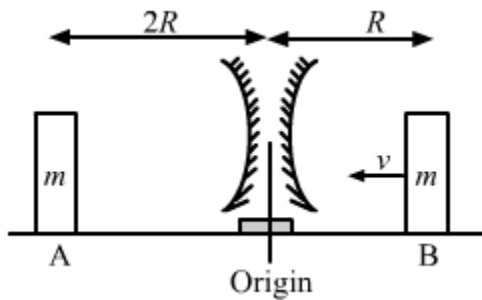


Two block A and B each mass m , are placed on the two sides of the stand. At $t=0$, the separation between A and the mirror is $2R$ and also the separation between B and the mirror is $2R$. The block B moves towards the mirror at a speed v . All collisions which takes place are elastic. Taking the original position of the mirror stand system to be $x=0$ and X-axis along AB, find the position of the images of A and B at $t=$

(a) R/v (b) $3R/v$ (c) $5R/v$

Solution 78

In time $t=R/v$ the mass B must have moved $v \times R/v = R$ towards the mirror
For block B



a)

$$u = -R \text{ and } f = \frac{-R}{2}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

On substituting we get

$$v = -R$$

For block A

$$u = -2R, f = -R/2$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

On substituting we get

$$v = \frac{-2R}{3}$$

Therefore with respect to given coordinate system

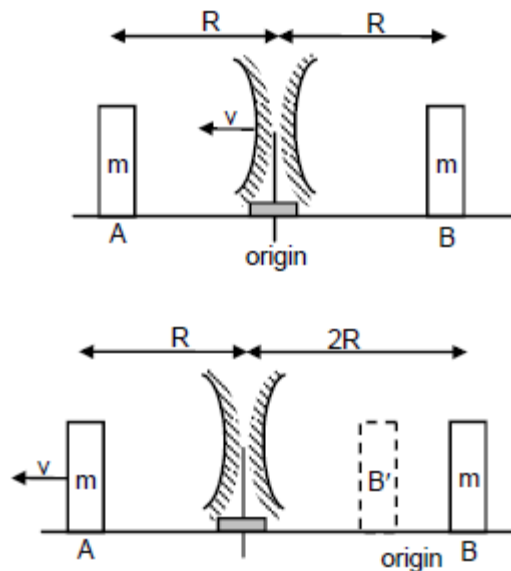
Position of A and B are $\frac{-2R}{3}$ and R respectively from origin

b)

When $t = \frac{3R}{v}$, the block B after colliding with mirror

stand must have come to rest and distance travelled by mirror should be R towards left from its initial position

so



For A

$u = -R$ and $f = -R/2$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

On substituting we get

$$v = -R$$

Hence position as per the coordinate system would be

$$x = -2R$$

For block B

Image is at same place as it is R distance from the mirror

Hence position of image is 0

Therefore

Position of images of A and B is $-2R$ and 0.

C)

Similarly it can be proved that at time $t = 5R/v$,

the position of the blocks will be

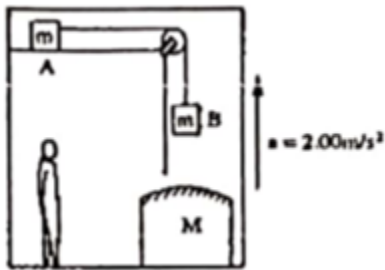
$-3R$ and $-4R/3$ respectively

Chapter 18 - Geometrical Optics Exercise 417

Question 79

Consider the situation shown in figure (18-E16). The elevator is going up with an acceleration of 2.00 m/s^2 and the focal length of the mirror is 12.0 cm . All the surfaces are smooth and the pulley is light. The mass pulley system is released from rest (with respect to the elevator) at $t=0$ when the

distance of B from the mirror is 42cm. Find the distance the image of the block B and the mirror at $t=0.200s$. Take $g=10m/s^2$.



Solution 79

According to given figure:

$$T - mg + ma - 2m = 0 \quad (1)$$

$$\text{And } T - ma = 0 \quad (2)$$

From eq (1) and (2)

$$2ma - mg - 2m = 0$$

$$2ma = m(g + 2)$$

$$A = 10 + 2/2 = 6m/s^2$$

And distance travelled is given as:

$$S = ut + at^2/2$$

$$S = 0 + 1/2 \times 6 \times (0.2)^2$$

$$S = 12cm$$

$$\text{Therefore } u = -(42 - 12)$$

$$U = -30cm$$

We know,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-30} + \frac{1}{v} = \frac{1}{12}$$

$$v = 8.57cm$$