

RD Sharma Solutions for Class 11 Maths Chapter 18 Binomial Theorem

1. Using binomial theorem, write down the expressions of the following:

(i) $(2x + 3y)^5$

(ii) $(2x - 3y)^4$

(iii) $\left(x - \frac{1}{x}\right)^6$

(iv) $(1 - 3x)^7$

(v) $\left(ax - \frac{b}{x}\right)^6$

(vi) $\left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)^6$

(vii) $\left(\sqrt[3]{x} - \sqrt[3]{a}\right)^6$

(viii) $(1 + 2x - 3x^2)^5$

(ix) $\left(x + 1 - \frac{1}{x}\right)^3$

(x) $(1 - 2x + 3x^2)^3$

Solution:

(i) $(2x + 3y)^5$

Let us solve the given expression:

$$\begin{aligned} (2x + 3y)^5 &= {}^5C_0 (2x)^5 (3y)^0 + {}^5C_1 (2x)^4 (3y)^1 + {}^5C_2 (2x)^3 (3y)^2 + {}^5C_3 (2x)^2 (3y)^3 + {}^5C_4 (2x)^1 (3y)^4 + {}^5C_5 (2x)^0 (3y)^5 \\ &= 32x^5 + 5(16x^4)(3y) + 10(8x^3)(9y^2) + 10(4x^2)(27y^3) + 5(2x)(81y^4) + 243y^5 \\ &= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5 \end{aligned}$$

(ii) $(2x - 3y)^4$

Let us solve the given expression:

$$\begin{aligned} (2x - 3y)^4 &= {}^4C_0 (2x)^4 (3y)^0 - {}^4C_1 (2x)^3 (3y)^1 + {}^4C_2 (2x)^2 (3y)^2 - {}^4C_3 (2x)^1 (3y)^3 + {}^4C_4 (2x)^0 (3y)^4 \\ &= 16x^4 - 4(8x^3)(3y) + 6(4x^2)(9y^2) - 4(2x)(27y^3) + 81y^4 \end{aligned}$$

$$= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$$

$$\text{(iii)} \left(x - \frac{1}{x} \right)^6$$

Let us solve the given expression:

$$\begin{aligned} & \left(x - \frac{1}{x} \right)^6 \\ &= {}^6C_0 x^6 \left(\frac{1}{x} \right)^0 - {}^6C_1 x^5 \left(\frac{1}{x} \right)^1 + {}^6C_2 x^4 \left(\frac{1}{x} \right)^2 - {}^6C_3 x^3 \left(\frac{1}{x} \right)^3 \\ &+ {}^6C_4 x^2 \left(\frac{1}{x} \right)^4 - {}^6C_5 x^1 \left(\frac{1}{x} \right)^5 + {}^6C_6 x^0 \left(\frac{1}{x} \right)^6 \\ &= x^6 - 6x^5 \times \frac{1}{x} + 15x^4 \times \frac{1}{x^2} - 20x^3 \times \frac{1}{x^3} + 15x^2 \times \frac{1}{x^4} - 6x \times \frac{1}{x^5} + \frac{1}{x^6} \\ &= x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6} \end{aligned}$$

$$\text{(iv)} (1 - 3x)^7$$

Let us solve the given expression:

$$\begin{aligned} (1 - 3x)^7 &= {}^7C_0 (3x)^0 - {}^7C_1 (3x)^1 + {}^7C_2 (3x)^2 - {}^7C_3 (3x)^3 + {}^7C_4 (3x)^4 - {}^7C_5 (3x)^5 + {}^7C_6 (3x)^6 - {}^7C_7 (3x)^7 \\ &= 1 - 7(3x) + 21(9x^2) - 35(27x^3) + 35(81x^4) - 21(243x^5) + 7(729x^6) - 2187x^7 \\ &= 1 - 21x + 189x^2 - 945x^3 + 2835x^4 - 5103x^5 + 5103x^6 - 2187x^7 \end{aligned}$$

$$\text{(v)} \left(ax - \frac{b}{x} \right)^6$$

Let us solve the given expression:

$$\begin{aligned} &= {}^6C_0 (ax)^6 \left(\frac{b}{x} \right)^0 - {}^6C_1 (ax)^5 \left(\frac{b}{x} \right)^1 + {}^6C_2 (ax)^4 \left(\frac{b}{x} \right)^2 - {}^6C_3 (ax)^3 \left(\frac{b}{x} \right)^3 \\ &+ {}^6C_4 (ax)^2 \left(\frac{b}{x} \right)^4 - {}^6C_5 (ax)^1 \left(\frac{b}{x} \right)^5 + {}^6C_6 (ax)^0 \left(\frac{b}{x} \right)^6 \\ &= a^6 x^6 - 6a^5 x^5 \times \frac{b}{x} + 15a^4 x^4 \times \frac{b^2}{x^2} - 20a^3 b^3 \times \frac{b^3}{x^3} + 15a^2 x^2 \times \frac{b^4}{x^4} - 6ax \times \frac{b^5}{x^5} + \frac{b^6}{x^6} \\ &= a^6 x^6 - 6a^5 x^4 b + 15a^4 x^2 b^2 - 20a^3 b^3 + 15 \frac{a^2 b^4}{x^2} - 6 \frac{ab^5}{x^4} + \frac{b^6}{x^6} \end{aligned}$$

$$\text{(vi)} \left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}} \right)^6$$

Let us solve the given expression:

$$\begin{aligned} &= {}^6C_0 \left(\sqrt{\frac{x}{a}} \right)^6 \left(\sqrt{\frac{a}{x}} \right)^0 - {}^6C_1 \left(\sqrt{\frac{x}{a}} \right)^5 \left(\sqrt{\frac{a}{x}} \right)^1 + {}^6C_2 \left(\sqrt{\frac{x}{a}} \right)^4 \left(\sqrt{\frac{a}{x}} \right)^2 - {}^6C_3 \left(\sqrt{\frac{x}{a}} \right)^3 \left(\sqrt{\frac{a}{x}} \right)^3 \\ &+ {}^6C_4 \left(\sqrt{\frac{x}{a}} \right)^2 \left(\sqrt{\frac{a}{x}} \right)^4 - {}^6C_5 \left(\sqrt{\frac{x}{a}} \right)^1 \left(\sqrt{\frac{a}{x}} \right)^5 + {}^6C_6 \left(\sqrt{\frac{x}{a}} \right)^0 \left(\sqrt{\frac{a}{x}} \right)^6 \\ &= \frac{x^3}{a^3} - 6 \frac{x^2}{a^2} + 15 \frac{x}{a} - 20 + 15 \frac{a}{x} - 6 \frac{a^2}{x^2} + \frac{a^3}{x^3} \end{aligned}$$

$$\text{(vii)} \left(\sqrt[3]{x} - \sqrt[3]{a} \right)^6$$

Let us solve the given expression:

$$\begin{aligned} &= {}^6C_0 (\sqrt[3]{x})^6 (\sqrt[3]{a})^0 - {}^6C_1 (\sqrt[3]{x})^5 (\sqrt[3]{a})^1 + {}^6C_2 (\sqrt[3]{x})^4 (\sqrt[3]{a})^2 - {}^6C_3 (\sqrt[3]{x})^3 (\sqrt[3]{a})^3 \\ &+ {}^6C_4 (\sqrt[3]{x})^2 (\sqrt[3]{a})^4 - {}^6C_5 (\sqrt[3]{x})^1 (\sqrt[3]{a})^5 + {}^6C_6 (\sqrt[3]{x})^0 (\sqrt[3]{a})^6 \\ &= x^2 - 6x^{5/3}a^{1/3} + 15x^{4/3}a^{2/3} - 20xa + 15x^{2/3}a^{4/3} - 6x^{1/3}a^{5/3} + a^2 \end{aligned}$$

$$\text{(viii)} (1 + 2x - 3x^2)^5$$

Let us solve the given expression:

Let us consider $(1 + 2x)$ and $3x^2$ as two different entities and apply the binomial theorem.

$$\begin{aligned} (1 + 2x - 3x^2)^5 &= {}^5C_0 (1 + 2x)^5 (3x^2)^0 - {}^5C_1 (1 + 2x)^4 (3x^2)^1 + {}^5C_2 (1 + 2x)^3 (3x^2)^2 - {}^5C_3 (1 + 2x)^2 (3x^2)^3 + {}^5C_4 (1 + 2x)^1 (3x^2)^4 - {}^5C_5 (1 + 2x)^0 (3x^2)^5 \\ &= (1 + 2x)^5 - 5(1 + 2x)^4 (3x^2) + 10(1 + 2x)^3 (9x^4) - 10(1 + 2x)^2 (27x^6) + 5(1 + 2x) (81x^8) - 243x^{10} \\ &= {}^5C_0 (2x)^0 + {}^5C_1 (2x)^1 + {}^5C_2 (2x)^2 + {}^5C_3 (2x)^3 + {}^5C_4 (2x)^4 + {}^5C_5 (2x)^5 - \\ &15x^2 [{}^4C_0 (2x)^0 + {}^4C_1 (2x)^1 + {}^4C_2 (2x)^2 + {}^4C_3 (2x)^3 + {}^4C_4 (2x)^4] + 90x^4 [1 + 8x^3 + 6x + 12x^2] - 270x^6 (1 + 4x^2 + 4x) + 405x^8 + 810x^9 - 243x^{10} \\ &= 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5 - 15x^2 - 120x^3 - 360x^4 - 480x^5 - 240x^6 + 90x^4 + 720x^7 + 540x^5 + \\ &1080x^6 - 270x^6 - 1080x^8 - 1080x^7 + 405x^8 + 810x^9 - 243x^{10} \\ &= 1 + 10x + 25x^2 - 40x^3 - 190x^4 + 92x^5 + 570x^6 - 360x^7 - 675x^8 + 810x^9 - 243x^{10} \end{aligned}$$

$$(ix) \left(x + 1 - \frac{1}{x}\right)^3$$

Let us solve the given expression:

$$\begin{aligned} &= {}^3C_0(x+1)^3\left(\frac{1}{x}\right)^0 - {}^3C_1(x+1)^2\left(\frac{1}{x}\right)^1 + {}^3C_2(x+1)^1\left(\frac{1}{x}\right)^2 - {}^3C_3(x+1)^0\left(\frac{1}{x}\right)^3 \\ &= (x+1)^3 - 3(x+1)^2 \times \frac{1}{x} + 3\frac{x+1}{x^2} - \frac{1}{x^3} \\ &= x^3 + 1 + 3x + 3x^2 - \frac{3x^2 + 3 + 6x}{x} + 3\frac{x+1}{x^2} - \frac{1}{x^3} \\ &= x^3 + 1 + 3x + 3x^2 - 3x - \frac{3}{x} - 6 + \frac{3}{x} + \frac{3}{x^2} - \frac{1}{x^3} \\ &= x^3 + 3x^2 - 5 + \frac{3}{x^2} - \frac{1}{x^3} \end{aligned}$$

$$(x) (1 - 2x + 3x^2)^3$$

Let us solve the given expression:

$$\begin{aligned} &= {}^3C_0(1-2x)^3 + {}^3C_1(1-2x)^2(3x^2) + {}^3C_2(1-2x)(3x^2)^2 + {}^3C_3(3x^2)^3 \\ &= (1-2x)^3 + 9x^2(1-2x)^2 + 27x^4(1-2x) + 27x^6 \\ &= 1 - 8x^3 + 12x^2 - 6x + 9x^2(1 + 4x^2 - 4x) + 27x^4 - 54x^5 + 27x^6 \\ &= 1 - 8x^3 + 12x^2 - 6x + 9x^2 + 36x^4 - 36x^3 + 27x^4 - 54x^5 + 27x^6 \\ &= 1 - 6x + 21x^2 - 44x^3 + 63x^4 - 54x^5 + 27x^6 \end{aligned}$$

2. Evaluate the following:

$$(i) (\sqrt{x+1} + \sqrt{x-1})^6 + (\sqrt{x+1} - \sqrt{x-1})^6$$

$$(ii) \left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$$

$$(iii) (1 + 2\sqrt{x})^5 + (1 - 2\sqrt{x})^5$$

$$(iv) (\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$$

$$(v) (3 + \sqrt{2})^5 - (3 - \sqrt{2})^5$$

$$(vi) (2 + \sqrt{3})^7 + (2 - \sqrt{3})^7$$

$$(vii) (\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$$

$$(viii) (0.99)^5 + (1.01)^5$$

$$(ix) (\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$$

$$(x) \left\{ a^2 + \sqrt{a^2 - 1} \right\}^4 + \left\{ a^2 - \sqrt{a^2 - 1} \right\}^4$$

Solution:

$$(i) (\sqrt{x+1} + \sqrt{x-1})^6 + (\sqrt{x+1} - \sqrt{x-1})^6$$

Let us solve the given expression:

$$\begin{aligned} &= 2 \left[{}^6C_0 (\sqrt{x+1})^6 (\sqrt{x-1})^0 + {}^6C_2 (\sqrt{x+1})^4 (\sqrt{x-1})^2 \right. \\ &\quad \left. + {}^6C_4 (\sqrt{x+1})^2 (\sqrt{x-1})^4 + {}^6C_6 (\sqrt{x+1})^0 (\sqrt{x-1})^6 \right] \\ &= 2 \left[(x+1)^3 + 15(x+1)^2(x-1) + 15(x+1)(x-1)^2 + (x-1)^3 \right] \\ &= 2 \left[x^3 + 1 + 3x + 3x^2 + 15(x^2 + 2x + 1)(x-1) + 15(x+1)(x^2 + 1 - 2x) + x^3 - 1 + 3x - 3x^2 \right] \\ &= 2 \left[2x^3 + 6x + 15x^3 - 15x^2 + 30x^2 - 30x + 15x - 15 + 15x^3 + 15x^2 - 30x^2 - 30x + 15x + 15 \right] \\ &= 2 \left[32x^3 - 24x \right] \\ &= 16x \left[4x^2 - 3 \right] \end{aligned}$$

$$(ii) \left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$$

Let us solve the given expression:

$$\begin{aligned} &= 2 \left[{}^6C_0 x^6 (\sqrt{x^2 - 1})^0 + {}^6C_2 x^4 (\sqrt{x^2 - 1})^2 + {}^6C_4 x^2 (\sqrt{x^2 - 1})^4 + {}^6C_6 x^0 (\sqrt{x^2 - 1})^6 \right] \\ &= 2 \left[x^6 + 15x^4 (x^2 - 1) + 15x^2 (x^2 - 1)^2 + (x^2 - 1)^3 \right] \\ &= 2 \left[x^6 + 15x^6 - 15x^4 + 15x^2 (x^4 - 2x^2 + 1) + (x^6 - 1 + 3x^2 - 3x^4) \right] \\ &= 2 \left[x^6 + 15x^6 - 15x^4 + 15x^6 - 30x^4 + 15x^2 + x^6 - 1 + 3x^2 - 3x^4 \right] \\ &= 2 \left[x^6 + 15x^6 - 15x^4 + 15x^2 (x^4 - 2x^2 + 1) + (x^6 - 1 + 3x^2 - 3x^4) \right] \\ &= 2 \left[x^6 + 15x^6 - 15x^4 + 15x^6 - 30x^4 + 15x^2 + x^6 - 1 + 3x^2 - 3x^4 \right] \\ &= 64x^6 - 96x^4 + 36x^2 - 2 \end{aligned}$$

$$(iii) (1 + 2\sqrt{x})^5 + (1 - 2\sqrt{x})^5$$

Let us solve the given expression:

$$\begin{aligned} &= 2 \left[{}^5C_0 (2\sqrt{x})^0 + {}^5C_2 (2\sqrt{x})^2 + {}^5C_4 (2\sqrt{x})^4 \right] \\ &= 2 \left[1 + 10 (4x) + 5 (16x^2) \right] \\ &= 2 \left[1 + 40x + 80x^2 \right] \end{aligned}$$

$$(iv) (\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$$

Let us solve the given expression:

$$\begin{aligned} &= 2 \left[{}^6C_0 (\sqrt{2})^6 + {}^6C_2 (\sqrt{2})^4 + {}^6C_4 (\sqrt{2})^2 + {}^6C_6 (\sqrt{2})^0 \right] \\ &= 2 \left[8 + 15 (4) + 15 (2) + 1 \right] \\ &= 2 \left[99 \right] \\ &= 198 \end{aligned}$$

$$(v) (3 + \sqrt{2})^5 - (3 - \sqrt{2})^5$$

Let us solve the given expression:

$$\begin{aligned} &= 2 [{}^5C_1 (3^4) (\sqrt{2})^1 + {}^5C_3 (3^2) (\sqrt{2})^3 + {}^5C_5 (3^0) (\sqrt{2})^5] \\ &= 2 [5 (81) (\sqrt{2}) + 10 (9) (2\sqrt{2}) + 4\sqrt{2}] \\ &= 2\sqrt{2} (405 + 180 + 4) \\ &= 1178\sqrt{2} \end{aligned}$$

$$(vi) (2 + \sqrt{3})^7 + (2 - \sqrt{3})^7$$

Let us solve the given expression:

$$\begin{aligned} &= 2 [{}^7C_0 (2^7) (\sqrt{3})^0 + {}^7C_2 (2^5) (\sqrt{3})^2 + {}^7C_4 (2^3) (\sqrt{3})^4 + {}^7C_6 (2^1) (\sqrt{3})^6] \\ &= 2 [128 + 21 (32)(3) + 35(8)(9) + 7(2)(27)] \\ &= 2 [128 + 2016 + 2520 + 378] \\ &= 2 [5042] \\ &= 10084 \end{aligned}$$

$$(vii) (\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$$

Let us solve the given expression:

$$\begin{aligned} &= 2 [{}^5C_1 (\sqrt{3})^4 + {}^5C_3 (\sqrt{3})^2 + {}^5C_5 (\sqrt{3})^0] \\ &= 2 [5 (9) + 10 (3) + 1] \\ &= 2 [76] \\ &= 152 \end{aligned}$$

$$(viii) (0.99)^5 + (1.01)^5$$

Let us solve the given expression:

$$\begin{aligned} &= (1 - 0.01)^5 + (1 + 0.01)^5 \\ &= 2 [{}^5C_0 (0.01)^0 + {}^5C_2 (0.01)^2 + {}^5C_4 (0.01)^4] \\ &= 2 [1 + 10 (0.0001) + 5 (0.00000001)] \\ &= 2 [1.00100005] \\ &= 2.0020001 \end{aligned}$$

$$(ix) (\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$$

Let us solve the given expression:

$$= 2 [{}^6C_1 (\sqrt{3})^5 (\sqrt{2})^1 + {}^6C_3 (\sqrt{3})^3 (\sqrt{2})^3 + {}^6C_5 (\sqrt{3})^1 (\sqrt{2})^5]$$

$$\begin{aligned}
&= 2 [6 (9\sqrt{3}) (\sqrt{2}) + 20 (3\sqrt{3}) (2\sqrt{2}) + 6 (\sqrt{3}) (4\sqrt{2})] \\
&= 2 [\sqrt{6} (54 + 120 + 24)] \\
&= 396 \sqrt{6}
\end{aligned}$$

$$(x) \left\{ a^2 + \sqrt{a^2 - 1} \right\}^4 + \left\{ a^2 - \sqrt{a^2 - 1} \right\}^4$$

Let us solve the given expression:

$$\begin{aligned}
&= 2 \left[{}^4C_0 (a^2)^4 (\sqrt{a^2 - 1})^0 + {}^4C_2 (a^2)^2 (\sqrt{a^2 - 1})^2 + {}^4C_4 (a^2)^0 (\sqrt{a^2 - 1})^4 \right] \\
&= 2 \left[a^8 + 6a^4 (a^2 - 1) + (a^2 - 1)^2 \right] \\
&= 2 [a^8 + 6a^6 - 6a^4 + a^4 + 1 - 2a^2] \\
&= 2a^8 + 12a^6 - 10a^4 - 4a^2 + 2
\end{aligned}$$

3. Find $(a + b)^4 - (a - b)^4$. Hence, evaluate $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$.

Solution:

Firstly, let us solve the given expression:

$$(a + b)^4 - (a - b)^4$$

The above expression can be expressed as,

$$\begin{aligned}
(a + b)^4 - (a - b)^4 &= 2 [{}^4C_1 a^3 b^1 + {}^4C_3 a^1 b^3] \\
&= 2 [4a^3 b + 4ab^3] \\
&= 8 (a^3 b + ab^3)
\end{aligned}$$

Now,

Let us evaluate the expression:

$$(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$$

So consider, $a = \sqrt{3}$ and $b = \sqrt{2}$ we get,

$$\begin{aligned}
(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 &= 8 (a^3 b + ab^3) \\
&= 8 [(\sqrt{3})^3 (\sqrt{2}) + (\sqrt{3}) (\sqrt{2})^3] \\
&= 8 [(3\sqrt{6}) + (2\sqrt{6})] \\
&= 8 (5\sqrt{6}) \\
&= 40\sqrt{6}
\end{aligned}$$

4. Find $(x + 1)^6 + (x - 1)^6$. Hence, or otherwise evaluate $(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$.

Solution:

Firstly, let us solve the given expression:

$$(x + 1)^6 + (x - 1)^6$$

The above expression can be expressed as,

$$\begin{aligned}(x+1)^6 + (x-1)^6 &= 2 [{}^6C_0 x^6 + {}^6C_2 x^4 + {}^6C_4 x^2 + {}^6C_6 x^0] \\ &= 2 [x^6 + 15x^4 + 15x^2 + 1]\end{aligned}$$

Now,

Let us evaluate the expression:

$$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$$

So consider, $x = \sqrt{2}$ then we get,

$$\begin{aligned}(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 &= 2 [x^6 + 15x^4 + 15x^2 + 1] \\ &= 2 [(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1] \\ &= 2 [8 + 15(4) + 15(2) + 1] \\ &= 2 [8 + 60 + 30 + 1] \\ &= 198\end{aligned}$$

5. Using binomial theorem evaluate each of the following:

(i) $(96)^3$

(ii) $(102)^5$

(iii) $(101)^4$

(iv) $(98)^5$

Solution:

(i) $(96)^3$

We have,

$$(96)^3$$

Let us express the given expression as two different entities and apply the binomial theorem.

$$\begin{aligned}(96)^3 &= (100 - 4)^3 \\ &= {}^3C_0 (100)^3 (4)^0 - {}^3C_1 (100)^2 (4)^1 + {}^3C_2 (100)^1 (4)^2 - {}^3C_3 (100)^0 (4)^3 \\ &= 1000000 - 120000 + 4800 - 64 \\ &= 884736\end{aligned}$$

(ii) $(102)^5$

We have,

$$(102)^5$$

Let us express the given expression as two different entities and apply the binomial theorem.

$$\begin{aligned}(102)^5 &= (100 + 2)^5 \\ &= {}^5C_0 (100)^5 (2)^0 + {}^5C_1 (100)^4 (2)^1 + {}^5C_2 (100)^3 (2)^2 + {}^5C_3 (100)^2 (2)^3 + {}^5C_4 (100)^1 (2)^4 + {}^5C_5 (100)^0 (2)^5 \\ &= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32\end{aligned}$$

$$= 11040808032$$

$$\text{(iii)} (101)^4$$

We have,

$$(101)^4$$

Let us express the given expression as two different entities and apply the binomial theorem.

$$(101)^4 = (100 + 1)^4$$

$$= {}^4C_0 (100)^4 + {}^4C_1 (100)^3 + {}^4C_2 (100)^2 + {}^4C_3 (100)^1 + {}^4C_4 (100)^0$$

$$= 100000000 + 4000000 + 60000 + 400 + 1$$

$$= 104060401$$

$$\text{(iv)} (98)^5$$

We have,

$$(98)^5$$

Let us express the given expression as two different entities and apply the binomial theorem.

$$(98)^5 = (100 - 2)^5$$

$$= {}^5C_0 (100)^5 (2)^0 - {}^5C_1 (100)^4 (2)^1 + {}^5C_2 (100)^3 (2)^2 - {}^5C_3 (100)^2 (2)^3 + {}^5C_4 (100)^1 (2)^4 - {}^5C_5 (100)^0 (2)^5$$

$$= 10000000000 - 1000000000 + 40000000 - 800000 + 8000 - 32$$

$$= 9039207968$$

6. Using binomial theorem, prove that $2^{3n} - 7n - 1$ is divisible by 49, where $n \in \mathbb{N}$.

Solution:

Given:

$$2^{3n} - 7n - 1$$

$$\text{So, } 2^{3n} - 7n - 1 = 8^n - 7n - 1$$

Now,

$$8^n - 7n - 1$$

$$8^n = 7n + 1$$

$$= (1 + 7)^n$$

$$= {}^nC_0 + {}^nC_1 (7)^1 + {}^nC_2 (7)^2 + {}^nC_3 (7)^3 + {}^nC_4 (7)^4 + {}^nC_5 (7)^5 + \dots + {}^nC_n (7)^n$$

$$8^n = 1 + 7n + 49 [{}^nC_2 + {}^nC_3 (7^1) + {}^nC_4 (7^2) + \dots + {}^nC_n (7)^{n-2}]$$

$$8^n - 1 - 7n = 49 (\text{integer})$$

So now,

$$8^n - 1 - 7n \text{ is divisible by } 49$$

Or

$$2^{3n} - 1 - 7n \text{ is divisible by } 49.$$

Hence proved.

EXERCISE 18.2 PAGE NO: 18.37

1. Find the 11th term from the beginning and the 11th term from the end in the expansion of $(2x - 1/x^2)^{25}$.

Solution:

Given:

$$(2x - 1/x^2)^{25}$$

The given expression contains 26 terms.

So, the 11th term from the end is the $(26 - 11 + 1)^{\text{th}}$ term from the beginning.

In other words, the 11th term from the end is the 16th term from the beginning.

Then,

$$\begin{aligned} T_{16} &= T_{15+1} = {}^{25}C_{15} (2x)^{25-15} (-1/x^2)^{15} \\ &= {}^{25}C_{15} (2^{10}) (x)^{10} (-1/x^{30}) \\ &= - {}^{25}C_{15} (2^{10} / x^{20}) \end{aligned}$$

Now we shall find the 11th term from the beginning.

$$\begin{aligned} T_{11} &= T_{10+1} = {}^{25}C_{10} (2x)^{25-10} (-1/x^2)^{10} \\ &= {}^{25}C_{10} (2^{15}) (x)^{15} (1/x^{20}) \\ &= {}^{25}C_{10} (2^{15} / x^5) \end{aligned}$$

2. Find the 7th term in the expansion of $(3x^2 - 1/x^3)^{10}$.

Solution:

Given:

$$(3x^2 - 1/x^3)^{10}$$

Let us consider the 7th term as T_7

So,

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^{10}C_6 (3x^2)^{10-6} (-1/x^3)^6 \\ &= {}^{10}C_6 (3)^4 (x)^8 (1/x^{18}) \\ &= [10 \times 9 \times 8 \times 7 \times 6 \times 5] / [4 \times 3 \times 2 \times 1] \times 81 / x^{10} \\ &= 17010 / x^{10} \end{aligned}$$

$\therefore$ The 7th term of the expression $(3x^2 - 1/x^3)^{10}$ is $17010 / x^{10}$.

3. Find the 5th term in the expansion of $(3x - 1/x^2)^{10}$.

Solution:

Given:

$$(3x - 1/x^2)^{10}$$

The 5th term from the end is the $(11 - 5 + 1)$ th, is., 7th term from the beginning.

So,

$$T_7 = T_{6+1}$$

$$= {}^{10}C_6 (3x)^{10-6} (-1/x^2)^6$$

$$= {}^{10}C_6 (3)^4 (x)^4 (1/x^{12})$$

$$= [10 \times 9 \times 8 \times 7 \times 81] / [4 \times 3 \times 2 \times x^8]$$

$$= 17010 / x^8$$

∴ The 5th term of the expression $(3x - 1/x^2)^{10}$ is $17010 / x^8$.

4. Find the 8th term in the expansion of $(x^{3/2} y^{1/2} - x^{1/2} y^{3/2})^{10}$.

Solution:

Given:

$$(x^{3/2} y^{1/2} - x^{1/2} y^{3/2})^{10}$$

Let us consider the 8th term as T_8

So,

$$T_8 = T_{7+1}$$

$$= {}^{10}C_7 (x^{3/2} y^{1/2})^{10-7} (-x^{1/2} y^{3/2})^7$$

$$= -[10 \times 9 \times 8] / [3 \times 2] x^{9/2} y^{3/2} (x^{7/2} y^{21/2})$$

$$= -120 x^8 y^{12}$$

∴ The 8th term of the expression $(x^{3/2} y^{1/2} - x^{1/2} y^{3/2})^{10}$ is $-120 x^8 y^{12}$.

5. Find the 7th term in the expansion of $(4x/5 + 5/2x)^8$.

Solution:

Given:

$$(4x/5 + 5/2x)^8$$

Let us consider the 7th term as T_7

So,

$$T_7 = T_{6+1}$$

$$= {}^8C_6 \left(\frac{4x}{5}\right)^{8-6} \left(\frac{5}{2x}\right)^6$$

$$= \frac{8 \times 7 \times 4 \times 4 \times 125 \times 125}{2 \times 1 \times 25 \times 64} x^2 \left(\frac{1}{x^6}\right)$$

$$= \frac{4375}{x^4}$$

∴ The 7th term of the expression $(4x/5 + 5/2x)^8$ is $4375/x^4$.

6. Find the 4th term from the beginning and 4th term from the end in the expansion of $(x + 2/x)^9$.

Solution:

Given:

$$(x + 2/x)^9$$

Let T_{r+1} be the 4th term from the end.

Then, T_{r+1} is $(10 - 4 + 1)$ th, i.e., 7th, term from the beginning.

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^9C_6 (x^{9-6}) \left(\frac{2}{x}\right)^6 \\ &= \frac{9 \times 8 \times 7}{3 \times 2} (x^3) \left(\frac{64}{x^6}\right) \\ &= \frac{5376}{x^3} \end{aligned}$$

4th term from the beginning = $T_4 = T_{3+1}$

$$\begin{aligned} T_4 &= {}^9C_3 (x^{9-3}) \left(\frac{2}{x}\right)^3 \\ &= \frac{9 \times 8 \times 7}{3 \times 2} (x^6) \left(\frac{8}{x^3}\right) \\ &= 672 x^3 \end{aligned}$$

7. Find the 4th term from the end in the expansion of $(4x/5 - 5/2x)^9$.

Solution:

Given:

$$(4x/5 - 5/2x)^9$$

Let T_{r+1} be the 4th term from the end of the given expression.

Then, T_{r+1} is $(10 - 4 + 1)$ th term, i.e., 7th term, from the beginning.

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^9C_6 \left(\frac{4x}{5}\right)^{9-6} \left(\frac{5}{2x}\right)^6 \\ &= \frac{9 \times 8 \times 7}{3 \times 2} \left(\frac{64}{125} x^3\right) \left(\frac{125 \times 125}{64 x^6}\right) \\ &= \frac{10500}{x^3} \end{aligned}$$

$\therefore$ The 4th term from the end is $10500/x^3$.

8. Find the 7th term from the end in the expansion of $(2x^2 - 3/2x)^8$.

Solution:

Given:

$$(2x^2 - 3/2x)^8$$

Let T_{r+1} be the 4th term from the end of the given expression.

Then, T_{r+1} is $(9 - 7 + 1)$ th term, i.e., 3rd term, from the beginning.

$$T_3 = T_{2+1}$$

$$\begin{aligned} &= {}^8C_2 \left(2x^2\right)^{8-2} \left(-\frac{3}{2x}\right)^2 \\ &= \frac{8 \times 7}{2 \times 1} (64x^{12}) \frac{9}{4x^2} \\ &= 4032 x^{10} \end{aligned}$$

$\therefore$ The 7th term from the end is $4032 x^{10}$.

9. Find the coefficient of:

(i) x^{10} in the expansion of $(2x^2 - 1/x)^{20}$

(ii) x^7 in the expansion of $(x - 1/x^2)^{40}$

(iii) x^{-15} in the expansion of $(3x^2 - a/3x^3)^{10}$

(iv) x^9 in the expansion of $(x^2 - 1/3x)^9$

(v) x^m in the expansion of $(x + 1/x)^n$

(vi) x in the expansion of $(1 - 2x^3 + 3x^5)(1 + 1/x)^8$

(vii) a^5b^7 in the expansion of $(a - 2b)^{12}$

(viii) x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$

Solution:

(i) x^{10} in the expansion of $(2x^2 - 1/x)^{20}$

Given:

$$(2x^2 - 1/x)^{20}$$

If x^{10} occurs in the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{20}C_r \left(2x^2\right)^{20-r} \left(\frac{-1}{x}\right)^r \\ &= (-1)^r {}^{20}C_r (2^{20-r}) (x^{40-2r-r}) \end{aligned}$$

For this term to contain x^{10} , we must have:

$$40 - 3r = 10$$

$$3r = 30$$

$$r = 10$$

$$\therefore \text{Coefficient of } x^{10} = (-1)^{10} {}^{20}C_{10} (2^{20-10}) = {}^{20}C_{10} (2^{10})$$

(ii) x^7 in the expansion of $(x - 1/x^2)^{40}$

Given:

$$(x - 1/x^2)^{40}$$

If x^7 occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{40}C_r x^{40-r} \left(\frac{-1}{x^2} \right)^r \\ &= (-1)^r {}^{40}C_r x^{40-r-2r} \end{aligned}$$

For this term to contain x^7 , we must have:

$$40 - 3r = 7$$

$$3r = 40 - 7$$

$$3r = 33$$

$$r = 33/3$$

$$= 11$$

$$\therefore \text{Coefficient of } x^7 = (-1)^{11} {}^{40}C_{11} = -{}^{40}C_{11}$$

$$40 - 3r = 7$$

$$3r = 40 - 7$$

$$3r = 33$$

$$r = 33/3$$

$$= 11$$

$$\therefore \text{Coefficient of } x^7 = (-1)^{11} {}^{40}C_{11} = -{}^{40}C_{11}$$

(iii) x^{-15} in the expansion of $(3x^2 - a/3x^3)^{10}$

Given:

$$(3x^2 - a/3x^3)^{10}$$

If x^{-15} occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$T_{r+1} = {}^{10}C_r (3x^2)^{10-r} \left(\frac{-a}{3x^3}\right)^r$$

$$= (-1)^r {}^{10}C_r (3^{10-r-r}) (x^{20-2r-3r}) (a^r)$$

For this term to contain x^{-15} , we must have:

$$20 - 5r = -15$$

$$5r = 20 + 15$$

$$5r = 35$$

$$r = 35/5$$

$$= 7$$

$$\therefore \text{Coefficient of } x^{-15} = (-1)^7 {}^{10}C_7 3^{10-14} (a^7) = -\frac{10 \times 9 \times 8}{3 \times 2 \times 9 \times 9} a^7 = -\frac{40}{27} a^7$$

(iv) x^9 in the expansion of $(x^2 - 1/3x)^9$

Given:

$$(x^2 - 1/3x)^9$$

If x^9 occurs at the $(r + 1)$ th term in the above expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$T_{r+1} = {}^9C_r (x^2)^{9-r} \left(\frac{-1}{3x}\right)^r$$

$$= (-1)^r {}^9C_r (x^{18-2r-r}) \left(\frac{1}{3^r}\right)$$

For this term to contain x^9 , we must have:

$$18 - 3r = 9$$

$$3r = 18 - 9$$

$$3r = 9$$

$$r = 9/3$$

$$= 3$$

$$\therefore \text{Coefficient of } x^9 = (-1)^3 {}^9C_3 \frac{1}{3^3} = -\frac{9 \times 8 \times 7}{2 \times 9 \times 9} = -\frac{28}{9}$$

For this term to contain x^9 , we must have:

$$18 - 3r = 9$$

$$3r = 18 - 9$$

$$3r = 9$$

$$r = 9/3$$

$$= 3$$

$$\therefore \text{Coefficient of } x^9 = (-1)^3 {}^9C_3 \frac{1}{3^3} = -\frac{9 \times 8 \times 7}{2 \times 9 \times 9} = -\frac{28}{9}$$

(v) x^m in the expansion of $(x + 1/x)^n$

Given:

$$(x + 1/x)^n$$

If x^m occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} \frac{1}{x^r}$$

$$T_{r+1} = {}^nC_r x^{n-r} \frac{1}{x^r} \\ = {}^nC_r x^{n-2r}$$

For this term to contain x^m , we must have:

$$n - 2r = m$$

$$2r = n - m$$

$$r = (n - m)/2$$

$$\therefore \text{Coefficient of } x^m = {}^nC_{(n-m)/2} = \frac{n!}{\left(\frac{n-m}{2}\right)! \left(\frac{n+m}{2}\right)!}$$

(vi) x in the expansion of $(1 - 2x^3 + 3x^5)(1 + 1/x)^8$

Given:

$$(1 - 2x^3 + 3x^5)(1 + 1/x)^8$$

If x occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$(1 - 2x^3 + 3x^5)(1 + 1/x)^8 = (1 - 2x^3 + 3x^5)({}^8C_0 + {}^8C_1(1/x) + {}^8C_2(1/x)^2 + {}^8C_3(1/x)^3 + {}^8C_4(1/x)^4 + {}^8C_5(1/x)^5 + {}^8C_6(1/x)^6 + {}^8C_7(1/x)^7 + {}^8C_8(1/x)^8)$$

So, ' x ' occurs in the above expression at $-2x^3 \cdot {}^8C_2(1/x^2) + 3x^5 \cdot {}^8C_4(1/x^4)$

$$\therefore \text{Coefficient of } x = -2(8!/(2!6!)) + 3(8!/(4!4!))$$

$$= -56 + 210$$

$$= 154$$

(vii) a^5b^7 in the expansion of $(a - 2b)^{12}$

Given:

$$(a - 2b)^{12}$$

If a^5b^7 occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$T_{r+1} = {}^{12}C_r a^{12-r} (-2b)^r$$

$$= (-1)^r {}^{12}C_r (a^{12-r}) (b^r) (2^r)$$

For this term to contain a^5b^7 , we must have:

$$12 - r = 5$$

$$r = 12 - 5$$

$$= 7$$

$$\therefore \text{Required coefficient} = (-1)^7 {}^{12}C_7 (2^7)$$

$$= - \frac{12 \times 11 \times 10 \times 9 \times 8 \times 128}{5 \times 4 \times 3 \times 2}$$

$$= -101376$$

(viii) x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$

Given:

$$(1 - 3x + 7x^2)(1 - x)^{16}$$

If x occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$(1 - 3x + 7x^2)(1 - x)^{16} = (1 - 3x + 7x^2)({}^{16}C_0 + {}^{16}C_1(-x) + {}^{16}C_2(-x)^2 + {}^{16}C_3(-x)^3 + {}^{16}C_4(-x)^4 + {}^{16}C_5(-x)^5 + {}^{16}C_6(-x)^6 + {}^{16}C_7(-x)^7 + {}^{16}C_8(-x)^8 + {}^{16}C_9(-x)^9 + {}^{16}C_{10}(-x)^{10} + {}^{16}C_{11}(-x)^{11} + {}^{16}C_{12}(-x)^{12} + {}^{16}C_{13}(-x)^{13} + {}^{16}C_{14}(-x)^{14} + {}^{16}C_{15}(-x)^{15} + {}^{16}C_{16}(-x)^{16})$$

So, ' x ' occurs in the above expression at ${}^{16}C_1(-x) - 3x{}^{16}C_0$

$$\therefore \text{Coefficient of } x = -(16!/(1! 15!)) - 3(16!/(0! 16!))$$

$$= -16 - 3$$

$$= -19$$

10. Which term in the expansion of $\left\{ \left(\frac{x}{\sqrt{y}} \right)^{1/3} + \left(\frac{y}{x^{1/3}} \right)^{1/2} \right\}^{21}$ contains x and y to one and the same power.

Solution:

Let us consider T_{r+1} th term in the given expansion contains x and y to one and the same power.

Then we have,

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$T_{r+1} = {}^{21}C_r \left[\left(\frac{x}{\sqrt{y}} \right)^{1/3} \right]^{21-r} \left[\left(\frac{y}{x^{1/3}} \right)^{1/2} \right]^r$$

$$= {}^{21}C_r \left(\frac{x^{(21-r)/3}}{y^{r/6}} \right) \left(\frac{y^{r/2}}{x^{r/6}} \right)$$

$$= {}^{21}C_r (x)^{7-r/2} (y)^{2r/3-r/2}$$

$$= {}^{21}C_r (x)^{7-r/2} (y)^{2r/3-7/2}$$

If x and y have the same power, then

$$7 - r/2 = 2r/3 - 7/2$$

$$2r/3 + r/2 = 7 + 7/2$$

$$(4r + 3r)/6 = (14+7)/2$$

$$7r/6 = 21/2$$

$$r = (21 \times 6)/(2 \times 7)$$

$$= 3 \times 3$$

$$= 9$$

Hence, the required term is the 10th term.

11. Does the expansion of $(2x^2 - 1/x)$ contain any term involving x^9 ?

Solution:

Given:

$$(2x^2 - 1/x)$$

If x^9 occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{20}C_r (2x^2)^{20-r} \left(\frac{-1}{x}\right)^r \\ &= (-1)^r {}^{20}C_r (2)^{20-r} (x)^{40-2r-r} \end{aligned}$$

For this term to contain x^9 , we must have

$$40 - 3r = 9$$

$$3r = 40 - 9$$

$$3r = 31$$

$$r = 31/3$$

It is not possible, since r is not an integer.

Hence, there is no term with x^9 in the given expansion.

12. Show that the expansion of $(x^2 + 1/x)^{12}$ does not contain any term involving x^{-1} .

Solution:

Given:

$$(x^2 + 1/x)^{12}$$

If x^{-1} occurs at the $(r + 1)$ th term in the given expression.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} T_{r+1} &= {}^{12}C_r (x^2)^{12-r} \left(\frac{1}{x}\right)^r \\ &= {}^{12}C_r x^{24-2r-r} \end{aligned}$$

For this term to contain x^{-1} , we must have

$$24 - 3r = -1$$

$$3r = 24 + 1$$

$$3r = 25$$

$$r = 25/3$$

It is not possible, since r is not an integer.

Hence, there is no term with x^{-1} in the given expansion.

13. Find the middle term in the expansion of:

(i) $(2/3x - 3/2x)^{20}$

(ii) $(a/x + bx)^{12}$

(iii) $(x^2 - 2/x)^{10}$

(iv) $(x/a - a/x)^{10}$

Solution:

(i) $(2/3x - 3/2x)^{20}$

We have,

$(2/3x - 3/2x)^{20}$ where, $n = 20$ (even number)

So the middle term is $(n/2 + 1) = (20/2 + 1) = (10 + 1) = 11$. ie., 11th term

Now,

$$\begin{aligned} T_{11} &= T_{10+1} \\ &= {}^{20}C_{10} (2/3x)^{20-10} (3/2x)^{10} \\ &= {}^{20}C_{10} 2^{10}/3^{10} \times 3^{10}/2^{10} x^{10-10} \\ &= {}^{20}C_{10} \end{aligned}$$

Hence, the middle term is ${}^{20}C_{10}$.

(ii) $(a/x + bx)^{12}$

We have,

$(a/x + bx)^{12}$ where, $n = 12$ (even number)

So the middle term is $(n/2 + 1) = (12/2 + 1) = (6 + 1) = 7$. ie., 7th term

Now,

$$T_7 = T_{6+1}$$

$$\begin{aligned}
&= {}^{12}C_6 \left(\frac{a}{x}\right)^{12-6} (bx)^6 \\
&= {}^{12}C_6 a^6 b^6 \\
&= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2} a^6 b^6 \\
&= 924 a^6 b^6
\end{aligned}$$

$$= 924 a^6 b^6$$

Hence, the middle term is $924 a^6 b^6$.

(iii) $(x^2 - 2/x)^{10}$

We have,

$(x^2 - 2/x)^{10}$ where, $n = 10$ (even number)

So the middle term is $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$. ie., 6th term

Now,

$$T_6 = T_{5+1}$$

$$\begin{aligned}
&= {}^{10}C_5 (x^2)^{10-5} \left(\frac{-2}{x}\right)^5 \\
&= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \times 32x^5 \\
&= -8064 x^5
\end{aligned}$$

Hence, the middle term is $-8064x^5$.

(iv) $(x/a - a/x)^{10}$

We have,

$(x/a - a/x)^{10}$ where, $n = 10$ (even number)

So the middle term is $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$. ie., 6th term

Now,

$$T_6 = T_{5+1}$$

$$\begin{aligned}
&= {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(\frac{-a}{x}\right)^5 \\
&= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \\
&= -252
\end{aligned}$$

Hence, the middle term is -252 .

14. Find the middle terms in the expansion of:

(i) $(3x - x^3/6)^9$

(ii) $(2x^2 - 1/x)^7$

(iii) $(3x - 2/x^2)^{15}$

(iv) $(x^4 - 1/x^3)^{11}$

Solution:

(i) $(3x - x^3/6)^9$

We have,

$(3x - x^3/6)^9$ where, $n = 9$ (odd number)

So the middle terms are $((n+1)/2) = ((9+1)/2) = 10/2 = 5$ and

$((n+1)/2 + 1) = ((9+1)/2 + 1) = (10/2 + 1) = (5 + 1) = 6$

The terms are 5th and 6th.

Now,

$T_5 = T_{4+1}$

$$\begin{aligned} &= {}^9C_4 \left(3x\right)^{9-4} \left(\frac{-x^3}{6}\right)^4 \\ &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 27 \times 9 \times \frac{1}{36 \times 36} x^{17} \\ &= \frac{189}{8} x^{17} \end{aligned}$$

And,

$T_6 = T_{5+1}$

$$\begin{aligned} &= {}^9C_5 \left(3x\right)^{9-5} \left(\frac{-x^3}{6}\right)^5 \\ &= -\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 81 \times \frac{1}{216 \times 36} x^{19} \\ &= -\frac{21}{16} x^{19} \end{aligned}$$

Hence, the middle term are $189/8 x^{17}$ and $-21/16 x^{19}$.

(ii) $(2x^2 - 1/x)^7$

We have,

$(2x^2 - 1/x)^7$ where, $n = 7$ (odd number)

So the middle terms are $((n+1)/2) = ((7+1)/2) = 8/2 = 4$ and

$((n+1)/2 + 1) = ((7+1)/2 + 1) = (8/2 + 1) = (4 + 1) = 5$

The terms are 4th and 5th.

Now,

$$\begin{aligned}
T_4 &= T_{3+1} \\
&= {}^7C_3 \left(2x^2\right)^{7-3} \left(\frac{-1}{x}\right)^3 \\
&= -\frac{7 \times 6 \times 5}{3 \times 2} \times 16 x^8 \times \frac{1}{x^3} \\
&= -560 x^5
\end{aligned}$$

And,

$$\begin{aligned}
T_5 &= T_{4+1} \\
&= {}^7C_4 \left(2x^2\right)^{7-4} \left(\frac{-1}{x}\right)^4 \\
&= 35 \times 8 \times x^6 \times \frac{1}{x^4} \\
&= 280 x^2
\end{aligned}$$

Hence, the middle term are $-560x^5$ and $280x^2$.

(iii) $(3x - 2/x^2)^{15}$

We have,

$(3x - 2/x^2)^{15}$ where, $n = 15$ (odd number)

So the middle terms are $((n+1)/2) = ((15+1)/2) = 16/2 = 8$ and

$((n+1)/2 + 1) = ((15+1)/2 + 1) = (16/2 + 1) = (8 + 1) = 9$

The terms are 8^{th} and 9^{th} .

Now,

$$\begin{aligned}
T_8 &= T_{7+1} \\
&= {}^{15}C_7 \left(3x\right)^{15-7} \left(\frac{-2}{x^2}\right)^7 \\
&= -\frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9}{7 \times 6 \times 5 \times 4 \times 3 \times 2} \times 3^8 \times 2^7 x^{8-14} \\
&= \frac{-6435 \times 3^8 \times 2^7}{x^6}
\end{aligned}$$

And,

$$\begin{aligned}
T_9 &= T_{8+1} \\
&= {}^{15}C_8 \left(3x\right)^{15-8} \left(\frac{-2}{x^2}\right)^8 \\
&= \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9}{7 \times 6 \times 5 \times 4 \times 3 \times 2} \times 3^7 \times 2^8 \times x^{7-16} \\
&= \frac{6435 \times 3^7 \times 2^8}{x^9}
\end{aligned}$$

Hence, the middle term are $(-6435 \times 3^8 \times 2^7)/x^6$ and $(6435 \times 3^7 \times 2^8)/x^9$.

(iv) $(x^4 - 1/x^3)^{11}$

We have,

$$(x^4 - 1/x^3)^{11}$$

where, $n = 11$ (odd number)

So the middle terms are $((n+1)/2) = ((11+1)/2) = 12/2 = 6$ and

$$((n+1)/2 + 1) = ((11+1)/2 + 1) = (12/2 + 1) = (6 + 1) = 7$$

The terms are 6th and 7th.

Now,

$$\begin{aligned} T_6 &= T_{5+1} \\ &= {}^{11}C_5 \left(x^4\right)^{11-5} \left(\frac{-1}{x^3}\right)^5 \\ &= -\frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2} \times (x)^{24-15} \\ &= -462 x^9 \end{aligned}$$

And,

$$\begin{aligned} T_7 &= T_{6+1} \\ &= {}^{11}C_6 \left(x^4\right)^{11-6} \left(\frac{-1}{x^3}\right)^6 \\ &= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2} (x)^{20-18} \\ &= 462 x^2 \end{aligned}$$

Hence, the middle term are $-462x^9$ and $462x^2$.

15. Find the middle terms in the expansion of:

(i) $(x - 1/x)^{10}$

(ii) $(1 - 2x + x^2)^n$

(iii) $(1 + 3x + 3x^2 + x^3)^{2n}$

(iv) $(2x - x^2/4)^9$

(v) $(x - 1/x)^{2n+1}$

(vi) $(x/3 + 9y)^{10}$

(vii) $(3 - x^3/6)^7$

(viii) $(2ax - b/x^2)^{12}$

(ix) $(p/x + x/p)^9$

(x) $(x/a - a/x)^{10}$

Solution:

(i) $(x - 1/x)^{10}$

We have,

$(x - 1/x)^{10}$ where, $n = 10$ (even number)

So the middle term is $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$. ie., 6th term

Now,

$$T_6 = T_{5+1}$$

$$= {}^{10}C_5 x^{10-5} \left(\frac{-1}{x} \right)^5$$

$$= - \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2}$$

$$= -252$$

Hence, the middle term is -252.

(ii) $(1 - 2x + x^2)^n$

We have,

$(1 - 2x + x^2)^n = (1 - x)^{2n}$ where, n is an even number.

So the middle term is $(2n/2 + 1) = (n + 1)$ th term.

Now,

$$T_n = T_{n+1}$$

$$= {}^{2n}C_n (-1)^n (x)^n$$

$$= (2n)!/(n!)^2 (-1)^n x^n$$

Hence, the middle term is $(2n)!/(n!)^2 (-1)^n x^n$.

(iii) $(1 + 3x + 3x^2 + x^3)^{2n}$

We have,

$(1 + 3x + 3x^2 + x^3)^{2n} = (1 + x)^{6n}$ where, n is an even number.

So the middle term is $(n/2 + 1) = (6n/2 + 1) = (3n + 1)$ th term.

Now,

$$T_{2n} = T_{3n+1}$$

$$= {}^{6n}C_{3n} x^{3n}$$

$$= (6n)!/(3n!)^2 x^{3n}$$

Hence, the middle term is $(6n)!/(3n!)^2 x^{3n}$.

(iv) $(2x - x^2/4)^9$

We have,

$(2x - x^2/4)^9$ where, $n = 9$ (odd number)

So the middle terms are $((n+1)/2) = ((9+1)/2) = 10/2 = 5$ and

$((n+1)/2 + 1) = ((9+1)/2 + 1) = (10/2 + 1) = (5 + 1) = 6$

The terms are 5th and 6th.

Now,

$$T_5 = T_{4+1}$$

$$\begin{aligned} &= {}^9C_4 \left(2x\right)^{9-4} \left(\frac{-x^2}{4}\right)^4 \\ &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 2^5 \frac{1}{4^4} x^{5+8} \\ &= \frac{63}{4} x^{13} \end{aligned}$$

And,

$$T_6 = T_{5+1}$$

$$\begin{aligned} &= {}^9C_5 \left(2x\right)^{9-5} \left(\frac{-x^2}{4}\right)^5 \\ &= -\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \times 2^4 \frac{1}{4^5} x^{4+10} \\ &= -\frac{63}{32} x^{14} \end{aligned}$$

Hence, the middle term is $\frac{63}{4} x^{13}$ and $-\frac{63}{32} x^{14}$.

$$(v) (x - 1/x)^{2n+1}$$

We have,

$(x - 1/x)^{2n+1}$ where, $n = (2n + 1)$ is an (odd number)

So the middle terms are $((n+1)/2) = ((2n+1+1)/2) = (2n+2)/2 = (n + 1)$ and

$((n+1)/2 + 1) = ((2n+1+1)/2 + 1) = ((2n+2)/2 + 1) = (n + 1 + 1) = (n + 2)$

The terms are $(n + 1)^{th}$ and $(n + 2)^{th}$.

Now,

$$T_n = T_{n+1}$$

$$\begin{aligned} &= {}^{2n+1}C_n x^{2n+1-n} \times \frac{(-1)^n}{x^n} \\ &= (-1)^n {}^{2n+1}C_n x \end{aligned}$$

And,

$$T_{n+2} = T_{n+1+1}$$

$$\begin{aligned} &= {}^{2n+1}C_n x^{2n+1-n-1} \frac{(-1)^{n+1}}{x^{n+1}} \\ &= (-1)^{n+1} {}^{2n+1}C_n \times \frac{1}{x} \end{aligned}$$

Hence, the middle term is $(-1)^n \cdot {}^{2n+1}C_n x$ and $(-1)^{n+1} \cdot {}^{2n+1}C_n (1/x)$.

(vi) $(x/3 + 9y)^{10}$

We have,

$(x/3 + 9y)^{10}$ where, $n = 10$ is an even number.

So the middle term is $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$. i.e., 6th term.

Now,

$$T_6 = T_{5+1}$$

$$\begin{aligned} &= {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} (9y)^5 \\ &= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \times \frac{1}{3^5} \times 9^5 \times x^5 y^5 \\ &= 61236 x^5 y^5 \end{aligned}$$

Hence, the middle term is $61236x^5y^5$.

(vii) $(3 - x^3/6)^7$

We have,

$(3 - x^3/6)^7$ where, $n = 7$ (odd number).

So the middle terms are $((n+1)/2) = ((7+1)/2) = 8/2 = 4$ and

$((n+1)/2 + 1) = ((7+1)/2 + 1) = (8/2 + 1) = (4 + 1) = 5$

The terms are 4th and 5th.

Now,

$$T_4 = T_{3+1}$$

$$= {}^7C_3 (3)^{7-3} (-x^3/6)^3$$

$$= -105/8 x^9$$

And,

$$T_5 = T_{4+1}$$

$$= {}^9C_4 (3)^{9-4} (-x^3/6)^4$$

$$= \frac{7 \times 6 \times 5}{3 \times 2} \times 3^5 \times \frac{1}{6^4} x^{12}$$

$$= \frac{35}{48} x^{12}$$

Hence, the middle terms are $-105/8 x^9$ and $35/48 x^{12}$.

(viii) $(2ax - b/x^2)^{12}$

We have,

$(2ax - b/x^2)^{12}$ where, $n = 12$ is an even number.

So the middle term is $(n/2 + 1) = (12/2 + 1) = (6 + 1) = 7$. i.e., 7th term.

Now,

$$\begin{aligned}
 T_7 &= T_{6+1} \\
 &= {}^{12}C_6 (2ax)^{12-6} \left(\frac{-b}{x^2}\right)^6 \\
 &= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1} \times \left(\frac{2ab}{x}\right)^6 \\
 &= \frac{59136 a^6 b^6}{x^6}
 \end{aligned}$$

Hence, the middle term is $(59136a^6b^6)/x^6$.

(ix) $(p/x + x/p)^9$

We have,

$(p/x + x/p)^9$ where, $n = 9$ (odd number).

So the middle terms are $((n+1)/2) = ((9+1)/2) = 10/2 = 5$ and

$((n+1)/2 + 1) = ((9+1)/2 + 1) = (10/2 + 1) = (5 + 1) = 6$

The terms are 5th and 6th.

Now,

$$\begin{aligned}
 T_5 &= T_{4+1} \\
 &= {}^9C_4 \left(\frac{p}{x}\right)^{9-4} \left(\frac{x}{p}\right)^4 \\
 &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times \left(\frac{p}{x}\right) \\
 &= \frac{126 p}{x}
 \end{aligned}$$

And,

$$\begin{aligned}
 T_6 &= T_{5+1} \\
 &= {}^9C_5 (p/x)^{9-5} (x/p)^5 \\
 &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times \left(\frac{x}{p}\right) \\
 &= \frac{126 x}{p}
 \end{aligned}$$

Hence, the middle terms are $126p/x$ and $126x/p$.

(x) $(x/a - a/x)^{10}$

We have,

$(x/a - a/x)^{10}$ where, $n = 10$ (even number)

So the middle term is $(n/2 + 1) = (10/2 + 1) = (5 + 1) = 6$. ie., 6th term

Now,

$$T_6 = T_{5+1}$$

$$\begin{aligned}
&= {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(\frac{-a}{x}\right)^5 \\
&= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \\
&= -252
\end{aligned}$$

Hence, the middle term is -252.

16. Find the term independent of x in the expansion of the following expressions:

(i) $(\frac{3}{2}x^2 - \frac{1}{3x})^9$

(ii) $(2x + \frac{1}{3x^2})^9$

(iii) $(2x^2 - 3/x^3)^{25}$

(iv) $(3x - 2/x^2)^{15}$

(v) $((\sqrt{x}/3) + \sqrt{3/2x^2})^{10}$

(vi) $(x - 1/x^2)^{3n}$

(vii) $(\frac{1}{2}x^{1/3} + x^{-1/5})^8$

(viii) $(1 + x + 2x^3)(\frac{3}{2}x^2 - \frac{3}{3x})^9$

(ix) $(\sqrt[3]{x} + \frac{1}{2}\sqrt[3]{x})^{18}, x > 0$

(x) $(\frac{3}{2}x^2 - \frac{1}{3x})^6$

Solution:

(i) $(\frac{3}{2}x^2 - \frac{1}{3x})^9$

Given:

$$(\frac{3}{2}x^2 - \frac{1}{3x})^9$$

If $(r + 1)$ th term in the given expression is independent of x.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$= {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(\frac{-1}{3x}\right)^r$$

$$= (-1)^r {}^9C_r \cdot \frac{3^{9-2r}}{2^{9-r}} \times x^{18-2r-r}$$

For this term to be independent of x, we must have

$$18 - 3r = 0$$

$$3r = 18$$

$$r = 18/3$$

$$= 6$$

So, the required term is 7th term.

We have,

$$\begin{aligned}
 T_7 &= T_{6+1} \\
 &= {}^9C_6 \times (3^{9-12})/(2^{9-6}) \\
 &= (9 \times 8 \times 7)/(3 \times 2) \times 3^{-3} \times 2^{-3} \\
 &= 7/18
 \end{aligned}$$

Hence, the term independent of x is 7/18.

(ii) $(2x + 1/3x^2)^9$

Given:

$$(2x + 1/3x^2)^9$$

If (r + 1)th term in the given expression is independent of x.

Then, we have:

$$\begin{aligned}
 T_{r+1} &= {}^nC_r x^{n-r} a^r \\
 &= {}^9C_r \left(2x \right)^{9-r} \left(\frac{1}{3x^2} \right)^r \\
 &= {}^9C_r \cdot \frac{2^{9-r}}{3^r} x^{9-r-2r}
 \end{aligned}$$

For this term to be independent of x, we must have

$$9 - 3r = 0$$

$$3r = 9$$

$$r = 9/3$$

$$= 3$$

So, the required term is 4th term.

We have,

$$\begin{aligned}
 T_4 &= T_{3+1} \\
 &= {}^9C_3 \times (2^6)/(3^3) \\
 &= {}^9C_3 \times 64/27
 \end{aligned}$$

Hence, the term independent of x is ${}^9C_3 \times 64/27$.

(iii) $(2x^2 - 3/x^3)^{25}$

Given:

$$(2x^2 - 3/x^3)^{25}$$

If (r + 1)th term in the given expression is independent of x.

Then, we have:

$$\begin{aligned}
 T_{r+1} &= {}^nC_r x^{n-r} a^r \\
 &= {}^{25}C_r (2x^2)^{25-r} (-3/x^3)^r
 \end{aligned}$$

$$= (-1)^r {}^{25}C_r \times 2^{25-r} \times 3^r x^{50-2r-3r}$$

For this term to be independent of x, we must have

$$50 - 5r = 0$$

$$5r = 50$$

$$r = 50/5$$

$$= 10$$

So, the required term is 11th term.

We have,

$$T_{11} = T_{10+1}$$

$$= (-1)^{10} {}^{25}C_{10} \times 2^{25-10} \times 3^{10}$$

$$= {}^{25}C_{10} (2^{15} \times 3^{10})$$

Hence, the term independent of x is ${}^{25}C_{10} (2^{15} \times 3^{10})$.

$$\text{(iv)} (3x - 2/x^2)^{15}$$

Given:

$$(3x - 2/x^2)^{15}$$

If (r + 1)th term in the given expression is independent of x.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$= {}^{15}C_r (3x)^{15-r} (-2/x^2)^r$$

$$= (-1)^r {}^{15}C_r \times 3^{15-r} \times 2^r x^{15-r-2r}$$

For this term to be independent of x, we must have

$$15 - 3r = 0$$

$$3r = 15$$

$$r = 15/3$$

$$= 5$$

So, the required term is 6th term.

We have,

$$T_6 = T_{5+1}$$

$$= (-1)^5 {}^{15}C_5 \times 3^{15-5} \times 2^5$$

$$= -3003 \times 3^{10} \times 2^5$$

Hence, the term independent of x is $-3003 \times 3^{10} \times 2^5$.

$$\text{(v)} ((\sqrt{x}/3) + \sqrt{3}/2x^2)^{10}$$

Given:

$$((\sqrt{x/3}) + \sqrt{3/2}x^2)^{10}$$

If $(r + 1)$ th term in the given expression is independent of x .

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^nC_r x^{n-r} a^r \\ &= {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{3}{2x^2}\right)^r \\ &= {}^{10}C_r \cdot \frac{3^{\frac{r}{2} - \frac{10-r}{2}}}{2^r} x^{\frac{10-r}{2} - 2r} \end{aligned}$$

For this term to be independent of x , we must have

$$(10-r)/2 - 2r = 0$$

$$10 - 5r = 0$$

$$5r = 10$$

$$r = 10/5$$

$$= 2$$

So, the required term is 3rd term.

We have,

$$\begin{aligned} T_3 &= T_{2+1} \\ &= {}^{10}C_2 \times \frac{3^{\frac{2}{2} - \frac{10-2}{2}}}{2^2} \\ &= \frac{10 \times 9}{2 \times 4 \times 9} \\ &= 90/72 \\ &= 15/12 \\ &= 5/4 \end{aligned}$$

Hence, the term independent of x is $5/4$.

$$\text{(vi)} (x - 1/x^2)^{3n}$$

Given:

$$(x - 1/x^2)^{3n}$$

If $(r + 1)$ th term in the given expression is independent of x .

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^nC_r x^{n-r} a^r \\ &= {}^{3n}C_r x^{3n-r} (-1/x^2)^r \\ &= (-1)^r {}^{3n}C_r x^{3n-r-2r} \end{aligned}$$

For this term to be independent of x , we must have

$$3n - 3r = 0$$

$$r = n$$

So, the required term is (n+1)th term.

We have,

$$(-1)^n {}^{3n}C_n$$

Hence, the term independent of x is $(-1)^n {}^{3n}C_n$

$$\text{(vii)} \quad \left(\frac{1}{2} x^{1/3} + x^{-1/5}\right)^8$$

Given:

$$\left(\frac{1}{2} x^{1/3} + x^{-1/5}\right)^8$$

If (r + 1)th term in the given expression is independent of x.

Then, we have:

$$\begin{aligned} T_{r+1} &= {}^nC_r x^{n-r} a^r \\ &= {}^8C_r \left(\frac{1}{2} x^{1/3}\right)^{8-r} \left(x^{-1/5}\right)^r \\ &= {}^8C_r \cdot \frac{1}{2^{8-r}} x^{\frac{8-r}{3} - \frac{r}{5}} \end{aligned}$$

For this term to be independent of x, we must have

$$(8-r)/3 - r/5 = 0$$

$$(40 - 5r - 3r)/15 = 0$$

$$40 - 5r - 3r = 0$$

$$40 - 8r = 0$$

$$8r = 40$$

$$r = 40/8$$

$$= 5$$

So, the required term is 6th term.

We have,

$$\begin{aligned} T_6 &= T_{5+1} \\ &= {}^8C_5 \times 1/(2^{8-5}) \\ &= (8 \times 7 \times 6)/(3 \times 2 \times 1) \\ &= 7 \end{aligned}$$

Hence, the term independent of x is 7.

$$\text{(viii)} \quad (1 + x + 2x^3) (3/2x^2 - 3/3x)^9$$

Given:

$$(1 + x + 2x^3) (3/2x^2 - 3/3x)^9$$

If $(r + 1)$ th term in the given expression is independent of x .

Then, we have:

$$(1 + x + 2x^3) (3/2x^2 - 3/3x)^9 =$$

$$= (1 + x + 2x^3) \left[\left(\frac{3}{2}x^2\right)^9 - {}^9C_1 \left(\frac{3}{2}x^2\right)^8 \frac{1}{3x} \dots + {}^9C_6 \left(\frac{3}{2}x^2\right)^3 \left(\frac{1}{3x}\right)^6 - {}^9C_7 \left(\frac{3}{2}x^2\right)^2 \left(\frac{1}{3x}\right)^7 \right]$$

By computing we get,

The term independent of x

$$\begin{aligned} &= 1 \left[{}^9C_6 \frac{3^3}{2^3} \times \frac{1}{3^6} \right] - 2x^3 \left[{}^9C_7 \frac{3^3}{2^3} \times \frac{1}{3^7} \times \frac{1}{x^3} \right] \\ &= \left[\frac{9 \times 8 \times 7}{1 \times 2 \times 3} \times \frac{1}{8 \times 27} \right] - 2 \left[\frac{9 \times 8}{1 \times 2} - \frac{1}{4 \times 243} \right] \\ &= \frac{7}{18} - \frac{2}{27} \\ &= \frac{17}{54} \end{aligned}$$

$$= 7/18 - 2/27$$

$$= (189 - 36)/486$$

$$= 153/486 \text{ (divide by 9)}$$

$$= 17/54$$

Hence, the term independent of x is $17/54$.

$$\text{(ix) } (\sqrt[3]{x} + 1/2\sqrt[3]{x})^{18}, x > 0$$

Given:

$$(\sqrt[3]{x} + 1/2\sqrt[3]{x})^{18}, x > 0$$

If $(r + 1)$ th term in the given expression is independent of x .

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$\begin{aligned} &= {}^{18}C_r \left(x^{1/3} \right)^{18-r} \left(\frac{1}{2x^{1/3}} \right)^r \\ &= {}^{18}C_r \times \frac{1}{2^r} x^{\frac{18-r}{3} - \frac{r}{3}} \end{aligned}$$

For this term to be independent of r , we must have

$$(18-r)/3 - r/3 = 0$$

$$(18 - r - r)/3 = 0$$

$$18 - 2r = 0$$

$$2r = 18$$

$$r = 18/2$$

$$= 9$$

So, the required term is 10th term.

We have,

$$T_{10} = T_{9+1}$$

$$= {}^{18}C_9 \times 1/2^9$$

Hence, the term independent of x is ${}^{18}C_9 \times 1/2^9$.

$$(x) (3/2x^2 - 1/3x)^6$$

Given:

$$(3/2x^2 - 1/3x)^6$$

If $(r + 1)$ th term in the given expression is independent of x.

Then, we have:

$$T_{r+1} = {}^nC_r x^{n-r} a^r$$

$$= {}^6C_r \left(\frac{3}{2}x^2\right)^{6-r} \left(\frac{-1}{3x}\right)^r$$

$$= (-1)^r {}^6C_r \times \frac{3^{6-r-r}}{2^{6-r}} x^{12-2r-r}$$

For this term to be independent of r, we must have

$$12 - 3r = 0$$

$$3r = 12$$

$$r = 12/3$$

$$= 4$$

So, the required term is 5th term.

We have,

$$T_5 = T_{4+1}$$

$$= {}^6C_4 \times \frac{3^{6-4-4}}{2^{6-4}}$$

$$= \frac{6 \times 5}{2 \times 1 \times 4 \times 9}$$

$$= \frac{5}{12}$$

Hence, the term independent of x is 5/12.

17. If the coefficients of $(2r + 4)$ th and $(r - 2)$ th terms in the expansion of $(1 + x)^{18}$ are equal, find r.

Solution:

Given:

$$(1 + x)^{18}$$

We know, the coefficient of the r term in the expansion of $(1 + x)^n$ is ${}^nC_{r-1}$

So, the coefficients of the $(2r + 4)$ and $(r - 2)$ terms in the given expansion are ${}^{18}C_{2r+4-1}$ and ${}^{18}C_{r-2-1}$

For these coefficients to be equal, we must have

$${}^{18}C_{2r+4-1} = {}^{18}C_{r-2-1}$$

$${}^{18}C_{2r+3} = {}^{18}C_{r-3}$$

$$2r + 3 = r - 3 \text{ (or) } 2r + 3 + r - 3 = 18 \text{ [Since, } {}^nC_r = {}^nC_s \Rightarrow r = s \text{ (or) } r + s = n]$$

$$2r - r = -3 - 3 \text{ (or) } 3r = 18 - 3 + 3$$

$$r = -6 \text{ (or) } 3r = 18$$

$$r = -6 \text{ (or) } r = 18/3$$

$$r = -6 \text{ (or) } r = 6$$

$\therefore r = 6$ [since, r should be a positive integer.]

18. If the coefficients of $(2r + 1)$ th term and $(r + 2)$ th term in the expansion of $(1 + x)^{43}$ are equal, find r .

Solution:

Given:

$$(1 + x)^{43}$$

We know, the coefficient of the r term in the expansion of $(1 + x)^n$ is ${}^nC_{r-1}$

So, the coefficients of the $(2r + 1)$ and $(r + 2)$ terms in the given expansion are ${}^{43}C_{2r+1-1}$ and ${}^{43}C_{r+2-1}$

For these coefficients to be equal, we must have

$${}^{43}C_{2r+1-1} = {}^{43}C_{r+2-1}$$

$${}^{43}C_{2r} = {}^{43}C_{r+1}$$

$$2r = r + 1 \text{ (or) } 2r + r + 1 = 43 \text{ [Since, } {}^nC_r = {}^nC_s \Rightarrow r = s \text{ (or) } r + s = n]$$

$$2r - r = 1 \text{ (or) } 3r + 1 = 43$$

$$r = 1 \text{ (or) } 3r = 43 - 1$$

$$r = 1 \text{ (or) } 3r = 42$$

$$r = 1 \text{ (or) } r = 42/3$$

$$r = 1 \text{ (or) } r = 14$$

$\therefore r = 14$ [since, value '1' gives the same term]

19. Prove that the coefficient of $(r + 1)$ th term in the expansion of $(1 + x)^{n+1}$ is equal to the sum of the coefficients of r th and $(r + 1)$ th terms in the expansion of $(1 + x)^n$.

Solution:

We know, the coefficients of $(r + 1)$ th term in $(1 + x)^{n+1}$ is ${}^{n+1}C_r$

So, sum of the coefficients of the r th and $(r + 1)$ th terms in $(1 + x)^n$ is

$$(1 + x)^n = {}^nC_{r-1} + {}^nC_r$$

$$= {}^{n+1}C_r \text{ [since, } {}^nC_{r+1} + {}^nC_r = {}^{n+1}C_{r+1}]$$

Hence proved.