

HC VERMA Solutions for Class 11 Physics Chapter 17 Light Waves

Question 1

Find the range of frequency of light that is visible to an average human being ($400 \text{ nm} < \lambda < 700 \text{ nm}$).

Solution 1

$$v = f \times \lambda$$

$$f_{\max} = \frac{v}{\lambda_{\min}}$$

$$f_{\max} = \frac{3 \times 10^8}{400 \times 10^{-9}} = 7.5 \times 10^{14} \text{ Hz}$$

$$f_{\min} = \frac{v}{\lambda_{\max}}$$

$$f_{\min} = \frac{3 \times 10^8}{700 \times 10^{-9}} = 4.3 \times 10^{14} \text{ Hz}$$

So, Range of frequency

$$4.3 \times 10^{14} \text{ Hz to } 7.5 \times 10^{14} \text{ Hz}$$

Question 2

The wavelength of sodium light in air is 589 nm. (a) Find its frequency in air. (b) Find its wavelength in water (refractive index=1.33). (c) Find its frequency in water. (d) Find its speed in the water.

Solution 2

$$(a) v = f \times \lambda$$

$$3 \times 10^8 = f_{\text{air}} \times 589 \times 10^{-9}$$

$$f_{\text{air}} = 5.09 \times 10^{14} \text{ Hz}$$

$$(b) \lambda_{\text{med}} = \frac{\lambda_{\text{air}}}{\mu}$$

$$= \frac{589}{1.33} \text{ nm}$$

$$\lambda_{\text{med}} = 443 \text{ nm}$$

$$(c) f_{\text{air}} = f_{\text{water}} [\because \text{frequency depends only upon source, not an medium}]$$

$$f_{\text{water}} = 5.09 \times 10^{14} \text{ Hz}$$

$$(d) v = \frac{c}{\mu}$$

$$= \frac{3 \times 10^8}{1.33}$$

$$v = 2.25 \times 10^8 \text{ m/s}$$

Question 3

The index of refraction of fused quartz is 1.472 for light of wavelength 400 nm and is 1.452 for light of wavelength 760 nm. Find the speeds of light of these wavelengths in fused quartz.

Solution 3

$$v = \frac{c}{\lambda}$$

For $\lambda = 400 \text{ nm}$; $\mu = 1.472$

$$v = \frac{3 \times 10^8}{1.472}$$

$$v = 2.04 \times 10^8 \text{ m/s}$$

For $\lambda = 760 \text{ nm}$; $\mu = 1.472$

$$v' = \frac{3 \times 10^8}{1.452}$$

$$v' = 2.07 \times 10^8 \text{ m/s}$$

Question 4

The speed of the yellow light in the certain liquid is $2.4 \times 10^8 \text{ m/s}$. Find the refractive index of the liquid.

Solution 4

$$\mu = \frac{c}{v}$$

$$\mu = \frac{3 \times 10^8}{2.4 \times 10^8}$$

$$\mu = 1.25$$

Question 5

Two narrow slits emitting light in phase are separated by a distance of 1.0 cm . The wavelength of the light is $5.0 \times 10^{-7} \text{ m}$. The interference pattern is observed on a screen placed at a distance of 1.0 m . (a) Find the separation between the consecutive maxima. Can you expect to distinguish between these maxima? (b) Find the separation between the sources which will give a separation of 1.0 mm between the consecutive maxima.

Solution 5

$$(a) d = 1 \text{ cm} = 1 \times 10^{-2} \text{ m}$$

$$\lambda = 5 \times 10^{-7} \text{ m}$$

$$D = 1 \text{ m}$$

$$\beta = \frac{D\lambda}{d}$$

$$= \frac{1 \times 5 \times 10^{-7}}{1 \times 10^{-2}}$$

$$\beta = 5 \times 10^{-5} \text{ m}$$

$$\beta = 0.05 \text{ mm}$$

$$(b) \beta = 1 \text{ mm}; D = 1 \text{ m}; \lambda = 5 \times 10^{-7} \text{ m}; d = ??$$

$$\beta = \frac{D\lambda}{d}$$

$$1 \times 10^{-3} = \frac{1 \times 5 \times 10^{-7}}{d}$$

$$d = 5 \times 10^{-4} \text{ m}$$

$$d = 0.5 \text{ mm}$$

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Question 6

The separation between the consecutive dark fringes in a Young's double slit experiment is 1.0 mm. The screen is placed at a distance of 2.5 m from the slits and the separation between the slits is 1.0 mm. Calculate the wavelength of light used for the experiment.

Solution 6

$$\beta = 1 \text{ mm}; D = 2.5 \text{ m}; d = 1 \text{ mm}$$

$$\beta = \frac{D\lambda}{d}$$

$$1 \times 10^{-3} = \frac{2.5 \times \lambda}{1 \times 10^{-3}}$$

$$\lambda = 4 \times 10^{-7} \text{ m}$$

$$\lambda = 400 \text{ nm}$$

Question 7

In a double slit interference experiment, the separation between the slits is 1.0 mm, the wavelength of light used is $5.0 \times 10^{-7} \text{ m}$ and the distance of the screen from the slits is 1.0m. (a) Find the distance of the centre of the central maximum. (b) How many bright fringes are formed in one centimeter width on the screen?

Solution 7

$$d = 1 \text{ mm}; \lambda = 5 \times 10^{-7} \text{ m}; D = 1 \text{ m}$$

$$\frac{\beta}{2} = \frac{D\lambda}{2d}$$

$$= \frac{1 \times 5 \times 10^{-7}}{2 \times 1 \times 10^{-3}}$$

$$= 2.5 \times 10^{-4} \text{ m}$$

$$= 0.25 \text{ mm}$$

$$\beta = \frac{D\lambda}{d} = \frac{1 \times 5 \times 10^{-7}}{10^{-3}} = 0.5 \text{ mm}$$

$$\text{No. of fringes obtained in 1 cm} = \frac{1 \times 10^{-2}}{0.5 \times 10^{-3}} = 20 \text{ fringes.}$$

Question 8

In a Young's double slit experiment, two narrow vertical slits placed 0.800 mm apart are illuminated by the same source of yellow light of wavelength 589 nm. How far are the adjacent bright bands in the interference pattern observed on a screen 2.00 m away?

Solution 8

$$d = 0.8 \text{ mm}; \lambda = 589 \text{ nm}; D = 2 \text{ m}$$

Distance between consecutive bright

$$\beta = \frac{D\lambda}{d}$$

$$= \frac{2 \times 589 \times 10^{-9}}{0.8 \times 10^{-3}}$$

$$= 1.47 \times 10^{-3} \text{ m}$$

$$= 1.47 \text{ mm}$$

Question 9

Find the angular separation between the consecutive bright fringes in a Young's double slit experiment with blue-green light of wavelength 500 nm. The separation between the slits is $2.0 \times 10^{-3} \text{ m}$.

Solution 9

$$\begin{aligned}\text{Angular fringe width, } \varphi &= \frac{\lambda}{d} \\ &= \frac{500 \times 10^{-9}}{2 \times 10^{-3}} \\ \varphi &= 5 \times 10^{-5} \text{ radian} \\ &= 25 \times 10^{-5} \times \frac{180^\circ}{\pi} \\ \varphi &= 0.014^\circ\end{aligned}$$

Question 10

A source of light emitting light of wavelengths 480 nm and 600 nm is used in a double slit interference experiment. The separation between the slits is 0.25 mm and the interference is observed on a screen placed at 150 cm from the slits. Find the linear separation between the first maximum (next to the central maximum) corresponding to the two wavelengths.

Solution 10

$$\begin{aligned}\lambda_1 &= 480 \text{ nm}; \lambda_2 = 600 \text{ nm}; d = 0.25 \text{ mm} \text{ \& } D = 1.5 \text{ m} \\ \text{Distance of first maxima from center} &= \frac{D\lambda}{d} \\ \text{Linear separation between first maxima, } y_2 - y_1 &= \frac{D}{d} (\lambda_2 - \lambda_1) \\ &= \frac{1.5(600 - 480) \times 10^{-9}}{0.25 \times 10^{-3}} \\ y_2 - y_1 &= 0.72 \text{ mm}\end{aligned}$$

Question 11

White light is used in a Young's double slit experiment. Find the minimum order of the violet fringe ($\lambda=400 \text{ nm}$) which overlaps with a red fringe ($\lambda=700 \text{ nm}$).

Solution 11

Central maxima for both wavelength forms at same position at centre of screen.

Let m^{th} maxima of violet coincides with n^{th} maxima of red light.

$$\begin{aligned}m \frac{D\lambda_v}{d} &= n \frac{D\lambda_R}{d} \\ m\lambda_v &= n\lambda_R \\ m \times 400 &= n \times 700 \\ \frac{m}{n} &= \frac{7}{4}\end{aligned}$$

Least value of m is 7.

Question 12

Find the thickness of a plate which will produce a change in optical path equal to half the wavelength λ of the light passing through it normally. The refractive index of the plate is μ .

Solution 12

$$\Delta = (\mu - 1)t$$

$$\frac{\lambda}{2} = (\mu - 1)t$$

$$t = \frac{\lambda}{2(\mu - 1)}$$

Question 13

A plate of thickness t made of a material of refractive index μ is placed in front of one of the slits in a double slit experiment. (a) Find the change in the optical path due to introduction of the plate. (b) What should be the minimum thickness t which will make the intensity at the centre of the fringe pattern zero? Wavelength of the light used is λ . Neglect any absorption of light in the plate.

Solution 13

(a) Change in optical path = $(\mu - 1)t$

(b) $\Delta = (\mu - 1)t$

$$\frac{(2n - 1)\lambda}{2} = (\mu - 1)t$$

For minimum thickness, D-1 will be formed.

$$\frac{(2 \times 1 - 1)\lambda}{2} = (\mu - 1)t$$

$$t = \frac{\lambda}{2(\mu - 1)}$$

Question 14

A transparent paper (refractive index = 1.45) of thickness 0.02 mm is pasted on one of the slits of a Young's slit experiment which uses monochromatic light of wavelength 620 nm. How many fringes will cross through the centre if the paper is removed?

Solution 14

$$\Delta = (\mu - 1)t = \frac{d}{D} \cdot y$$

$$(1.45 - 1)(0.02 \times 10^{-3}) = \frac{d}{D} \cdot y \times \frac{\lambda}{\lambda}$$

$$0.45 \times 0.02 \times 10^{-3} = \frac{y \times \lambda}{\beta}$$

$$y = \frac{0.45 \times 0.02 \times 10^{-3}}{620 \times 10^{-9}} \cdot \beta$$

$$y = 14.5 \beta$$

Hence, 14.5 fringes will shift.

Question 15

In a Young's double slit experiment using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 micron (1 micron = 10^{-5} m) is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the screen and slits is doubled. It is found that the distance between the successive maxima now is the same as the observed fringe-shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in the experiment.

Solution 15

Distance moved by fringes after introduction of slab

$$\Delta = (\mu - 1)t = \frac{d}{D} \cdot y$$

$$y = \frac{(\mu - 1)tD}{d}$$

Now, distance of screen is doubled, new fringe width

$$D' = 2D$$

$$\beta' = \frac{D'\lambda}{d}$$

$$\beta' = \frac{2D\lambda}{d}$$

Given,

$$y = \beta'$$

$$\frac{(\mu - 1)tD}{d} = \frac{2D\lambda}{d}$$

$$\lambda = \frac{(\mu - 1)t}{2}$$

$$\lambda = \frac{(1.6 - 1)(1.964 \times 10^{-6})}{2}$$

$$\lambda = 589.2 \text{ nm}$$

Question 16

A mica strip and a polysterene strip are fitted on the two slits of a double slit apparatus. The thickness of the strips is 0.50 mm and the separation between the slits is 0.12 cm. The refractive index of mica and polysterene are 1.58 and 1.55 respectively for the light of wavelength 590 nm which is used in the experiment. The interference is observed on a screen a distance one metre away. (a) What would be the fringe-width? (b) At what distance from the centre will be the first maximum be located?

Solution 16

$$\begin{aligned} \text{(a)} \quad \beta &= \frac{D\lambda}{d} \\ &= \frac{1 \times 590 \times 10^{-9}}{0.12 \times 10^{-2}} \end{aligned}$$

$$\beta = 4.9 \times 10^{-4} \text{ m}$$

(b) No. of fringes shifted

Path difference due to sheets,

$$\Delta = (\mu_m - 1)t - (\mu_p - 1)t$$

$$= (\mu_m - \mu_p)t$$

$$= (1.58 - 1.55) \times 0.5 \times 10^{-3}$$

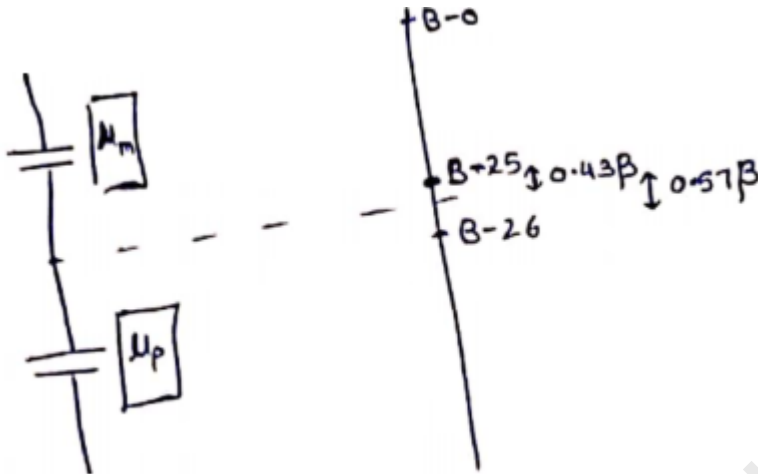
$$\frac{d}{D} \cdot y = 0.015 \times 10^{-3} \text{ m}$$

$$y = \frac{0.015 \times 10^{-3}}{\lambda} \times \frac{D\lambda}{d}$$

$$y = \frac{0.015 \times 10^{-3}}{590 \times 10^{-9}} \cdot \beta$$

$$y = 25.43\beta$$

25 Fringes will be shifted completely



Position of first maxima on one side = 0.43β

$$= 0.43 \times 4.91 \times 10^{-4}$$

$$= 2.1113 \times 10^{-4} \text{ m}$$

$$= 0.021 \text{ cm}$$

and on other side = 0.57β

$$= 0.57 \times 4.91 \times 10^{-4}$$

$$= 0.028 \text{ cm}$$

Question 17

Two transparent slabs having equal thickness but different refractive indices μ_1 and μ_2 are pasted side by side to form a composite slab. This slab is placed just after the double slit in a Young's experiment so that the light from one slit goes through one material and other slit goes through the other material. What should be the minimum thickness of the slab so that there is a minimum at the point p_0 which is equidistant from the slits?

Solution 17

Path difference at centre of screen

$$\Delta = (\mu_1 - 1)t - (\mu_2 - 1)t$$

$$\Delta = (\mu_1 - \mu_2)t$$

Minima is formed at the centre of screen

$$\Delta = \frac{(2n - 1)\lambda}{2} = (\mu_1 - \mu_2)t$$

$$t = \frac{(2n - 1)\lambda}{2(\mu_1 - \mu_2)}$$

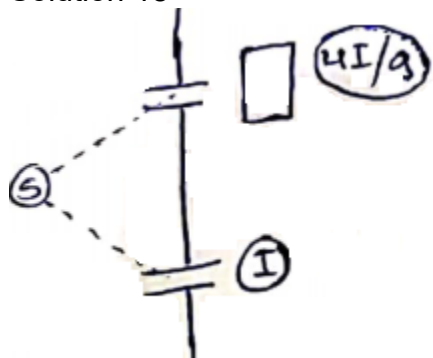
For minimum thickness, $n=1$

$$t = \frac{\lambda}{2(\mu_1 - \mu_2)}$$

Question 18

A thin paper of thickness 0.02 mm having a refractive index 1.45 is pasted across one of the slits in a Young's double slit experiment. The paper transmits $4/9$ of the light energy falling on it. (a) Find the ratio of the maximum intensity to the minimum intensity in the fringe pattern. (b) How many fringes will cross through the centre if an identical paper piece is pasted on the other slit also? The wavelength of the light used is 600 nm .

Solution 18



$$(a) \quad I_1 = \frac{4I}{9} ; I_2 = I$$

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{\frac{4I}{9}} + \sqrt{I})^2}{(\sqrt{\frac{4I}{9}} - \sqrt{I})^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{25}{1}$$

$$(b) \text{ Initially path difference at center of screen} = (\mu - 1)t$$

$$\Delta = (1.45 - 1)(0.02 \times 10^{-3})$$

$$\frac{d}{D} \cdot y = 0.45 \times 0.02 \times 10^{-3}$$

$$y = \frac{90 \times 10^{-7} D \lambda}{\lambda d}$$

$$y = \frac{90 \times 10^{-7}}{600 \times 10^{-9}} \beta$$

$$y = 15\beta$$

So, B-15 is formed.

Now, when identical paper piece is pasted, path difference at center will be zero. So, B-0 will be formed.

Hence, 15 fringes will shift.

Question 19

A Young's double slit apparatus has slits separated by 0.28 mm and a screen 48 cm away from the slits. The whole apparatus is immersed in water and the slits are illuminated by the red light ($\lambda = 700 \text{ nm}$ in vacuum). Find the fringe-width of the pattern formed on the screen.

Solution 19

Fringe width

$$\beta = \frac{D\lambda}{\mu d}$$

$$= \frac{(0.48)(700 \times 10^{-9})}{(1.33)(0.28 \times 10^{-3})}$$

$$= 9 \times 10^{-4} \text{ m}$$

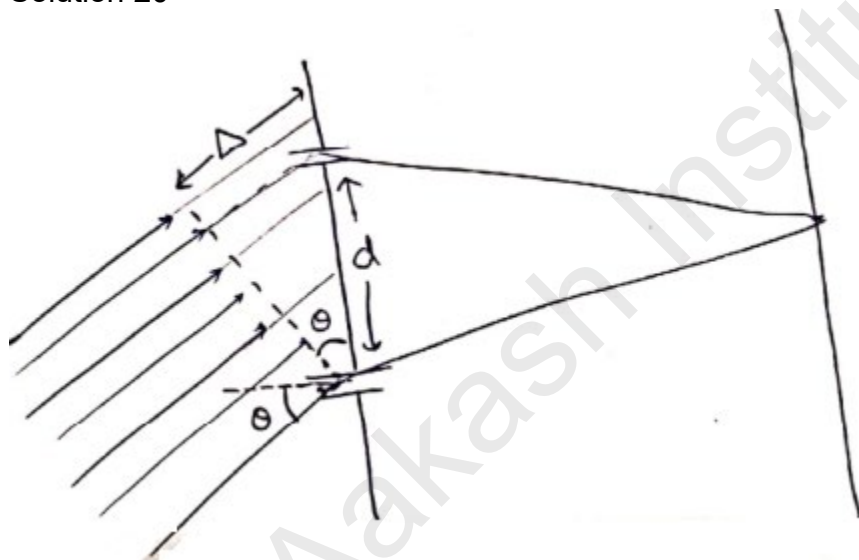
$$\beta = 0.9 \text{ mm}$$

Question 20

A parallel beam of monochromatic light is used in a Young's double slit experiment. The slits are separated by a distance d and the screen is placed parallel to the plane of the slits. Show that if the

incident beam makes an angle $\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right)$ with the normal to the plane of the slits, there will be a dark fringe at the centre P_0 of the pattern.

Solution 20



Path difference between waves meeting at center of screen will be due to pre-slit path difference.

$$\Delta = d \sin \theta$$

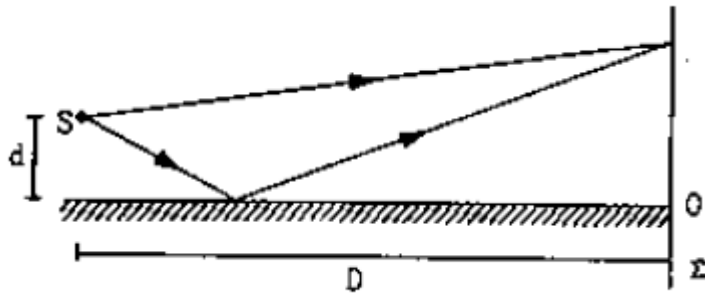
$$= d \left(\frac{\lambda}{2d} \right)$$

$$\Delta = \frac{\lambda}{2}$$

So, dark fringe is obtained at centre of screen.

Question 21

A narrow slit S transmitting light of wavelength λ is placed a distance d above a large plane mirror as shown in figure. The light coming directly from the slit and that coming after the reflection interface at a screen Σ placed at a distance D from the slit. (a) What will be the intensity at a point just above the mirror, i.e., just above O ? (b) At what distance from O does the first maximum occur?



Solution 21

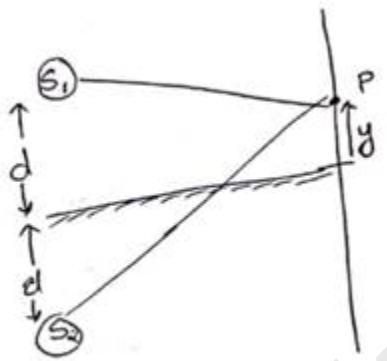
(a) The phase of a light wave reflecting from plane mirror will change by π .

At point just above O, the $\varphi_{\text{path}} = 0$ and $\varphi_{\text{ref}} = \pi$

$$\varphi_{\text{net}} = \varphi_{\text{path}} + \varphi_{\text{ref}}$$

$$\varphi_{\text{net}} = \pi$$

So, minima is formed i.e., zero intensity.



(b) Let y distance above O, maxima is obtained at point P.

$$\varphi_{\text{net}} = \varphi_{\text{path}} + \varphi_{\text{ref}}$$

$$\varphi_{\text{net}} = \frac{2\pi}{\lambda} (S_2P - S_1P) + \pi$$

$$\varphi_{\text{net}} = \frac{2\pi}{\lambda} \left(\frac{2d}{D} \cdot y \right) + \pi$$

For maxima, $\varphi_{\text{net}} = 2n\pi$

$$2n\pi = \frac{4\pi d}{D\lambda} \cdot y + \pi$$

Put $n=1$ for y_{min}

$$2(1)\pi = \frac{4\pi d}{D\lambda} y_{\text{min}} + \pi$$

$$y_{\text{min}} = \frac{D\lambda}{4d}$$

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Question 22

A long narrow horizontal slit is placed 1 mm above a horizontal plane mirror. The interference between the light coming directly from the slit and that after reflection is seen on a screen 1.0 m away from the slit. Find the fringe-width if the light used has a wavelength of 700 nm.

Solution 22

Fringe width,

$$\beta = \frac{D\lambda}{d}$$

$$\beta = \frac{(1)(700 \times 10^{-9})}{(2 \times 1 \times 10^{-3})}$$

$$\beta = 0.35 \text{ mm}$$

Question 23

Consider the situation of the previous problem. If the mirror reflects only 64% of the light energy falling on it, what will be the ratio of the maximum to the minimum intensity in the interference pattern observed on the screen?

Solution 23

Intensity of light from source $I_1 = I$

Intensity of light after reflection $I_2 = \frac{64}{100}I = \frac{16I}{25}$

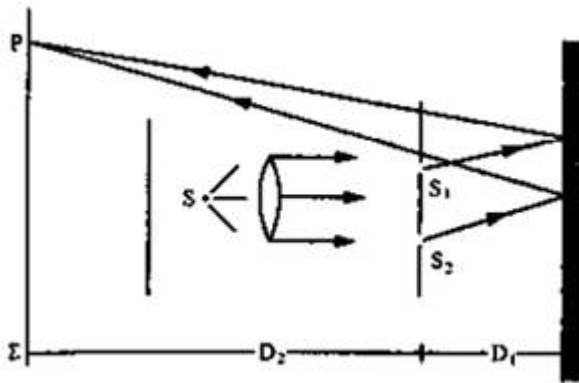
$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I} + \sqrt{\frac{16I}{25}})^2}{(\sqrt{I} - \sqrt{\frac{16I}{25}})^2}$$

$$\frac{I_{\max}}{I_{\min}} = \frac{81}{1}$$

Question 24

A double slit S_1 - S_2 is illuminated by a coherent light of wavelength λ . The slits are separated by a distance d . A plane mirror is placed in front of the double slit at a distance D_1 from it and a screen Σ is placed behind the double slit at a distance D_2 from it. The screen Σ receives only the light reflected by the mirror. Find the fringe-width of the interference pattern on the screen.



Solution 24

Distance between slits = d

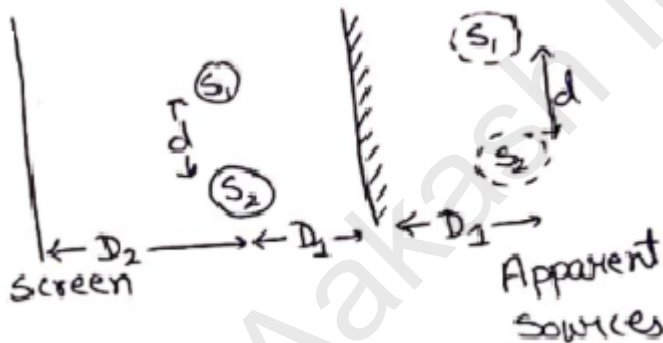
Distance between apparent sources and screen

$$= D_1 + D_1 + D_2$$

$$= 2D_1 + D_2$$

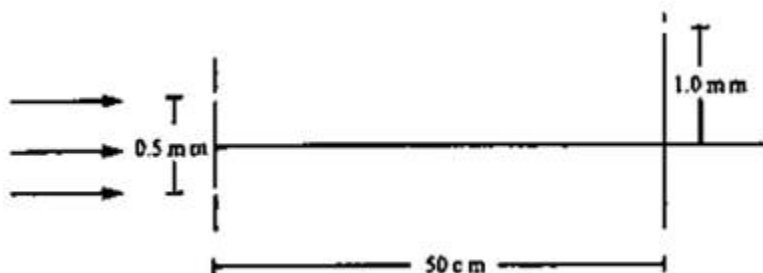
$$\text{Fringe width } \beta = \frac{D\lambda}{d}$$

$$\beta = \frac{(2D_1 + D_2)\lambda}{d}$$



Question 25

White coherent light (400 nm-700 nm) is sent through the slits of a Young's double slit experiment. The separation between the slits is 0.5 mm and the screen is 50 cm away from the slits. There is a hole in the screen at a point 1.0 mm away (along the width of the fringes) from the central line. (a) Which wavelength(s) will be absent in the light coming from the hole? (b) Which wavelength(s) will have a strong intensity?



Solution 25

(a) The wavelength which forms dark (minima) at hole will be absent.

$$\Delta = \frac{d}{D} \cdot y$$

$$\frac{(2n-1)\lambda}{2} = \frac{d}{D} \cdot y$$

$$\lambda = \frac{2}{(2n-1)} \frac{(0.5 \times 10^{-3})(1 \times 10^{-3})}{(50 \times 10^{-2})}$$

$$\lambda = \frac{2}{(2n-1)} \times 1000 \text{ nm}$$

For $n=1$, $\lambda=2000 \text{ nm}$

For $n=2$, $\lambda=667 \text{ nm}$

For $n=3$, $\lambda=400 \text{ nm}$

Thus, 667 nm and 400 nm are absent in the range of (400 nm-700 nm).

(b) The wavelength which forms bright (maxima) will have strong intensity.

$$\Delta = \frac{d}{D} \cdot y$$

$$n\lambda = \frac{d}{D} \cdot y$$

$$\lambda = \frac{1}{n} \frac{d}{D} \cdot y$$

$$= \frac{1}{n} \frac{(0.5 \times 10^{-3})(1 \times 10^{-3})}{(50 \times 10^{-2})}$$

$$\lambda = \frac{1000}{n} \text{ (in nm)}$$

For $n=1$; $\lambda=1000 \text{ nm}$

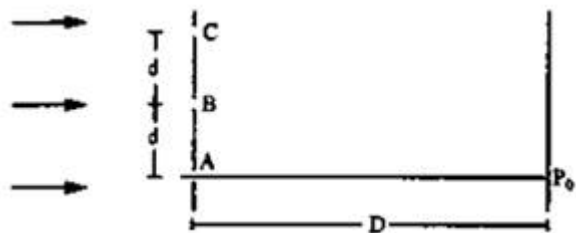
For $n=2$; $\lambda=500 \text{ nm}$

For $n=3$; $\lambda=334 \text{ nm}$

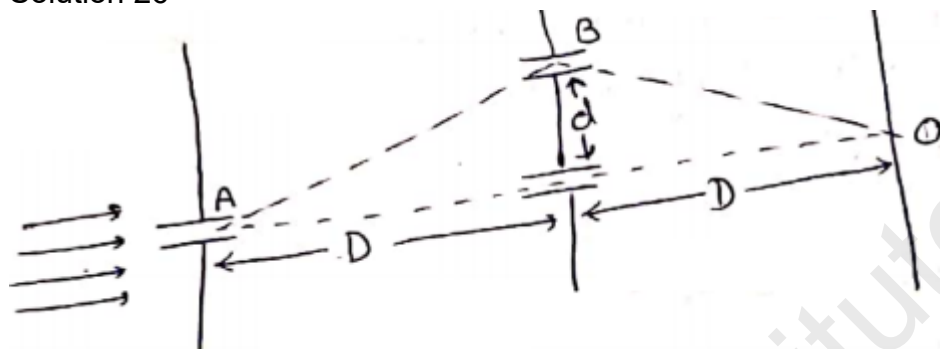
So, in Range of (400-700 nm) , light of wavelength 500 nm will have maxima.

Question 26

Consider the arrangement shown in figure. The distance D is large compared to the separation d between the slits. (a) Find the minimum value of d so that there is a dark fringe at O . (b) Suppose d has this value. Find the distance x at which the next bright fringe is formed. (c) Find the fringe-width.



Solution 26



(a) For minima at O

$$\Delta = \frac{(2n-1)\lambda}{2}$$

$$(AB + BO) - (AO) = \frac{(2n-1)\lambda}{2}$$

$$2\left(\sqrt{D^2 + d^2} - D\right) = \frac{(2n-1)\lambda}{2}$$

$$2D\left[\left(1 + \frac{d^2}{D^2}\right)^{\frac{1}{2}} - 1\right] = \frac{(2n-1)\lambda}{2}$$

By binomial expansion

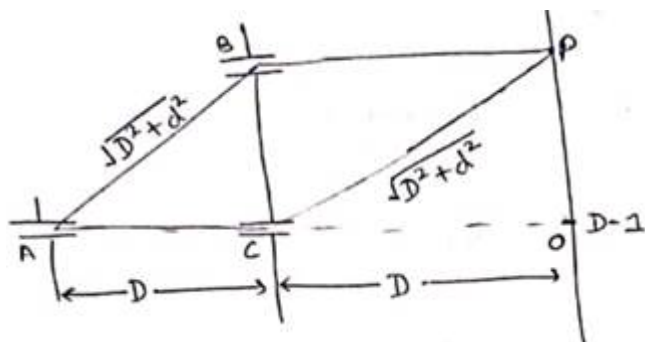
$$2D\left[\left(1 + \frac{1}{2}\frac{d^2}{D^2}\right) - 1\right] = \left(\frac{2n-1}{2}\right)\lambda$$

$$\frac{d^2}{D} = \left(\frac{2n-1}{2}\right)\lambda$$

For minima d, n=1

$$d_{\min} = \sqrt{\frac{D\lambda}{2}}$$

(b) At point O, D-1 is formed



Bright fringe next to D-1 will be B-0.

So, point at which path difference is zero B-0 will be formed.

By symmetry the point is in front of slit B.

So, minimum distance $x=d$.

(c) distance between B-0 and D-1 is d

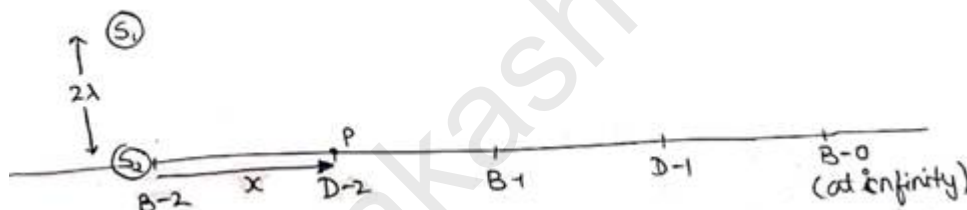
$$\frac{\beta}{2} = d$$

$$\beta = 2d$$

Question 27

Two coherent point sources S_1 and S_2 vibrating in phase emit light of wavelength λ . The separation between the sources is 2λ . Consider a line passing through S_2 and perpendicular to the line S_1, S_2 . What is the smallest distance from S_2 where a minimum of intensity occurs?

Solution 27



Let point P at distance x will have minimum intensity i.e., D-2

$$\text{Path difference} = \frac{3\lambda}{2}$$

$$S_1P - S_2P = \frac{3\lambda}{2}$$

$$\sqrt{(2\lambda)^2 + x^2} - x = \frac{3\lambda}{2}$$

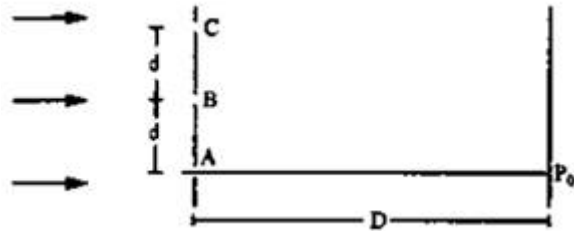
$$4\lambda^2 + x^2 = \left(\frac{3\lambda}{2} + x\right)^2$$

$$4\lambda^2 + x^2 = \frac{9\lambda^2}{4} + x^2 + 3\lambda x$$

$$x = \frac{7\lambda}{12}$$

Question 28

Figure shows equidistant slits being illuminated by a monochromatic parallel beam of light. Let $BP_0 - AP_0 = \lambda/3$ and $D \gg \lambda$. (a) Show that in this case $d = \sqrt{2\lambda D/3}$. (b) Show that the intensity at P_0 is three times the intensity due to any of the three slits individually.



Solution 28

(a) Path difference $BP_0 - AP_0 = \lambda/3$

$$\sqrt{D^2 + d^2} - D = \frac{\lambda}{3}$$

$$D \left[\left(1 + \frac{d^2}{D^2} \right)^{\frac{1}{2}} - 1 \right] = \frac{\lambda}{3}$$

By binomial expansion

$$D \left[\left(1 + \frac{1}{2} \frac{d^2}{D^2} \right) - 1 \right] = \frac{\lambda}{3}$$

$$\frac{d^2}{2D} = \frac{\lambda}{3}$$

$$d = \sqrt{\frac{2\lambda D}{3}}$$

(b) Path difference between wave from slit A and C is

$$\Delta_{CA} = CP_0 - AP_0$$

$$\Delta_{CA} = \sqrt{D^2 + (2d)^2} - D$$

$$\Delta_{CA} = D \left[\left(1 + \frac{4d^2}{D^2} \right)^{\frac{1}{2}} - 1 \right]$$

By binomial expansion

$$\Delta_{CA} = D \left[\left(1 + \frac{1}{2} \frac{4d^2}{D^2} \right) - 1 \right]$$

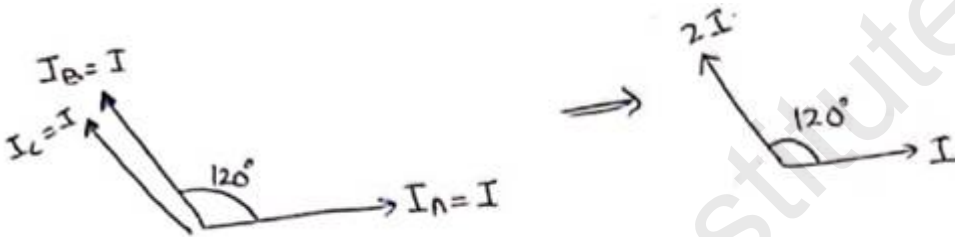
$$\Delta_{CA} = \frac{2d^2}{D}$$

$$\Delta_{CA} = 2 \left(\frac{2\lambda}{3} \right)$$

$$\phi_{CA} = \frac{2\pi}{\lambda} \cdot \Delta_{CA}$$

$$\begin{aligned}
 &= \frac{2\pi}{\lambda} \cdot \frac{4\lambda}{3} \\
 \phi_{CA} &= \frac{8\pi}{3} \quad \text{or} \quad \left(2\pi + \frac{2\pi}{3}\right) \\
 \phi_{CA} &= \frac{2\pi}{3} \\
 \phi_{BA} &= \frac{2\pi}{\lambda} (\Delta_{BA}) \\
 &= \frac{2\pi}{\lambda} \left(\frac{\lambda}{3}\right) \\
 \phi_{BA} &= \frac{2\pi}{3}
 \end{aligned}$$

By superposition principle



$$\begin{aligned}
 I_R &= (I)^2 + (2I)^2 + 2(I)(2I) \cos 120^\circ \\
 I_R &= 3I
 \end{aligned}$$

Thus, the resultant intensity is three times the intensity due to the individual slits.

Question 29

In a Young's double slit experiment, the separation between the slits = 2.0 mm, the wavelength of the light = 600 nm and the distance of the screen from the slits = 2.0 m. If the intensity at the centre of the central maximum is 0.20 W/m², what will be the intensity at a point 0.5 cm away from this centre along the width of the fringes?

Solution 29

$$I_0 = 0.2 \frac{\text{W}}{\text{m}^2}; d = 2\text{mm} = 2 \times 10^{-3}\text{m}; \lambda = 600\text{ nm}; D = 2\text{m}$$

For point at position, $y = 0.5\text{cm}$

$$\begin{aligned}
 \text{Path difference} &= \frac{d}{D} \cdot y \\
 &= \left(\frac{2 \times 10^{-3}}{2}\right)(0.5 \times 10^{-2})
 \end{aligned}$$

$$\Delta = 5 \times 10^{-6}\text{m}$$

$$\begin{aligned}
 \text{Phase difference, } \phi &= \frac{2\pi}{\lambda} \cdot \Delta \\
 &= \frac{2\pi}{6 \times 10^{-7}} (5 \times 10^{-6}) \\
 &= \frac{50\pi}{3} = 16\pi + \frac{2\pi}{3}
 \end{aligned}$$

$$\phi = \frac{2\pi}{3}$$

$$\begin{aligned} I_R &= I_0 \cos^2\left(\frac{\phi}{2}\right) \\ &= 0.2 \cos^2\left(\frac{2\pi}{3 \times 2}\right) \\ &= \frac{0.2}{4} \\ I_R &= 0.05 \text{ W/m}^2 \end{aligned}$$

Question 30

In a Young's double slit interference experiment the fringe pattern is observed on a screen placed at a distance D from the slits. The slits are separated by a distance d and are illuminated by monochromatic light of wavelength λ . Find the distance from the central point where the intensity falls to (a) half the maximum, (b) one fourth of the maximum.

Solution 30

$$(a) \quad I_N = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{I_0}{2} = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{1}{\sqrt{2}} = \cos\left(\frac{\phi}{2}\right)$$

$$\frac{\phi}{2} = \frac{\pi}{4}$$

$$\phi = \frac{\pi}{2} = \frac{2\pi}{\lambda} \cdot \Delta$$

$$\Delta = \frac{\lambda}{4}$$

$$\frac{d}{D} \cdot y = \frac{\lambda}{4}$$

$$y = \frac{D\lambda}{4d}$$

$$(b) \quad I_N = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{I_0}{4} = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{1}{2} = \cos\left(\frac{\phi}{2}\right)$$

$$\frac{\phi}{2} = \frac{\pi}{3}$$

$$\phi = \frac{2\pi}{3} = \frac{2\pi}{\lambda} \cdot \Delta$$

$$\Delta = \frac{\lambda}{3}$$

$$\frac{d}{D} \cdot y = \frac{\lambda}{3}$$

$$y = \frac{D\lambda}{3d}$$

Question 31

In a Young's double slit experiment $\lambda=500$ nm, $d=1.0$ mm and $D=1.0$ m. Find the minimum distance from the central maximum for which the intensity is half of the maximum intensity.

Solution 31

$$I_N = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{I_0}{2} = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{1}{\sqrt{2}} = \cos\left(\frac{\phi}{2}\right)$$

$$\frac{\phi}{2} = \frac{\pi}{4}$$

$$\phi = \frac{\pi}{2} = \frac{2\pi}{\lambda} \cdot \Delta$$

$$\Delta = \frac{\lambda}{4}$$

$$\frac{d}{D} \cdot y = \frac{\lambda}{4}$$

$$y = \frac{D\lambda}{4d}$$

$$= \frac{(1)(500 \times 10^{-9})}{4(1 \times 10^{-3})}$$

$$y = 1.25 \times 10^{-4} \text{ m}$$

Question 32

The linewidth of a bright fringe is sometimes defined as the separation between the points on the two sides of the central line where the intensity falls to half the maximum. Find the linewidth of a bright fringe in a Young's double slit experiment in terms of λ , d and D where the symbols have their usual meanings.

Solution 32

Distance of the point from central maxima where intensity falls by half.

$$I_N = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{I_0}{2} = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{1}{\sqrt{2}} = \cos\left(\frac{\phi}{2}\right)$$

$$\frac{\phi}{2} = \frac{\pi}{4}$$

$$\phi = \frac{\pi}{2} = \frac{2\pi}{\lambda} \cdot \Delta$$

$$\Delta = \frac{\lambda}{4}$$

$$\frac{d}{D} \cdot y = \frac{\lambda}{4}$$

$$y = \frac{D\lambda}{4d}$$

$$\text{Line-width} = 2y$$

$$= 2\left(\frac{D\lambda}{4d}\right)$$

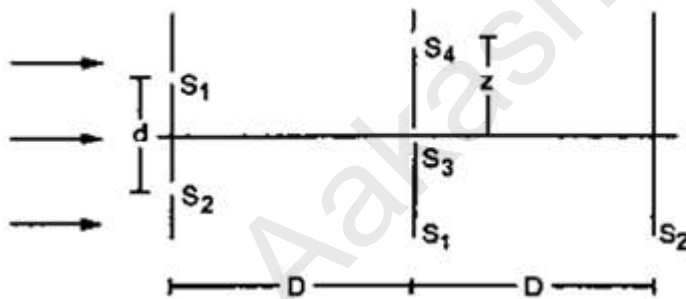
$$= \frac{D\lambda}{2d}$$

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Question 33

Consider the situation given in the figure. The two slits S_1 and S_2 placed symmetrically around the central line are illuminated by a monochromatic light of wavelength λ . The separation between the slits is d . The light transmitted by the slits falls on a screen Σ_1 placed at a distance D from the slits. The slit S_3 is at the central line and the slit S_4 is at a distance z from S_3 . Another screen Σ_2 is placed a further distance D away from Σ_1 . Find the ratio of the maximum to minimum intensity observed on Σ_2 if z is equal to

$$(a) \ z = \frac{\lambda D}{2d}, (b) \ \frac{\lambda D}{d}, (c) \ \frac{\lambda D}{4d}$$



Solution 33

$$(a) \ Z = \frac{D\lambda}{2d} = \frac{\beta}{2}$$

At $\beta/2$ distance from central maxima, minima will be obtained.

So, intensity from $S_4 = 0$

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{4I} + \sqrt{0})^2}{(\sqrt{4I} - \sqrt{0})^2} = 1:1$$

$$(b) \ Z = \frac{D\lambda}{d} = \beta$$

At β distance from central maxima, first order bright will be obtained.

So, intensity from $S_4 = 4I$

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{4I} + \sqrt{4I})^2}{(\sqrt{4I} - \sqrt{4I})^2} = \infty$$

$$(c) \quad z = \frac{D\lambda}{4d}$$

$$\Delta = \frac{d}{D} \cdot y = \frac{d}{D} \left(\frac{D\lambda}{4d} \right)$$

$$\Delta = \frac{\lambda}{4}$$

$$\phi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

So, intensity at $S_4 = I + I + 2\sqrt{I \cdot I} \cos \pi/2 = 2I$

Now,

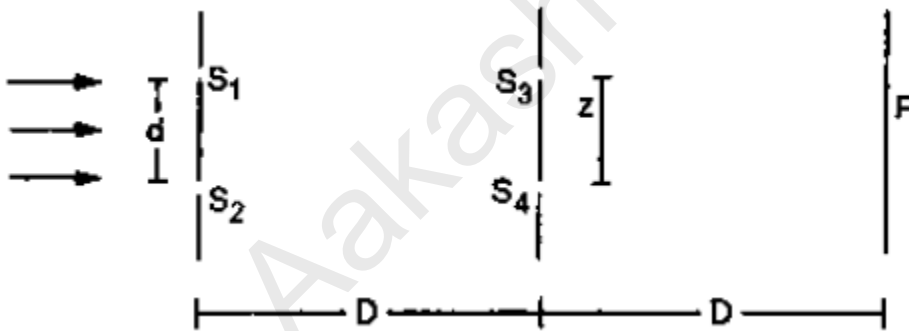
$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{4I} + \sqrt{2I})^2}{(\sqrt{4I} - \sqrt{2I})^2} = 34$$

Question 34

Consider the arrangement shown in figure. By some mechanism, the separation between the slits S_3 and S_4 can be changed. The intensity is measured at the point P which is common perpendicular

bisector of $S_1 S_2$ and $S_3 S_4$. When $z = \frac{D\lambda}{2d}$, the intensity measured at P is I. Find this intensity when z is equal to

(a) $\frac{D\lambda}{2d}$, (b) $\frac{3D\lambda}{2d}$, (c) $\frac{2D\lambda}{d}$



Solution 34

$$(a) \quad z = \frac{D\lambda}{2d}$$

Distance of slit S_3 and S_4 from perpendicular bisector is $y = \frac{D\lambda}{2d}$.

Path difference from S_1 and S_2 wave at $S_3 = \frac{d}{D} \cdot y = \frac{d}{D} \left(\frac{D\lambda}{2d} \right) = \frac{\lambda}{2}$.

Similarly, at S_4 path difference = $\frac{\lambda}{2}$

i.e., dark fringes are formed at S_3 and S_4 .

So, intensity of light at S_3 and S_4 are zero.

Hence, intensity at P is also zero.

$$(b) Z = \frac{3D\lambda}{2d}$$

Distance of slit S_3 and S_4 from perpendicular bisector is $y = \frac{3D\lambda}{2d}$.

Path difference from S_1 and S_2 wave at $S_3 = \frac{d}{D} \cdot y = \frac{d}{D} \left(\frac{3D\lambda}{2d} \right) = \frac{3\lambda}{2}$.

Similarly, at S_4 path difference = $\frac{3\lambda}{2}$

Phase difference at S_3 and S_4 will be $= \frac{2\pi}{\lambda} \cdot \left(\frac{3\lambda}{2} \right) = 3\pi$

When $Z = \frac{D\lambda}{2d}$, i.e., distance from perpendicular bisector is $\frac{D\lambda}{2(2d)}$

$$\Delta = \frac{d}{D} \cdot y = \frac{d}{D} \left(\frac{D\lambda}{4d} \right) = \frac{\lambda}{4}$$

$$\phi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \text{ intensity was } I \text{ at point P.}$$

At phase difference $\frac{3\pi}{2}$, intensity at point P is I .

$$(c) Z = \frac{2D\lambda}{d}$$

Distance of slit S_3 and S_4 from perpendicular bisector is $y = \frac{D\lambda}{d}$.

So, at S_3 and S_4 , bright fringe will be formed.

Intensity at P is $2I$.

Question 35

A soap film of thickness 0.0011 mm appears dark when seen by the reflected light of wavelength 580 nm . What is the index of refraction of the soap solution, if it is known to be between 1.2 and 1.5 ?

Solution 35

Minima is obtained in reflected light

$$2\mu t = n\lambda$$

$$\mu = \frac{n\lambda}{2t}$$

$$= \frac{n \times 580 \times 10^{-9}}{2 \times 0.0011 \times 10^{-3}}$$

$$\mu = 0.264_n$$

For refractive index between 1.2 and 1.5 , put $n=5$

$$\mu = 0.264 \times 5$$

$$\mu = 1.32$$

Question 36

A parallel beam of light of wavelength 560 nm falls on a thin film of oil (refractive index $= 1.4$). What should be the minimum thickness of the film so that it strongly reflects the light?

Solution 36

Maxima is obtained in reflected light.

$$2\mu t = (2n + 1) \frac{\lambda}{2}$$

for $t_{\min}$, put $n=0$

$$t_{\min} = \frac{\lambda}{4\mu}$$

$$= \frac{560 \times 10^{-9}}{4 \times 1.4}$$

$$t_{\min} = 100 \text{ nm}$$

Question 37

A parallel beam of white light is incident normally on a water film 1.0×10^{-4} cm thick. Find the wavelength in the visible range (400 nm-750 nm) which are strongly transmitted by the film. Refractive index of water=1.33.

Solution 37

Maxima in reflected light

$$2\mu t = n\lambda$$

$$\lambda = \frac{2 \times 1.33 \times 10^{-6}}{n}$$

$$\lambda = \frac{2660 \times 10^{-9}}{n}$$

For $n=4$, $\lambda=666$ nm

For $n=5$, $\lambda=532$ nm

For $n=6$, $\lambda=443$ nm

Question 38

A glass surface is coated by an oil film of uniform thickness 1.00×10^{-4} cm. The index of refraction of the oil is 1.25 and that of the glass is 1.50. Find the wavelengths of light in the visible region (400 nm-750 nm) which are completely transmitted by the oil film under normal incidence.

Solution 38

Condition for maxima in reflected light

$$\frac{(2n+1)\lambda}{2} = 2\mu t$$

$$\lambda = \frac{4\mu t}{(2n+1)}$$

$$\lambda = \frac{4 \times 1.25 \times 10^{-6}}{(2n+1)}$$

$$\lambda = \frac{5000}{(2n+1)} \text{ nm}$$

For $n=3$, $\lambda=714$ nm

For $n=4$, $\lambda=556$ nm

For $n=5$, $\lambda=455$ nm

Question 39

Plane microwaves are incident on long slit having a width of 5.0 cm. Calculate the wavelength of the microwaves if the first diffraction minimum is formed at $\theta=30^\circ$.

Solution 39

For minima, in diffraction

$$a \sin \theta = n\lambda$$

$$(5 \times 10^{-2}) \sin 30 = 1(\lambda)$$

$$\lambda = 2.5 \text{ cm}$$

Question 40

Light of wavelength 560 nm goes through a pinhole of diameter 0.20 mm and falls on a wall at a distance of 2.00 m. What will be the radius of the central bright spot formed on the wall?

Solution 40

$$R = 1.22 \frac{D\lambda}{d}$$

$$= \frac{1.22 \times 2 \times 560 \times 10^{-9}}{0.2 \times 10^{-3}}$$

$$= 0.683 \times 10^{-2} \text{ m}$$

$$R = 0.683 \text{ cm}$$

Diameter of central bright spot = $2R$

$$= 2(0.683)$$

$$= 1.37 \text{ cm}$$

Question 41

A convex lens of diameter 8.0 cm is used to focus a parallel beam of light of wavelength 620 nm. If the light be focused at a distance of 20 cm from the lens, what would be the radius of the central bright spot formed?

Solution 41

$$R = 1.22 \frac{D\lambda}{d}$$

$$= \frac{1.22 \times 20 \times 10^{-2} \times 620 \times 10^{-9}}{8 \times 10^{-2}}$$

$$= 1.9 \times 10^{-6} \text{ m}$$

∴ Diameter of the central bright spot, $2R = 3.8 \times 10^{-6} \text{ m}$.