

HC VERMA Solutions for Class 11 Physics Chapter 12 Simple Harmonic Motion

Question 1

A particle executes simple harmonic motion with an amplitude of 10cm and time period 6s. At $t=0$ it is at position $x=5\text{cm}$ going towards positive x -direction. Write the equation for the displacement x at time t . Find the magnitude of the acceleration of the particle at $t=4\text{s}$.

Solution 1

S.H.M equation is

$$X = A \sin(\omega t + \phi) \quad (1)$$

Amplitude, $A=10\text{cm}$

At $t=0$, position $x=5\text{cm}$

Time period $= 6\text{sec}$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ sec}^{-1}$$

Substitute in eq (1)

$$5 = 10 \sin\left(\frac{\pi}{3}(0) + \phi\right)$$

$$\frac{1}{2} = \sin(\phi)$$

$$\phi = \frac{\pi}{6}$$

Now, displacement equation is

$$X = A \sin(\omega t + \phi)$$

$$X = (10\text{cm}) \sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right)$$

At $t=4\text{sec}$

$$X = (10) \sin\left(\frac{\pi}{3}(4) + \frac{\pi}{6}\right)$$

$$X = 10 \sin\left(\frac{3\pi}{2}\right) \text{cm} = -10\text{cm}$$

On solving

Acceleration, $a = -\omega^2 x$

$$= -\left(\frac{\pi}{3}\right)^2 (10)$$

$$= -11\text{cm/s}^2$$

Question 2

The position, velocity and acceleration of a particle executing simple harmonic motion are found to have magnitudes 2cm, 1m/s and 10m/s^2 at a certain instant. Find the amplitude and the time period of the motion.

Solution 2

Taking only magnitude, $a = \omega^2 x$

$$\omega = \sqrt{\frac{a}{x}}$$

Acceleration, $a = 10\text{m/s}^2$

Position, $x = 2\text{cm} = 0.02\text{m}$

$$\omega = \sqrt{\frac{10}{0.02}} = 10\sqrt{5} \text{ sec}^{-1}$$

$$\text{Time period, } T = \frac{2\pi}{\omega} = \frac{2\pi}{10\sqrt{5}} = 0.28 \text{ sec}$$

$$\text{Velocity, } v = \omega \sqrt{A^2 - x^2} \quad \{A = \text{amplitude}\}$$

$$v^2 = \omega^2 (A^2 - x^2)$$

$$A = \sqrt{\frac{v^2}{\omega^2} + x^2}$$

$$= \sqrt{\frac{(1)^2}{(10\sqrt{5})^2} + (0.02)^2} = 0.049 \text{ m}$$

$$A = 4.9 \text{ cm}$$

Question 3

A particle executes simple harmonic motion with an amplitude of 10cm. At what distance from the mean position are the kinetic and potential energies are equal?

Solution 3

$$K.E = \frac{1}{2} m \omega^2 (A^2 - X^2) \quad \{A = \text{amplitude}\}$$

$$P.E = \frac{1}{2} m \omega^2 X^2$$

$$K.E = P.E$$

$$\frac{1}{2} m \omega^2 (A^2 - X^2) = \frac{1}{2} m \omega^2 X^2$$

$$A^2 - X^2 = X^2$$

$$A = \sqrt{2} X$$

$$A = 10 \text{ cm}$$

$$\frac{10}{\sqrt{2}} = X$$

$$X = 5\sqrt{2} \text{ cm}$$

$$\text{Particle distance from mean position} = 5\sqrt{2} \text{ cm}$$

Question 4

The maximum speed and acceleration of a particle executing simple harmonic motion are 10cm/s and 50 cm/s^2 . Find the position(s) of the particle when the speed is 8cm/s.

Solution 4

$$V_{\max} = \omega A \quad (1)$$

$$a_{\max} = \omega^2 A \quad (2)$$

{a=acceleration, A=amplitude}

$$\frac{(1)}{(2)} \Rightarrow \frac{V_{\max}}{a_{\max}} = \frac{\omega A}{\omega^2 A}$$

$$\frac{10}{50} = \frac{1}{\omega}$$

$$\omega = 5 \text{ sec}^{-1}$$

$$\text{From (1), } V_{\max} = \omega A$$

$$(10) = r(5)$$

$$r = 2 \text{ cm}$$

Now, speed=8cm/s, position=?

$$v = \omega \sqrt{A^2 - x^2}$$

$$8 = 5 \sqrt{4 - x^2}$$

$$\frac{64}{25} = 4 - x^2$$

$$x = \sqrt{4 - \frac{64}{25}}$$

$$x = \pm 1.2 \text{ cm from mean position.}$$

Question 5

A particle having mass 10g oscillates according to the

equation $x = (2.0 \text{ cm}) \sin \left[(100 \text{ s}^{-1})t + \frac{\pi}{6} \right]$. Find (a) the amplitude, the time period and the spring constant (b) the position, the velocity and the acceleration at $t=0$.

Solution 5

$$x = (2.0 \text{ cm}) \sin \left[(100 \text{ s}^{-1})t + \frac{\pi}{6} \right]$$

Compare it with, $x = A \sin(\omega t + \phi)$

(a) Amplitude(A)=2.0cm $\omega = 100 \text{ sec}^{-1}$

$$\text{Time period} = \frac{2\pi}{\omega} = \frac{2\pi}{50} = \frac{\pi}{50} = 0.063 \text{ sec}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

For spring constant,

$$\frac{\pi}{50} = 2\pi \sqrt{\frac{10g}{k}}$$

$$k = \frac{(m)(4\pi^2)}{T^2} = \frac{(10g)(4\pi^2)}{(0.063)^2}$$

$$k = 10^5 = 100 \text{ N/m}$$

(b) At $t=0$,

$$x = (2.0 \text{ cm}) \sin \left[0 + \frac{\pi}{6} \right] = (2.0 \text{ cm}) \left(\frac{1}{2} \right) = 1.0 \text{ cm}$$

$$\text{Velocity} = v = A\omega \cos(\omega t + \phi) = (2)(100) \cos \left(0 + \frac{\pi}{6} \right) = 1.73 \text{ m/s}$$

$$\text{Acceleration, } a = -A\omega^2 \sin(\omega t + \phi)$$

$$= -\omega^2 X$$

$$= -(100)^2 (1) \times 10^{-2} \text{ m/s}^2$$

$$a = 100 \text{ m/s}^2 \text{ \{considering only magnitude\}}$$

Question 6

The equation of motion of a particle started at $t=0$ is given by $x = 5 \sin(20t + \pi/3)$ where x is in centimeter and t in second. What does the particle

(a) first come to rest.

- (b) First have zero acceleration.
 (c) First have maximum speed?

Solution 6

Give SHM equation

$$x = 5 \sin(20t + \pi/3)$$

- (a) It comes to rest when velocity=0
 'v' becomes '0' at extreme position

At extreme position, $x = A = 5$ cm

$$5 = 5 \sin(20t + \pi/3)$$

$$1 = \sin(20t + \pi/3)$$

$$20t + \frac{\pi}{3} = \pi/2$$

$$20t = \pi/6$$

$$t = \pi/120 \text{ seconds}$$

- (b) We know that, $a = -\omega^2 x$

$$a = -\omega^2 \left[5 \sin \left(20t + \frac{\pi}{3} \right) \right]$$

$$0 = -\omega^2 \left[5 \sin \left(20t + \frac{\pi}{3} \right) \right]$$

$$5 \sin \left(20t + \frac{\pi}{3} \right) = 0$$

$$\sin \left(20t + \frac{\pi}{3} \right) = 0$$

$$20t + \frac{\pi}{3} = \pi$$

$$20t = 2\pi/3$$

$$t = \pi/30 \text{ seconds}$$

- (c) $v = 5(20) \left[5 \cos \left(20t + \frac{\pi}{3} \right) \right]$

$$\cos \left(20t + \frac{\pi}{3} \right) = 1 \text{ or } -1$$

'v' is maximum only when

$$\cos \left(20t + \frac{\pi}{3} \right) = \cos 0$$

$$20t + \frac{\pi}{3} = 0$$

$$t = -\frac{\pi}{60} \text{ sec}$$

Not possible because time won't be negative.

$$\cos \left(20t + \frac{\pi}{3} \right) = \cos \pi$$

$$20t = \frac{2\pi}{3}$$

$$t = \frac{\pi}{30} \text{ sec}$$

Question 7

Consider a particle moving in simple harmonic motion according to the equation.

$$x = 2.0 \cos(50\pi t + \tan^{-1} 0.75)$$

where x is in centimeter and t in second. The motion is started at t=0. (a) When does the particle come to rest for the first time? (b) When does the acceleration have its maximum magnitude for the first time? (c) When does the particle come to rest for the second time?

Solution 7

$$x = 2.0 \cos(50\pi t + \tan^{-1} 0.75) \\ = 2.0 \cos(50\pi t + 0.643)$$

$$(a) \text{ We know that velocity}(v) = \frac{dx}{dt} \\ v = -100\pi \sin[50\pi t + 0.643]$$

The particle velocity becomes zero for 1st time when $\sin[50\pi t + 0.643] = 0$

$$\text{It becomes '0' when } 50\pi t + 0.643 = \pi \\ 50\pi t = 3.14 - 0.643$$

$$t = 0.016 \text{ sec}$$

$$t = 1.6 \times 10^{-2} \text{ sec}$$

$$(a) = \frac{dv}{dt} = -(50\pi)(100\pi) \cos[50\pi t + 0.643]$$

(b) Acceleration, It is having negative sign. To have maximum acceleration,

$$\cos[50\pi t + 0.643] = -1 = \cos \pi$$

$$50\pi t + 0.643 = \pi$$

$$\text{On solving, } t = 1.6 \times 10^{-2} \text{ sec}$$

(c) Particle again comes to rest when

$$v = -100\pi \sin[50\pi t + 0.643] = 0$$

$$\sin[50\pi t + 0.643] = 0 = \sin 2\pi$$

$$50\pi t + 0.643 = 2\pi$$

$$\text{On solving, } t = 3.6 \times 10^{-2} \text{ s}$$

Question 8

Consider a simple harmonic motion of time period T. Calculate the time taken for the displacement to change value from half the amplitude to the amplitude.

Solution 8

Let the equation of SHM be

$$x = A \sin \omega t \text{ with time period } T$$

$$\text{For half-amplitude, } x = \frac{A}{2}, t = t_1$$

$$\frac{A}{2} = A \sin \omega t_1$$

$$t_1 = \frac{1}{\omega} \sin^{-1} \left(\frac{1}{2} \right) \quad \text{---(1)}$$

For $x = A, t = t_2$

$$A = A \sin(\omega t_2)$$

$$t_2 = \frac{1}{\omega} \sin^{-1}(1) \quad \text{---(2)}$$

$$\begin{aligned}
 t_2 - t_1 &= \frac{1}{\omega} \left(\frac{\pi}{2} \right) - \frac{1}{\omega} \left(\frac{\pi}{6} \right) \\
 \text{Now} \\
 &\Rightarrow \frac{\pi}{2\omega} \left[1 - \frac{1}{3} \right] \\
 &\Rightarrow \frac{\pi T}{2(2\pi)} \left[\frac{2}{3} \right] \\
 t_2 - t_1 &\Rightarrow \frac{T}{6} \text{sec}
 \end{aligned}$$

Question 9

The pendulum of a clock is replaced by a spring-mass system with the spring having spring constant 0.1N/m. What mass should be attached to the spring?

Solution 9

Length of the pendulum clock=1m

$$T = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{1}{9.8}} \approx 2 \text{ sec}$$

Given, k=0.1N/m

Time periods are same.

$$\begin{aligned}
 2\pi \sqrt{\frac{m}{k}} &= 2 \text{ sec} \\
 \Rightarrow 2\pi \sqrt{\frac{m}{0.1}} &= 2
 \end{aligned}$$

On solving, m=0.01kg=10gms

Question 10

A block suspended from a vertical spring is in equilibrium. Show that the extension of the spring equals the length of an equivalent simple pendulum i.e., a pendulum having frequency same as that of the block.

Solution 10

Given

Spring frequency= Pendulum frequency

$$\begin{aligned}
 \sqrt{\frac{g}{l}} &= \sqrt{\frac{k}{m}} \\
 \frac{g}{l} &= \frac{k}{m} \\
 \frac{1}{m} \left[\because \text{length of pendulum clock} = 1\text{m} \right] \\
 \frac{mg}{k} &= 1 \\
 \frac{F}{k} &= 1 \\
 x &= 1 \left[\because F = kx \right]
 \end{aligned}$$

$\therefore$ Extension=1m

Question 11

A block of mass 0.5kg hanging from a vertical spring executes simple harmonic motion of amplitude 0.1m and time period 0.314s. Find the maximum force exerted by the spring on the block.

Solution 11

Given,

Time period=0.314sec

Mass=0.5kg
Amplitude=0.1m

$$T = 2\pi \sqrt{\frac{m}{k}}$$

We know,

$$0.314 = 2\pi \sqrt{\frac{0.5}{k}}$$

On solving, $k = 200 \text{ N/m}$

$$\begin{aligned} F &= kx \\ &= (200)(0.1) = 20 \text{ N} \end{aligned}$$

Nor. Max. force=weight + force due to extension
 $= 5 + 20 = 25 \text{ N}$

Question 12

A body of mass 2kg suspended through a vertical spring executes simple harmonic motion of periods. If the oscillations are stopped and the body hangs in equilibrium, find the potential energy stored in the spring.

Solution 12

Given, $m=2\text{kg}$, $T=4\text{sec}$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Time period,

$$T^2 = 4\pi^2 \cdot \frac{m}{k}$$

$$k = 4\pi^2 \cdot \frac{m}{T^2}$$

$$= 4\pi^2 \cdot \left(\frac{2}{16}\right) \approx 5 \text{ N/m}$$

Now, $F = kx$ & $F = mg$

$$mg = kx$$

$$(2)(9.8) = (5)(x)$$

$$x \approx 4 \text{ m}$$

Now, P.E $= \frac{1}{2} kx^2 = \frac{1}{2} (5)(4)^2 = 40 \text{ J}$

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Question 13

A spring stores 5J of energy when stretched by 25cm. It is kept vertical with the lower end fixed. A block fastened to its other end is made to undergo small oscillations. If the block makes 5 oscillations each second, what is the mass of the block?

Solution 13

$$P.E = \frac{1}{2} kx^2$$

$$5 = \frac{1}{2} k(0.25)^2$$

$$k = 160 \text{ N/m}$$

In 1sec, it completes 5 revolutions.

So, time taken for 1 rotation = $\frac{1}{5} = 0.2$ sec

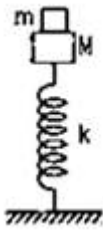
$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$0.2 = 2\pi \sqrt{\frac{m}{160}}$$

On solving, $m = 160 \text{ N/m}$

Question 14

A small block of mass m is kept on a bigger block of mass M which is attached to a vertical spring of spring constant k as shown in figure. The system oscillates vertically. (a) Find the resultant force on the smaller block when it is displaced through a distance x above its equilibrium position. (b) Find the normal force on the smaller block at this position. What is the force smallest in the magnitude? (c) What can be the maximum amplitude with which the two blocks may oscillate together?



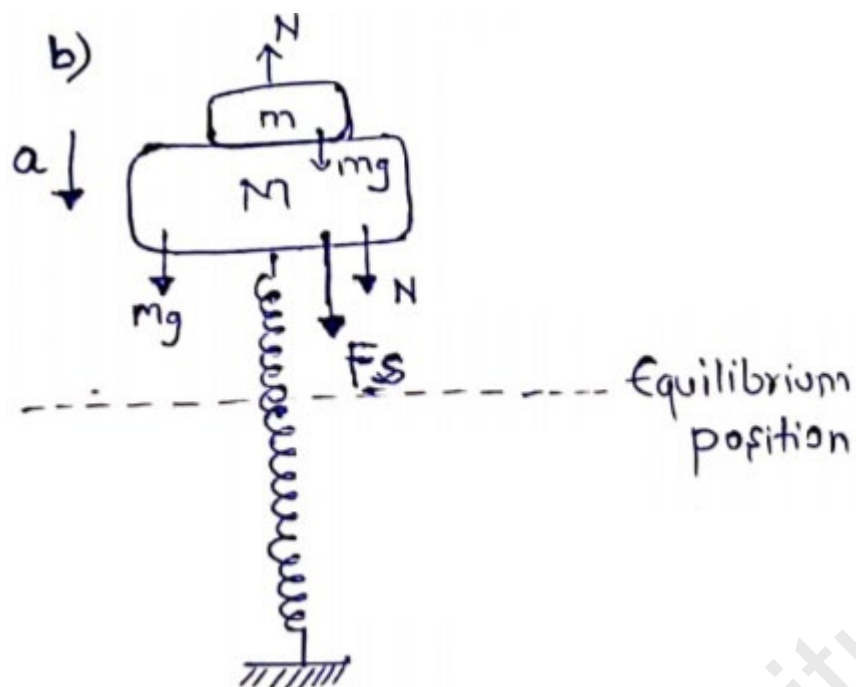
Solution 14

(a) $F_n = \text{mass} \times \text{acceleration}$
 $= m(-\omega^2 x)$

$$= -m \left(\sqrt{\frac{k}{M+m}} \right)^2 \cdot x \Rightarrow F_N = \frac{-mkx}{(M+m)}$$

Here -ve sign shows that force is towards equilibrium position.

(b)



For small block

$$F_N = mg - N$$

$$\frac{mkx}{(M+m)} = mg - N$$

$$N = mg - \frac{mkx}{(M+m)}$$

Normal contact will be minimum at highest point.

(c) For maximum amplitude, $x=A$, $N=0$

$$mg - \frac{mkA}{(M+m)} = 0$$

$$mg = \frac{mkA}{(M+m)}$$

$$A = \frac{(M+m)g}{k}$$

Question 15

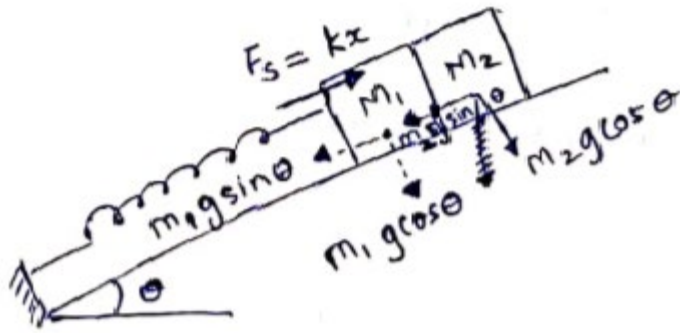
The block of mass m_1 shown in figure is fastened to the spring and the block of mass m_2 is placed against it. (a) Find the compression of the spring in the equilibrium position. (b) The blocks are

pushed a further distance $\left(\frac{2}{k}\right) (m_1 + m_2)g \sin \theta$ against the spring and released. Find the position where the two blocks separate. (c) What is common speed of blocks at the time of separation?



Solution 15

(a) Let compression in spring be x in equilibrium position.



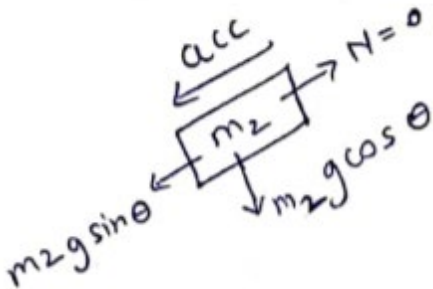
$$F_N = 0$$

$$kx = m_1 g \sin \theta + m_2 g \sin \theta$$

$$x = \frac{(m_1 + m_2) g \sin \theta}{k}$$

(b) Let blocks lose contact at y distance from equilibrium position.

For m_2 block



$$m_2 g \sin \theta - 0 = m_2 a$$

$$m_2 g \sin \theta = m_2 (\omega^2 y)$$

$$m_2 g \sin \theta = m_2 \left(\sqrt{\frac{k}{m_1 + m_2}} \right)^2 y$$

$$g \sin \theta = \frac{k}{(m_1 + m_2)} \cdot y$$

$$y = \frac{(m_1 + m_2) g \sin \theta}{k}$$

$y = x$ (At natural length of spring it loses contact)

(c) Spring is further compressed by $2x$ for extreme position

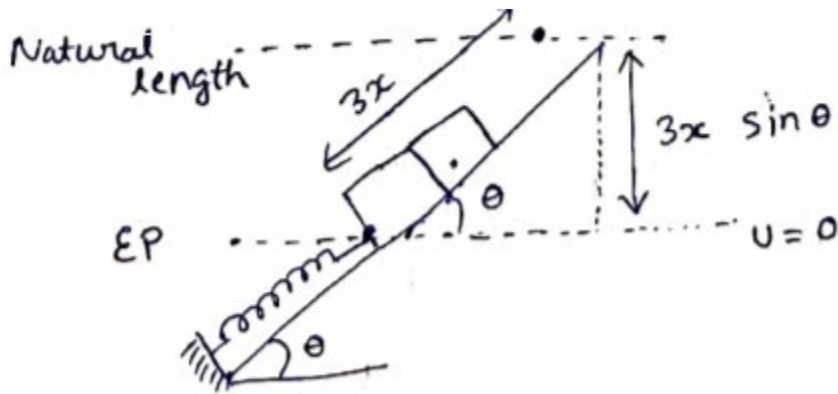
Total compression $= x + 2x = 3x$

Now,

Total energy conservation at extreme position and natural length of spring.

$$T\epsilon)_{EP} = T\epsilon)_{NL}$$

$$\frac{1}{2} k (3x)^2 + \frac{1}{2} (m_1 + m_2) (0)^2 + 0 = \frac{1}{2} k (0)^2 + \frac{1}{2} (m_1 + m_2) v^2 + (m_1 + m_2) g (3x \sin \theta)$$



$$\frac{1}{2} k \frac{(3(m_1+m_2)g \sin \theta)^2}{k} = \frac{1}{2} (m_1+m_2) v^2 + (m_1+m_2) g \frac{(3(m_1+m_2)g \sin \theta)}{k} \sin \theta$$

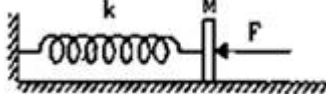
$$\frac{9 (m_1+m_2) g^2 \sin^2 \theta}{2 k} = \frac{1}{2} v^2 + \frac{9 (m_1+m_2) g^2 \sin^2 \theta}{k}$$

$$\frac{9 (m_1+m_2) g^2 \sin^2 \theta}{2 k} = \frac{1}{2} v^2$$

$$v = \sqrt{\frac{9 (m_1+m_2) g^2 \sin^2 \theta}{k}}$$

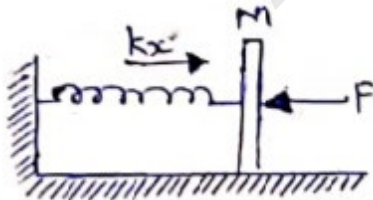
Question 16

In figure $k=100\text{N/m}$, $M=1\text{kg}$ and $F=10\text{N}$. (a) Find the compression of the spring in the equilibrium position. (b) A sharp blow by some external agent imparts a speed of 2m/s to the block towards left. Find the sum of the potential energy of the spring and the kinetic energy of the block at this instant. (c) Find the time period of the resulting simple harmonic motion. (d) Find the amplitude. (e) Write the potential energy of the spring when the block is at left extreme. (f) Write the potential energy of the spring when the block is at the right extreme. The answers of (b), (e) and (f) are different. Explain why this does not violate the principle of conservation of energy.



Solution 16

(a) Let spring is compressed by x at equilibrium position.



$$\therefore F_N = 0$$

$$F = kx$$

$$10 = 100 \times x$$

$$x = 0.1\text{cm} = 10_{\text{cm}}$$

(b) Total energy = K.E + P.E

$$\begin{aligned}
 &= \frac{1}{2} M v^2 + \frac{1}{2} k x^2 \\
 &= \frac{1}{2} (1)(2)^2 + \frac{1}{2} (100)(0.1)^2 \\
 &= 2 + 0.5 = 2.5 \text{ J}
 \end{aligned}$$

$$T = 2\pi \sqrt{\frac{M}{k}}$$

(c) Time period,

$$= 2\pi \sqrt{\frac{1}{100}} = \frac{\pi}{5} \text{ sec}$$

(d) At the time of maximum compression ($v=0$) and let spring is compressed by $(x+A)$

$$W_{ext} = \Delta K.E + \Delta P.E$$

$$F \times A = \left(0 - \frac{1}{2} M v^2\right) + \left(\frac{1}{2} k (x+A)^2 - \frac{1}{2} k x^2\right)$$

$$10A = -\frac{1}{2} \times 1 \times (2)^2 + \frac{1}{2} k (A^2 + 2Ax)$$

$$10A = -2 + \frac{1}{2} \times 100 (A^2 + 2A \times 0.1)$$

$$10A = -2 + 50(A^2 + 0.2A)$$

$$10A = -2 + 50A^2 + 10A$$

$$2 = 50A^2$$

$$A^2 = \frac{1}{25}$$

$$A = \frac{1}{5} \text{ m}$$

$$A = \frac{100}{5} = 20 \text{ cm}$$

$$(e) \quad PE = \frac{1}{2} k (x+A)^2$$

$$= \frac{1}{2} \times 100 (0.1 + 0.2)^2$$

$$PE = 4.5 \text{ J}$$

(f) At right extreme, extension in spring will be

$$A - x = 0.2 - 0.1 = 0.1 \text{ m}$$

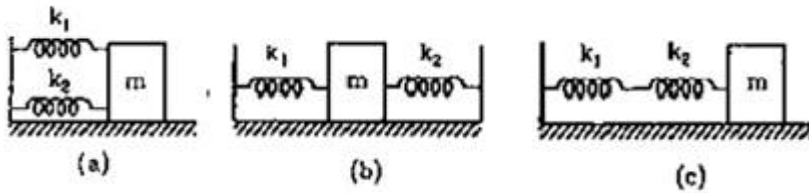
$$PE = \frac{1}{2} k (A - x)^2$$

$$= \frac{1}{2} \times 100 (0.1)^2 = 0.5 \text{ J}$$

Since, the external force is acting on block the energy will not be conserved.

Question 17

Find the time period of the oscillation of mass m in figures. What is the equivalent spring constant of the pair of strings in each case?



Solution 17

(a) The springs are parallel here.

Equivalent spring constant in parallel case is

$$k_{eq} = k_1 + k_2$$

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{k_1 + k_2}}$$

(b) We can consider the springs here as parallel. Because, they are not in linear and continuous connection. If the block moves, deformation in one spring increases. So, the force increases.

$$k = k_1 + k_2$$

$$T = 2\pi \sqrt{\frac{m}{k_1 + k_2}}$$

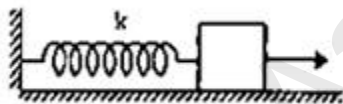
(c) Springs are in series,

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_1 + k_2}{k_1 k_2}$$

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

Question 18

The spring shown in figure is unstretched when a man starts pulling on the cord. The mass of the block is M . If the man exerts a constant force F , find (a) the amplitude and the time period of the motion of the block, (b) the energy stored in the spring when the block passes through the equilibrium and (c) the kinetic energy of the block at this position.



Solution 18

(a) $F = kx$

$$x = \frac{F}{k}$$

Maximum displacement is given by $x = \frac{F}{k}$

(b) Energy stored at equilibrium = $\frac{1}{2} kx^2$

$$= \frac{1}{2} k \left(\frac{F}{k} \right)^2$$

$$= \frac{1}{2} \frac{F^2}{k}$$

(c) At mean position,

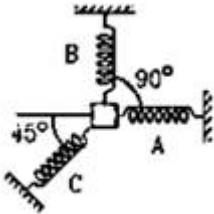
$$P.E.=0$$

$$K.E = \frac{1}{2} kx^2$$

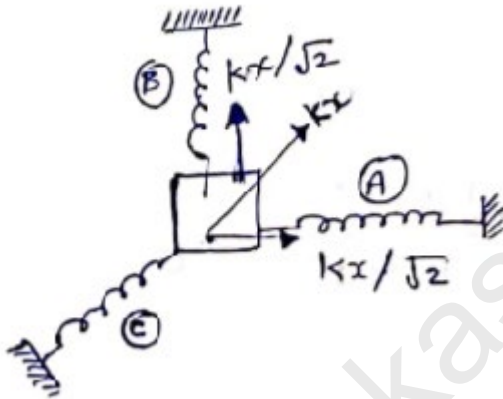
$$= \frac{1}{2} \frac{F^2}{k}$$

Question 19

A particle of mass m is attached to three springs A, B and C of equal force constants k as shown in figure. If the particle is pushed slightly against the spring C and released, find the time period of oscillation.



Solution 19



Let spring C is compressed by x at any instant,

then extension in spring $A = x \cos 45 = \frac{x}{\sqrt{2}}$

and in spring $B = x \sin 45 = \frac{x}{\sqrt{2}}$

Net force on block

$$F_N = kx + \sqrt{\left(\frac{kx}{\sqrt{2}}\right)^2 + \left(\frac{kx}{\sqrt{2}}\right)^2}$$

$$m \times a = 2kx$$

$$m \times \omega^2 x = 2kx$$

$$\omega^2 = \frac{2k}{m}$$

$$\omega = \sqrt{\frac{2k}{m}}$$

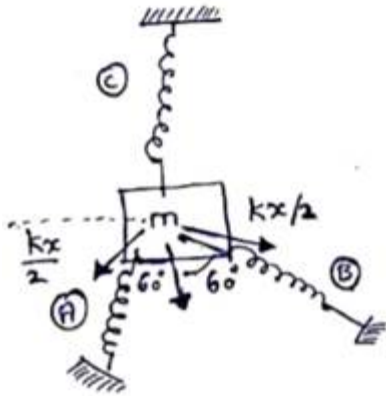
$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{2k}}$$

$$T = \pi \sqrt{\frac{2m}{k}}$$

Question 20

Repeat the exercise if the angle between each pair of springs is 120° initially.

Solution 20



Let spring C is compressed by x at any instant, then extension in

Spring A = $x \cos 60^\circ = x/2$

Spring B = $x \cos 60^\circ = x/2$

Net force on block

$$F_N = kx + \sqrt{\left(\frac{kx}{2}\right)^2 + \left(\frac{kx}{2}\right)^2 + 2\left(\frac{kx}{2}\right)\left(\frac{kx}{2}\right)\cos 120^\circ}$$

$$F_N = \frac{3kx}{2}$$

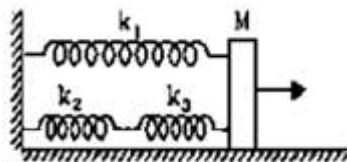
$$m\omega^2 x = \frac{3kx}{2}$$

$$\omega = \sqrt{\frac{3k}{2m}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{2m}{3k}}$$

Question 21

The springs shown in the figure are all unstretched in the beginning when a man starts pulling the block. The man exerts a constant force F on the block. Find the amplitude and the frequency of the motion of the block.



Solution 21

To find frequency and amplitude, first resolve it.

k_2 & k_3 are in series.

So, $k_{\text{equivalent}} = \frac{k_2 k_3}{k_2 + k_3} = k_{eq}$

Now k_{eq} & k_1 are in parallel.

Total, $K = k_{eq} + k_1 = \frac{k_2 k_3}{k_2 + k_3} + k_1 = \frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{k_2 + k_3}$

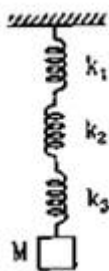
$$= \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{K}{M}} = \frac{1}{2\pi} \sqrt{\frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{M(k_2 + k_3)}}$$

Now, frequency

Amplitude, $X = \frac{F}{K} = \frac{F(k_2 + k_3)}{k_1 k_2 + k_2 k_3 + k_3 k_1} \quad [\because F = k(x)]$

Question 22

Find the elastic potential energy stored in each spring shown in figure, when the block is in equilibrium. Also find the time period of vertical oscillation of the block.



Solution 22

We know $PE = \frac{1}{2} kx^2$

For k_1 spring, $x_1 = \frac{F}{k_1} = \frac{Mg}{k_1}; PE_1 = k_1 x_1^2 = \frac{M^2 g^2}{2k_1}$

For k_2 spring, $x_2 = \frac{F}{k_2} = \frac{Mg}{k_2}; PE_2 = k_2 x_2^2 = \frac{M^2 g^2}{2k_2}$

For k_3 spring, $x_3 = \frac{F}{k_3} = \frac{Mg}{k_3}; PE_3 = k_3 x_3^2 = \frac{M^2 g^2}{2k_3}$

Now k_1, k_2 & k_3 are in series

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

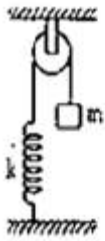
$$T = 2\pi \sqrt{\frac{M}{K}} = 2\pi \sqrt{M \cdot \left(\frac{1}{k_{eq}}\right)}$$

$$T = 2\pi \sqrt{M \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)}$$

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Question 23

The spring, the spring and the pulley shown in figure are in light. Find the time period of the mass m.



Solution 23

Initially there will be some tension in spring.

$$T_1 = kx_1$$

After applying some force 'F', tension becomes

$$T_2 = kx_1 + kx_2 = k(x_1 + x_2)$$

Resultant tension = $T = T_2 - T_1 = kx_2 + kx_1 - kx_1 = kx_2$

$$T = kx_2$$

$$T = 2\pi \sqrt{\frac{M}{k}}$$

$$\text{But } F = kx \Rightarrow Ma = kx$$

$$\frac{M}{k} = \frac{x}{a}$$

$$\frac{M}{k} = \frac{\text{Displacement}}{\text{acceleration}}$$

$$M(\text{acceleration}) = kx_2$$

$$\text{acceleration} = \frac{kx_2}{M}$$

$$= 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi \sqrt{\frac{x_2}{\frac{kx_2}{M}}}$$

Time period

$$T = 2\pi \sqrt{\frac{M}{k}}$$

Question 24

Solve the previous problem if the pulley has a moment of inertia I about its axis and the string does not slip over it.

Solution 24

Let at any instant the velocity of the block is v and spring is stretched by x .

So,

$$\text{Total energy} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

Total energy is constant

$$\frac{d}{dt}(T.E.) = 0$$

$$\frac{1}{2}m(2v)\frac{dv}{dt} + \frac{1}{2}I(2v)\frac{dv}{dt} + \frac{1}{2}k(2x)\frac{dx}{dt} = 0$$

$$\left(mv + \frac{I}{r^2}v\right)\frac{dv}{dt} + kxv = 0$$

$$a\left(m + \frac{I}{r^2}\right) = -kx$$

$$a = \frac{-k}{(m + \frac{I}{r^2})} x$$

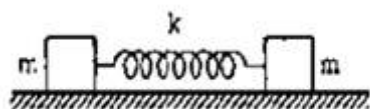
Comparing with $a = -\omega^2 x$

$$\omega = \sqrt{\frac{k}{(m + \frac{I}{r^2})}}$$

$$T = 2\pi \sqrt{\frac{(m + \frac{I}{r^2})}{k}}$$

Question 25

Consider the situation shown in figure. Show that if the blocks are displaced slightly in opposite directions and released, they will execute simple harmonic motion. Calculate the time period.



Solution 25

The velocity of center of mass is zero, so it will remain at rest during motion.

If left block stretches spring by x so, right block will also stretch spring by x .

$$\text{Total energy} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}k(2x)^2$$

TE remains constant

$$\begin{aligned} \frac{d}{dt}(TE) &= \frac{d}{dt}(2kx^2) + \frac{d}{dt}(mv^2) \\ &= 2k(2x) \frac{dx}{dt} + m(2v) \frac{dv}{dt} = 0 \end{aligned}$$

$$mv \frac{dv}{dt} = -2kx \frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} = -2 \frac{k}{m} x$$

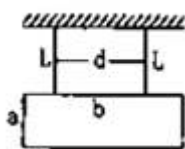
Comparing with $a = -\omega^2 x$

$$\omega = \sqrt{\frac{2k}{m}}$$

$$T = 2\pi \sqrt{\frac{m}{2k}}$$

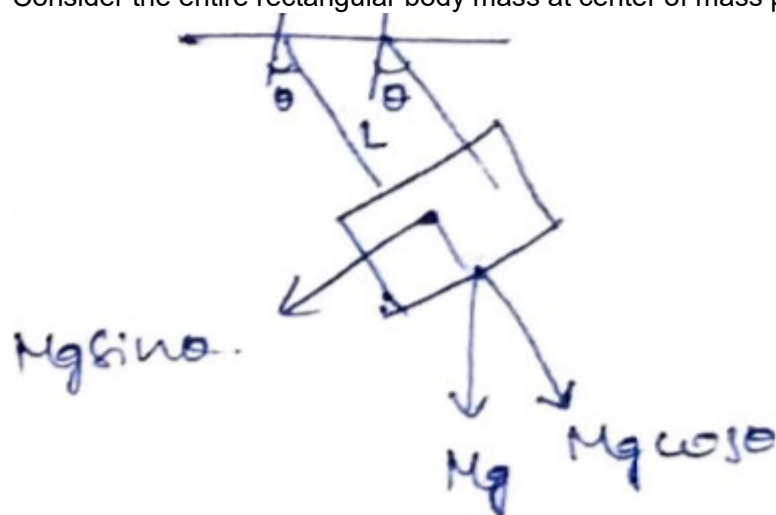
Question 26

A rectangular plate of sides a and b is suspended from a ceiling by two parallel strings of length L each. The separation between the strings is d . The plate is displaced slightly in its plane keeping the strings tight. Show that it will execute simple harmonic motion. Find the time period.



Solution 26

Consider the entire rectangular body mass at center of mass point.



As it is displaced, it will move.

Here $Mg \sin \theta$ gives the moment in actual direction we required.

Now, $F = Mg \sin \theta$

$$Ma = Mg \sin \theta$$

$$a = g \sin \theta$$

For small angles, $\sin \theta \approx \theta$

$$a = \theta$$

$$a = g \left(\frac{x}{L} \right)$$

$$a \propto x$$

So, this motion is SHM.

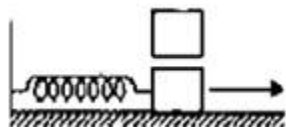


Displacement

$$T = 2\pi \sqrt{\frac{\text{Displacement}}{\text{acceleration}}} = 2\pi \sqrt{\frac{x}{\frac{gx}{L}}} = 2\pi \sqrt{\frac{L}{g}}$$

Question 27

A 1kg block is executing simple harmonic motion of amplitude 0.1m on a smooth horizontal surface under the restoring force of a spring of spring constant 100N/m. A block of mass 3kg is gently placed on it at instant it passes through the mean position. Assuming that the two blocks move together, find the frequency and amplitude of the motion.



Solution 27

Total mass of the system, $M = 3 + 1 = 4\text{kg}$

Time period of SHM,

$$T = 2\pi \sqrt{\frac{M}{k}}$$

$$T = 2\pi \sqrt{\frac{4}{100}} = \frac{2\pi}{5} \text{ sec}$$

So, frequency = $\frac{5}{2\pi}\text{Hz}$

Let v be the velocity of the 1kg block at mean position

KE = TE

$$\frac{1}{2}mv^2 = \frac{1}{2}KA^2$$

$$\frac{1}{2} \times 1 \times v^2 = \frac{1}{2} \times 100 \times (0.1)^2$$

$$v = 1\text{m/s}$$

Let v' be the velocity of combined blocks

$$P_i = P_f$$

$$1 \times v = 4v'$$

$$v' = \frac{1}{4}\text{m/s}$$

Now, KE' = TE'

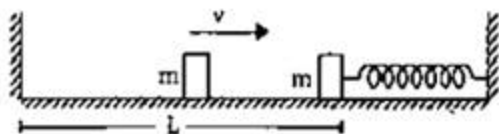
$$\frac{1}{2}mv'^2 = \frac{1}{2}KA'^2$$

$$\frac{1}{2} \times 4 \times \left(\frac{1}{4}\right)^2 = \frac{1}{2} \times 100 \times A'^2$$

$$A'^2 = 5\text{cm}$$

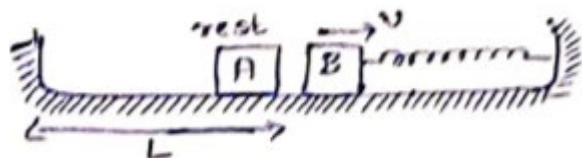
Question 28

The left in figure moves at a speed v towards the right block placed in equilibrium. All collisions take place are elastic and the surfaces are frictionless. Show that the motions of the two blocks are periodic. Find the time period of those periodic motions. Neglect the widths of the block.



Solution 28

For same mass and elastic collision, blocks will exchange velocity on collision. Now, block A comes to rest and block B moves and execute half SHM.



Now again block B collides and come to rest. Block A moves and execute periodic motion for distance $2L$ with speed v .

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

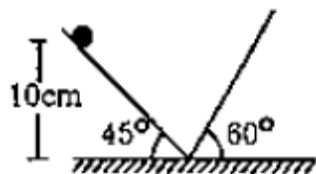
$$v = \frac{2L}{t_2}$$

$$t_2 = 2L/v$$

$$\text{Total time} = t_1 + t_2 = \pi \sqrt{\frac{m}{k}} + \frac{2L}{v}$$

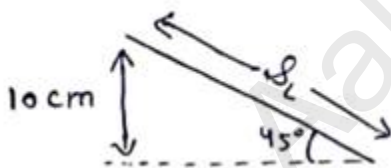
Question 29

Find the time period of the motion of the particle shown in figure. Neglect the small effect of the bend near the bottom.



Solution 29

The maximum height attained by ball on both inclined will be same as no energy loss. For left incline,



$$\sin 45 = \frac{10}{s_L}$$

$$s_L = 10\sqrt{2} \text{ cm}$$

$$a_L = g \sin 45$$

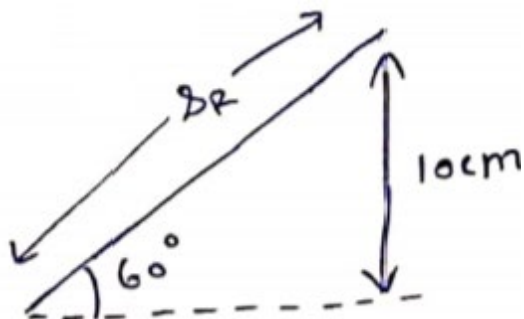
$$U=0$$

$$s = ut + \frac{1}{2}at^2$$

$$\frac{100\sqrt{2}}{100} = 0 + \frac{1}{2} \left(\frac{g}{\sqrt{2}} \right) t_1^2$$

$$t_1 = \sqrt{\frac{4}{10g}} = \frac{2}{10} = 0.2 \text{ sec}$$

For right incline



$$\sin 60^\circ = \frac{10}{s_R}$$

$$s_R = \frac{20}{\sqrt{3}} \text{ cm}$$

$$a = g \sin 60^\circ = \frac{\sqrt{3}g}{2}$$

$$U=0$$

$$s_R = ut + \frac{1}{2}at^2$$

$$\frac{20}{\sqrt{3}} \times \frac{1}{100} = 0 + \frac{1}{2} \frac{\sqrt{3}g}{2} t_2^2$$

$$\frac{8}{30g} = t_2^2$$

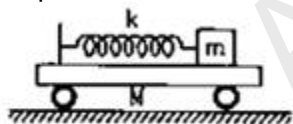
$$t_2 = 0.163 \text{ sec}$$

$$\text{Total time} = 2(t_1 + t_2)$$

$$= 2(0.2 + 0.163) = 0.73 \text{ sec}$$

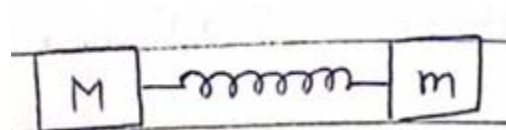
Question 30

All the surfaces shown in figure are frictionless. The mass of the car is M , that of the block is m and the spring has spring constant k . Initially, the car and the block are at rest and the spring is stretched through a length x_0 when the system is released. (a) Find the amplitudes of the simple harmonic motion of the block and of the car as seen from the road. (b) Find the time period(s) of the two simple harmonic motions.



Solution 30

The above situation can be drawn as



Considering both blocks and spring as a system, there is no external resultant force on the system. Hence, center of mass will remain at rest. The mean positions of the two SHM occur when the spring becomes unstretched. If the mass M moves towards right through a distance x and the mass m moves towards left through a distance X before the spring acquires natural length,

$$x + X = x_0 \quad (1)$$

As COM remains at rest; $mx = Mx_0$ -(2)

From (1) and (2)

$$x = \frac{Mx_0}{M+m} \quad \& \quad X = \frac{mx_0}{M+m}$$

Hence, the left block is $x = \frac{Mx_0}{M+m}$ distance away from its mean position in the beginning of the motion. The force by the spring on this block at this instant is equal to the tension of spring

$$\text{i.e., } = kx_0$$

$$T = \frac{k(M+m)x}{M}$$

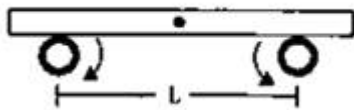
$$\text{acc} = \frac{I}{m} = \frac{k(M+m)x}{mM}$$

$$\therefore \text{Angular frequency } \omega = \sqrt{\frac{k(M+m)}{mM}}$$

$$T = 2\pi \sqrt{\frac{mM}{k(M+m)}}$$

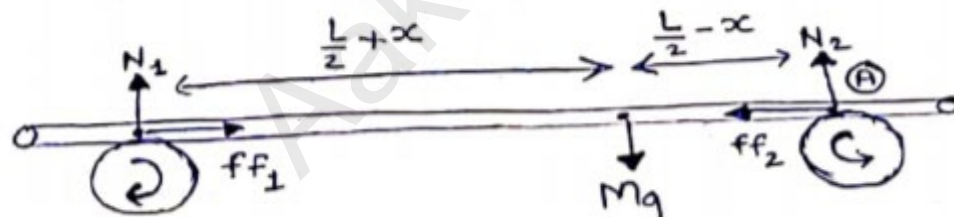
Question 31

A uniform plate of mass M stays horizontally and symmetrically on two wheels rotating in opposite directions. The separation between the wheels is L . The friction is co-efficient between each wheel and the plate is μ . Find the time period of oscillation of the plate if it is slightly displaced along its length and released.



Solution 31

Let plate is displaced by x



Vertical translatory equilibrium

$$N_1 + N_2 = Mg$$

Torque about A for rotational equilibrium.

$$\tau_{\text{clockwise}} = \tau_{\text{anticlockwise}}$$

$$N_1(L) = Mg \left(\frac{L}{2} - x \right)$$

$$N_1 = \frac{Mg}{L} \left(\frac{L}{2} - x \right)$$

$$\text{and } N_2 = Mg - N_1$$

$$\begin{aligned}
 &= Mg - \frac{Mg}{L} \left(\frac{L}{2} - x \right) \\
 N_2 &= \frac{Mg}{2} + \frac{Mgx}{L} \\
 ff_1 &= \mu N_1 = \mu \left(\frac{Mg}{2} - \frac{Mgx}{L} \right) \\
 ff_2 &= \mu N_2 = \mu \left(\frac{Mg}{2} + \frac{Mgx}{L} \right) \\
 \therefore ff_2 &> ff_1 \\
 \text{Net force in horizontal direction} &= ff_2 - ff_1 \\
 F_N &= \left[\mu \left(\frac{Mg}{2} + \frac{Mgx}{L} \right) - \mu \left(\frac{Mg}{2} - \frac{Mgx}{L} \right) \right] \\
 M \times a &= \frac{2\mu Mgx}{L} \\
 \omega^2 x &= \frac{2\mu gx}{L} \\
 \omega^2 &= \frac{2\mu g}{L} \\
 T = \frac{2\pi}{\omega} &= 2\pi \sqrt{\frac{L}{2\mu g}}
 \end{aligned}$$

Question 32

A pendulum having time period equal to two seconds is called a seconds pendulum. Those used in pendulum clocks are of this type. Find the length of a seconds pendulum at a place where $g = \pi^2 \text{ m/s}^2$.

Solution 32

$$\begin{aligned}
 T &= 2\pi \sqrt{\frac{l}{g}} \\
 \text{Given, } T &= 2 \text{ sec} \\
 2 &= 2\pi \sqrt{\frac{l}{\pi^2}} \\
 4 &= 4\pi^2 \times \frac{l}{\pi^2} \\
 l &= 1 \text{ m}
 \end{aligned}$$

Question 33

The angle made by the string of a simple pendulum with the vertical depends on the time as $\theta = \frac{\pi}{90} \sin[(\pi s^{-1})t]$. Find the length of the pendulum if $g = \pi^2 \text{ m/s}^2$.

Solution 33

$$\theta = \frac{\pi}{90} \sin[(\pi s^{-1})t]$$

This is similar to SHM equation.

$$\begin{aligned}
 \omega &= \pi \text{ sec}^{-1} \\
 \frac{2\pi}{T} &= \pi \text{ sec}^{-1}
 \end{aligned}$$

$$T = 2 \text{ sec}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$2 = 2\pi \sqrt{\frac{l}{g}}$$

$$4 = 4\pi^2 \left(\frac{l}{g} \right) \Rightarrow l = 1 \text{ m}$$

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Question 34

The pendulum of a certain clock has time period 2.04s. How fast or slow does the clock run during 24hours?

Solution 34

We know Time period of pendulum clock = 2sec

1 oscillation $\rightarrow$ 2.00 sec

x oscillations $\rightarrow$ 1 day = 24 $\times$ 3600 sec

$$x = \frac{24 \times 3600}{2} = 43200$$

Give T=2.04 sec

In each oscillation, it is slower by 2.04-2.00=0.04 sec

For 1 day = $43200 \times 0.04 = 28.8$ min

$\therefore$ The clock runs 28.8 min slower in one day.

Question 35

A pendulum clock giving correct time at a place where $g = 9.800 \text{ m/s}^2$ is taken to another place where it loses 24 seconds during 24 hours. Find the value of g at this new place.

Solution 35

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$\frac{\Delta T}{T} = \frac{1}{2} \cdot \frac{\Delta g}{g}$$

$$\frac{24}{24 \times 3600} = \frac{1}{2} \cdot \frac{\Delta g}{g}$$

$$\frac{\Delta g}{g} = \frac{1}{1800}$$

$$1 - \frac{g'}{g} = \frac{1}{1800}$$

$$\frac{g'}{g} = \frac{1799}{1800}$$

$$g' = \frac{1799}{1800} \times 9.8 = 9.795 \text{ m/s}^2$$

Question 36

A simple pendulum is constructed by hanging a heavy ball by a 5.0m long string. It undergoes small oscillations. (a) How many oscillations does it make per second? (b) What will be the frequency if the system is taken on the moon where acceleration due to gravitation of the moon 1.67 m/s^2 .

Solution 36

$$T = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{5}{10}} = 1.4\pi$$

(a) $n = \frac{1}{T} = \frac{1}{1.4\pi} = \frac{0.7}{\pi} \text{ sec}^{-1}$

Frequency =

(b) When it is on moon

$$T = 2\pi \sqrt{\frac{l}{g'}}$$

$$T = 2\pi \sqrt{\frac{5}{1.67}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi \sqrt{\frac{5}{1.67}}} = \frac{1}{2\pi \sqrt{3}}$$

$$f = \frac{1}{2\pi \sqrt{3}} \text{ sec}^{-1}$$

Question 37

The maximum tension in the spring of an oscillating pendulum is double of the minimum tension. Find the angular amplitude.

Solution 37

Change in K.E = change in P.E

$$\frac{1}{2}mv^2 - 0 = mgl - mgl \cos \theta$$

$$v^2 = 2gl(1 - \cos \theta)$$

$$T = mg + \frac{mv^2}{l}$$

Now,

$$mg = \text{due to weight}$$

$$\frac{mv^2}{l} = \text{due to oscillation (centrifugal force)}$$

$$T_{\max} = mg + \frac{m}{l} [2gl(1 - \cos \theta)]$$

$$= mg + 2mg(1 - \cos \theta)$$

$$T_{\min} = mg \cos \theta$$

$$\text{Now, } T_{\max} = 2T_{\min}$$

$$mg + 2mg(1 - \cos \theta) = 2mg \cos \theta$$

$$3mg = 4mg \cos \theta$$

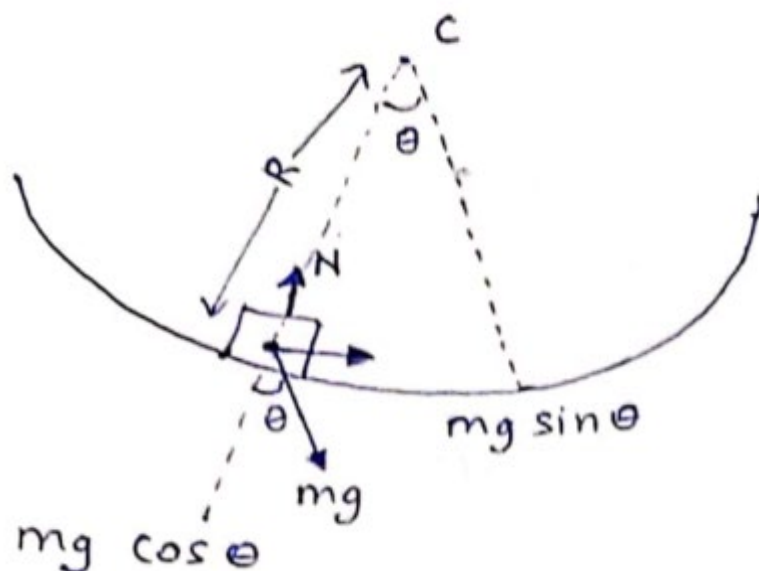
$$\theta = \cos^{-1} \left(\frac{3}{4} \right)$$

Question 38

A small block oscillates back and forth on a smooth concave surface of radius R. Find the time period of small oscillation.



Solution 38



Torque about center of concave surface

$$\tau = \tau_a$$

$$\tau = F_1 (\text{distance})$$

$$\tau_{rest} = -mg \sin \theta (R)$$

$$I_a = -mgR \sin \theta$$

$$[\theta \text{ is very small } \sin \theta \approx \theta]$$

$$mR^2 \alpha = -mgR \theta$$

$$\alpha = \frac{-g\theta}{R}$$

$$\text{Comparing with } \alpha = -\omega^2 \theta$$

$$\omega^2 = \frac{g}{R}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{R}{g}}$$

Question 39

A spherical ball of mass m and radius r rolls without slipping on a rough concave surface of large radius R . It makes small oscillations about the lowest point. Find the time period.

Solution 39

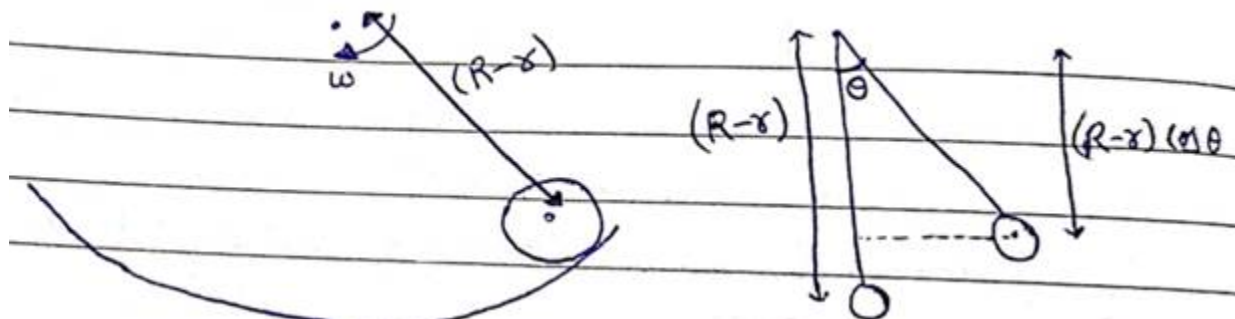
Let ω be the angular velocity of the system about the point of suspension at any point.

Velocity of the ball rolling on a rough concave surface (V_C) is given by,

$$V_C = (R - r)\omega$$

$$\text{Also, } V_C = r\omega_1$$

where ω_1 is the rotational velocity of the sphere.



$$\Rightarrow \omega_1 = \frac{V_C}{r} = \left(\frac{R-r}{r}\right) \omega \quad (1)$$

As total energy of a particle in SHM remains constant,

$$mg(R-r)(1 - \cos \theta) + \frac{1}{2} m V_C^2 + \frac{1}{2} I \omega_1^2 = \text{constant}$$

Substituting the values of V_C and ω in the above equation, we get:

$$mg(R-r)(1 - \cos \theta) + \frac{1}{2} m(R-r)^2 \omega^2 + \frac{1}{2} m r^2 \left(\frac{R-r}{r}\right) \omega^2 = \text{constant}$$

$$(\because I = m r^2)$$

$$mg(R-r)(1 - \cos \theta) + \frac{1}{2} m(R-r)^2 \omega^2 + \frac{1}{2} m r^2 \left(\frac{R-r}{r}\right) \omega^2 = \text{constant}$$

$$\Rightarrow g(R-r)(1 - \cos \theta) + (R-r)^2 \omega^2 \left[\frac{1}{2} + \frac{1}{2}\right] = \text{constant}$$

Taking derivative on both sides, we get:

$$g(R-r) \sin \theta \frac{d\theta}{dt} = \frac{7}{10} (R-r)^2 2\omega \frac{d\omega}{dt}$$

$$\Rightarrow g \sin \theta = 2 \times \left(\frac{7}{10}\right) (R-r) \propto \quad (\because \propto = \frac{d\omega}{dt})$$

$$\Rightarrow g \sin \theta = \left(\frac{7}{5}\right) (R-r) \propto$$

$$\propto = \frac{5g \sin \theta}{7(R-r)}$$

$$= \frac{5g \theta}{7(R-r)}$$

$$\therefore \frac{\propto}{\theta} = \omega^2 = \frac{5g}{7(R-r)} = \text{constant}$$

Therefore, the motion is S.H.M

$$\omega = \sqrt{\frac{5g}{7(R-r)}}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{7(R-r)}{5g}}$$

Time period is given by,

Question 40

A simple pendulum of length 40cm is taken inside a deep mine. Assume for the time being that the mine is 1600km deep. Calculate the time period of the pendulum there. Radius of the earth=6400km.

Solution 40

Acceleration due to gravity at a depth of 1600 mts is

$$g_d = g \left(1 - \frac{d}{R}\right)$$

$$= 9.8 \left(1 - \frac{1600}{6400} \right)$$

$$g_d = 7.35 \text{ m/s}^2$$

$$T_d = 2\pi \sqrt{\frac{l}{g_d}}$$

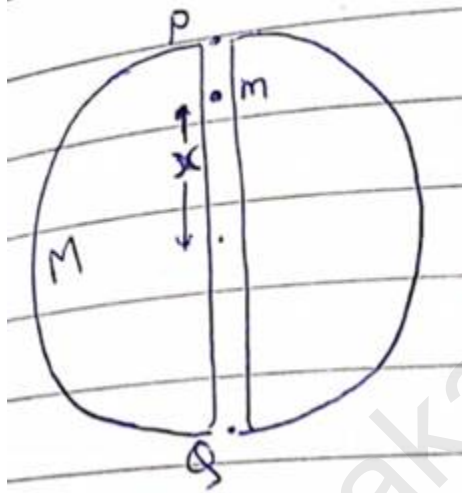
Time period at depth,
 $l=0.4\text{m}$

$$T_d = 2\pi \sqrt{\frac{0.4}{7.35}} = 1.47 \text{ sec}$$

Question 41

Assume that a tunnel is dug across the earth (radius= R) passing through its center. Find the time a particle takes to cover the length of the tunnel if (a) it is projected into the tunnel with a speed of $\sqrt{gR}$ (b) it is released from a height R above the tunnel (c) it is thrown vertically upward along the length of tunnel with a speed of $\sqrt{gR}$.

Solution 41



Force on particle of mass m at a distance x from center is

$$\left(\frac{GMx}{R^3} \right) m$$

$$F = \frac{-GMmx}{R^3}$$

$$-m\omega^2 x = \frac{GMmx}{R^3}$$

$$\omega^2 = \frac{g}{R}$$

$$\omega = \sqrt{\frac{g}{R}}$$

$$(a) \quad v = \omega \sqrt{A^2 - y^2}$$

$$\text{At } y=R; \quad v = \sqrt{gR}$$

$$\sqrt{gR} = \sqrt{\frac{g}{R}} (\sqrt{A^2 - R^2})$$

$$R = \sqrt{A^2 - R^2}$$

$$2R^2 = A^2$$

$$A = \sqrt{2}R$$

Displacement-time equation given by

$$y = \sqrt{2}R \sin \omega t$$

Let t_1 and t_2 be the time taken by the particle to reach the positions P and Q.

$$\frac{\pi}{2} < \text{Phase at P} < \pi \text{ and } \pi < \text{Phase at Q} < \frac{3\pi}{2}$$

At point P; $y=R$

$$R = \sqrt{2}R \sin \omega_1 t$$

$$\omega_1 t = \frac{3\pi}{4}$$

At point Q; $y=R$

$$R = \sqrt{2}R \sin \omega_2 t$$

$$\omega_2 t = \frac{5\pi}{4}$$

$$\text{So, } \omega(t_2 - t_1) = \frac{\pi}{2}$$

$$t_2 - t_1 = \frac{\pi}{2\omega} = \frac{\pi}{2} \sqrt{\frac{R}{g}}$$

(b) By energy conservation,

At height R above tunnel = At tunnel surface

$$0 + \left(\frac{-GMm}{2R} \right) = \frac{1}{2}mv^2 + \left(\frac{-GMm}{R} \right)$$

$$\frac{GMm}{R} - \frac{GMm}{2R} = \frac{1}{2}mv^2$$

$$\frac{GMm}{2R} = \frac{1}{2}mv^2$$

$$v = \sqrt{gR} \quad \because g = \frac{GM}{R^2}$$

$$\text{So, time to cover tunnel length} = \frac{\pi}{2} \sqrt{\frac{R}{g}}$$

(c) By energy conservation, the velocity of ball on reaching back again at projection point

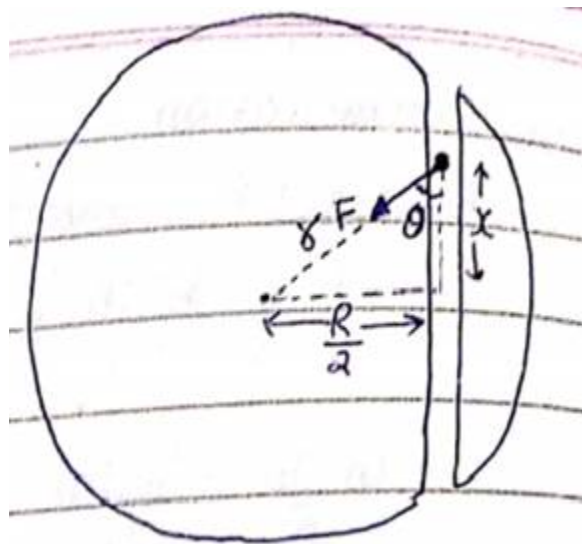
$$\text{will be } v = \sqrt{gR}.$$

$$\text{So, time to cover tunnel length} = \frac{\pi}{2} \sqrt{\frac{R}{g}}$$

Question 42

Assume that a tunnel is dug along a cord of the earth, at a perpendicular distance $R/2$ from the earth's center where R is the radius of the earth. The wall of the tunnel is frictionless. (a) Find the gravitational force exerted by the earth on a particle of mass m placed in the tunnel at a distance x from the center of the tunnel. (b) Find the component of this force along the tunnel and perpendicular to the tunnel. (c) Find the normal force exerted by the wall on the particle. (d) Find the resultant force on the particle. (e) Show that the motion of the particle in the tunnel is simple harmonic and find the time period.

Solution 42



(b) Force along tunnel = $F \cos \theta$

$$F_{\parallel} = \left(\frac{GMmr}{R^3} \right) \left(\frac{x}{r} \right)$$

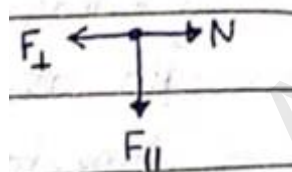
$$F_{\parallel} = \left(\frac{GMmx}{R^3} \right)$$

Force perpendicular to tunnel = $F \sin \theta$

$$F_{\perp} = \left(\frac{GMmr}{R^3} \right) \left(\frac{R}{2r} \right)$$

$$F_{\perp} = \left(\frac{GMm}{R^2} \right)$$

(c) Normal force exerted by wall, $N = F_{\perp}$



$$N = \frac{GMm}{R^2}$$

(d) Resultant force is $F_{\parallel}$

$$= \left(\frac{GMmx}{R^3} \right)$$

(e) $F_N = \text{mass} \times \text{acceleration}$

$$\frac{GMmx}{R^3} = m \times \text{acc}$$

$$\text{acc} = \frac{GMx}{R^3} \quad (\text{towards center})$$

$$a \propto x$$

So, body executes SHM.

$$\omega^2 x = \frac{GMx}{R^3}$$

$$\omega = \sqrt{\frac{GM}{R^3}}$$

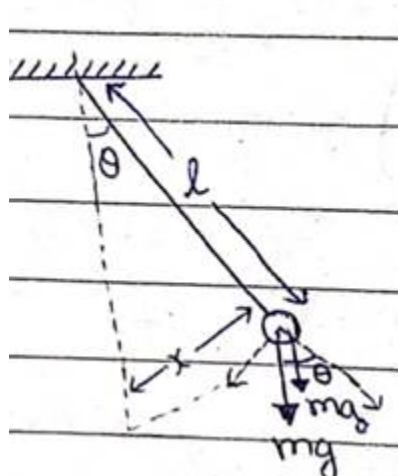
$$T = 2\pi \sqrt{\frac{R^3}{GM}}$$

Question 43

A simple pendulum of length l is suspended through the ceiling of an elevator. Find the time period of small oscillations if the elevator (a) is going up with an acceleration a_0 (b) is going down with an acceleration a_0 and (c) is moving with a uniform velocity.

Solution 43

(a) Restoring force



$$F_R = -(mg + ma_0) \sin \theta$$

$$m \times \text{acc} = -m(g + a_0) \sin \theta$$

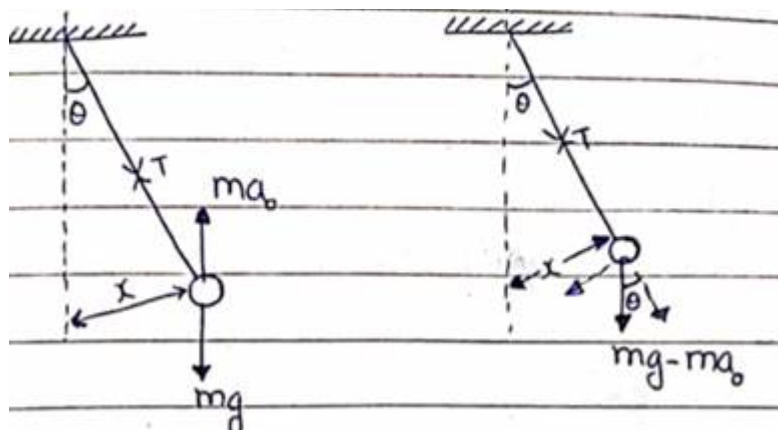
$$\text{acc} = -(g + a_0) \left(\frac{x}{l} \right)$$

$$-\omega^2 x = \frac{-(g + a_0)}{l} \cdot x$$

$$\omega = \sqrt{\frac{g + a_0}{l}}$$

$$T = 2\pi \sqrt{\frac{l}{(g + a_0)}}$$

(b)



$$F_R = -(mg - ma_0) \sin \theta$$

$$m \times \text{acc} = -m(g - a_0) \left(\frac{x}{l} \right)$$

$$-\omega^2 x = -(g - a_0) \left(\frac{x}{l} \right)$$

$$\omega = \sqrt{\frac{g - a_0}{l}}$$

$$T = 2\pi \sqrt{\frac{l}{(g - a_0)}}$$

(c) uniform velocity means $a=0$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Question 44

A simple pendulum of length 1 feet suspended from the ceiling of an elevator takes $\frac{\pi}{3}$ seconds to complete one oscillation. Find the acceleration of the elevator.

Solution 44

Time period of simple pendulum having acceleration 'a' upward

$$T = 2\pi \sqrt{\frac{l}{(g+a)}}$$

$$\frac{\pi}{3} = 2\pi \sqrt{\frac{1}{(32+a)}}$$

$$\frac{1}{6} = \sqrt{\frac{1}{(32+a)}}$$

Squaring,

$$32+a=36$$

$$a = 4 \text{ ft/sec}^2$$

Question 45

A simple pendulum fixed in a car has a time period of 4 seconds when the car is moving uniformly on a horizontal road. When the accelerator is passed, the time period changes to 3.99 seconds. Making an approximation analysis, find the acceleration of the car.

Solution 45

Here velocity is uniform.

So, acceleration = 0 m/s^2

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$4 = 2\pi \sqrt{\frac{l}{g}} \quad (1)$$

If car is moving with acceleration a_0 ,

$$T_2 = 2\pi \sqrt{\frac{l}{\sqrt{g^2 + a^2}}}$$

Then

$$3.99 = 2\pi \sqrt{\frac{l}{\sqrt{g^2 + a^2}}} \quad (2)$$

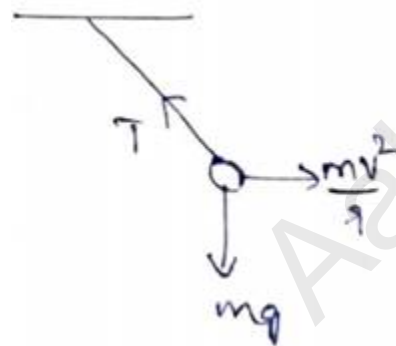
$$\frac{(1)}{(2)} = \frac{4}{3.99} = \frac{2\pi \sqrt{\frac{l}{g}}}{2\pi \sqrt{\frac{l}{\sqrt{g^2 + a^2}}}} = \frac{\sqrt{g^2 + a^2}}{\sqrt{g}}$$

$$\Rightarrow \frac{4}{3.99} = \frac{(g^2 + a_0^2)^{\frac{1}{4}}}{\sqrt{g}}$$

Question 46

A simple pendulum of length l is suspended from the ceiling of the car moving with a speed v on a circular horizontal road of radius r . (a) Find the tension in the string when it is at rest with respect to car. (b) Find the time period of small oscillation.

Solution 46



$$T = \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$$

$$ma = m \sqrt{g^2 + \frac{v^4}{r^2}}$$

$$a = \sqrt{g^2 + \frac{v^4}{r^2}}$$

$$T = 2\pi \sqrt{\frac{l}{\left(g^2 + \frac{v^4}{r^2}\right)^{\frac{1}{2}}}}$$

Question 47

The ear ring of a lady shown in figure has a 3cm long light suspension wire. (a) Find the time period of small oscillations if the lady is standing on the ground. (b) The lady now sits in a merry-go-round moving at 4m/s in a circle of radius 2cm. Find the time period of small oscillations of the ear-ring.



Solution 47

$$T = 2\pi \sqrt{\frac{l}{g}} \quad [\because l = 3\text{cm} = 0.03\text{m}]$$

$$(a) \quad = 2\pi \sqrt{\frac{0.03}{9.8}} = 0.34 \text{ sec}$$

(b) If she sits on a merry-go-round, she and her earrings will have centripetal acceleration.

$$T = \sqrt{\left(\frac{mv^2}{r}\right)^2 + (mg)^2}$$

$$ma = m \sqrt{\frac{v^2}{r} + g^2}$$

$$a = \sqrt{\frac{v^2}{r} + g^2}$$

$$= \sqrt{\frac{4^2}{2} + (10)^2} = 12.8\text{m/s}^2$$

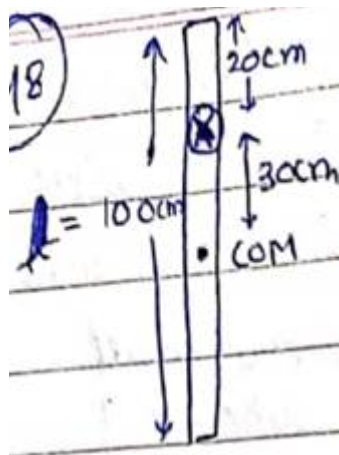
$$\text{Now, } T = 2\pi \sqrt{\frac{0.03}{12.8}} = 0.3 \text{ sec}$$

Question 48

Find the time period of small oscillations of the following systems. (a) A meter stick suspended through the 20cm mark. (b) A ring of mass m and radius r suspended through a point on its periphery. (c) A uniform square plate of edge a suspended through a corner. (d) A uniform disc of mass m and radius r suspended through a point $r/2$ away from the center.

Solution 48

(a)



$$T = 2\pi \sqrt{\frac{I}{mgl}}$$

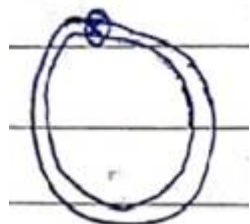
$$= 2\pi \sqrt{\frac{\left(\frac{ml^2}{12} + m(0.3)^2\right)}{mg(0.3)}}$$

$$= 2\pi \sqrt{\frac{\left[\frac{l^2}{12} + 0.09\right]}{g(0.3)}}$$

$$= 2\pi \sqrt{\frac{1 + 0.09 \times 12}{12(g)(0.3)}}$$

$$T = 1.52 \text{ sec}$$

(b)

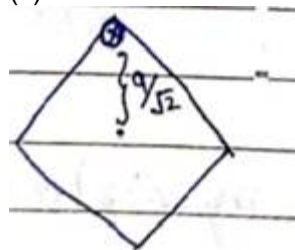


$$T = 2\pi \sqrt{\frac{I}{mgl}}$$

$$= 2\pi \sqrt{\frac{(mR^2 + mR^2)}{mgR}}$$

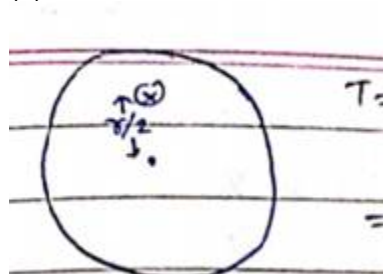
$$= 2\pi \sqrt{\frac{2R}{g}}$$

(c)



$$\begin{aligned}
 T &= 2\pi \sqrt{\frac{I}{mgl}} \\
 &= 2\pi \sqrt{\frac{\frac{ma^2}{6} + m\left(\frac{a}{\sqrt{2}}\right)^2}{mg\left(\frac{a}{\sqrt{2}}\right)}} \\
 &= 2\pi \sqrt{\frac{\frac{a^2}{6} + \frac{a^2}{2}}{\frac{ga}{\sqrt{2}}}} \\
 &= 2\pi \sqrt{\frac{\frac{2a^2}{3}}{\frac{ga}{\sqrt{2}}}} = 2\pi \sqrt{\frac{2\sqrt{2}a}{3g}}
 \end{aligned}$$

(d)



$$\begin{aligned}
 T &= 2\pi \sqrt{\frac{I}{mgl}} \\
 &= 2\pi \sqrt{\frac{\frac{mr^2}{2} + m\left(\frac{r}{2}\right)^2}{mg\left(\frac{r}{2}\right)}} \\
 &= 2\pi \sqrt{\frac{\frac{3mr^2}{2}}{\frac{mgr}{2}}} \\
 &= 2\pi \sqrt{\frac{3r}{2g}}
 \end{aligned}$$

Question 49

A uniform rod of length l is suspended by an end and is made to undergo small oscillations. Find the length of the simple pendulum having the time period equal to that of the rod.

Solution 49

Time period of rod

$$\begin{aligned}
 T_{rod} &= 2\pi \sqrt{\frac{I}{mgl}} \\
 &= 2\pi \sqrt{\frac{\frac{ml^2}{3}}{\frac{mgl}{2}}}
 \end{aligned}$$

$$T_{rod} = 2\pi \sqrt{\frac{2l}{3g}}$$

Now,

$$T_{rod} = T_{pend}$$

$$2\pi \sqrt{\frac{2l}{3g}} = 2\pi \sqrt{\frac{l_s}{g}}$$

$$l_s = \frac{2l}{3}$$

Question 50

A uniform disc of radius r is to be suspended through a small hole made in the disc. Find the minimum possible time period of the disc for small oscillations. What should be the distance of the hole from the center for it to have minimum time period?

Solution 50

Let distance of hole from center of disc be x

$$T = 2\pi \sqrt{\frac{I}{mgl}}$$

$$= 2\pi \sqrt{\frac{\frac{mr^2}{2} + mx^2}{mgx}}$$

$$= 2\pi \sqrt{\frac{r^2}{2gx} + \frac{x}{g}}$$

For minimum time period,

$$y = \frac{r^2}{2gx} + \frac{x}{g}$$

Minimize,

Differentiate with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{r^2}{2gx} \right) + \left(\frac{x}{g} \right)$$

$$= \frac{r^2}{2g} \left(\frac{-1}{x^2} \right) + \frac{1}{g} = 0$$

$$\frac{r^2}{2gx^2} = \frac{1}{g}$$

$$x = r/\sqrt{2}$$

Now, put x in T

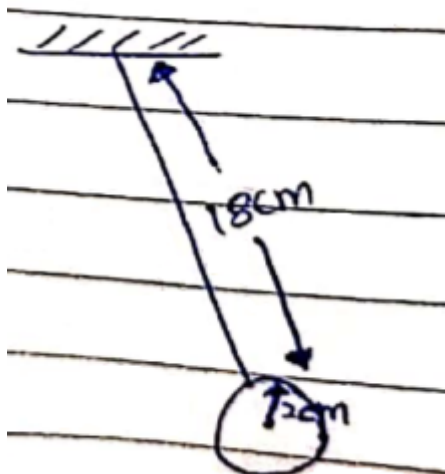
$$T = 2\pi \sqrt{\frac{r^2}{2g} \left(\frac{\sqrt{2}}{r} \right) + \frac{r}{\sqrt{2}g}}$$

$$T = 2\pi \sqrt{\frac{\sqrt{2}r}{g}}$$

Question 51

A hollow sphere of radius 2cm is attached to an 18cm long thread to make a pendulum. Find the time period of oscillation of this pendulum. How does it differ from the time period calculated using the formula for a simple pendulum?

Solution 51



$$T = 2\pi \sqrt{\frac{I}{mgl}}$$

$$= 2\pi \sqrt{\frac{\frac{1}{2}mr^2 + m(l+r)^2}{mg(l+r)}}$$

$$T = 2\pi \sqrt{\frac{\frac{1}{2}(0.02)^2 + (0.18+0.02)^2}{9.8(0.18+0.02)}} = 0.89 \text{ sec}$$

Time period by considering it as simple pendulum

$$T' = 2\pi \sqrt{\frac{l}{g}}$$

$$= 2\pi \sqrt{\frac{0.19}{9.8}} = 0.86 \text{ sec}$$

$$\% \text{ more} = \frac{T - T'}{T'} = \frac{0.89 - 0.86}{0.86} \cong 3.48\%$$

Question 52

A closed circular wire hung on a nail in a wall undergoes small oscillations of amplitude 2° and time period 2s. Find (a) the radius of the circular wire, (b) the speed of the particle farthest away from the point of suspension as it goes through its mean position, (c) the acceleration of this particle as it goes through its mean position and (d) the acceleration of this particle when it is at an extreme position. Take $g = \pi^2 \text{ m/s}^2$.

Solution 52

$$T = 2\pi \sqrt{\frac{I}{mgl}}$$

(a)

$$2 = 2\pi \sqrt{\frac{mR^2 + mR^2}{mgR}}$$

$$2 = 2\pi \sqrt{\frac{2R}{g}}$$

$$\frac{1}{\pi^2} = \frac{2R}{g}$$

$$R = \frac{g}{2\pi^2} = 0.5m = 50 \text{ cm}$$

(b) Total energy at extreme position = Total energy at mean position

$$0 + mgR(1 - \cos \theta) = \frac{1}{2} I \omega^2 + 0$$

$$mgR(1 - \cos \theta) = \frac{1}{2} (mR^2 + mR^2) \cdot \omega^2$$

$$gR(1 - \cos 2^\circ) = R^2 \omega^2$$

$$\omega^2 = \frac{g}{R} (1 - \cos 2^\circ)$$

$$\omega^2 = \frac{9.8}{0.5} (1 - 0.999)$$

$$\omega = 0.11 \text{ rad/s}$$

Speed of lowest point $\Rightarrow v = (\text{distance}) \omega$

$$v = R\omega = 2 \times 0.5 \times 0.11$$

$$v = 0.11 \frac{m}{s} = 11 \text{ cm/sec}$$

$$\vec{a}_{\text{radial}} = \frac{v^2}{(2R)} = \frac{(0.11)^2}{2 \times 0.5} = 0.0121 \frac{m}{s^2} = 1.21 \text{ cm/s}^2$$

$$(c) \vec{a}_t = 0$$

$$\vec{a}_N = \sqrt{a_t^2 + a_r^2} = 1.21 \text{ cm/s}^2$$

(d) At extreme position, $v=0$

$$\text{Radial acceleration, } a_r = \frac{v^2}{R} = 0 \text{ cm/s}^2$$

Tangential acceleration,

$$a_t = \frac{mg \sin 2^\circ}{m} = g \sin 2^\circ = 0.34 \frac{m}{s^2} = 34 \text{ cm/s}^2$$

$$\vec{a}_N = \sqrt{a_t^2 + a_r^2}$$

$$a_N = a_t = 34 \text{ cm/s}^2$$

Question 53

A uniform disc of mass m and radius r is suspended through a wire attached to its center. If the time period of the torsional constant of the wire.

Solution 53

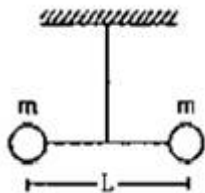
$$T = 2\pi \sqrt{\frac{I}{C}}$$

$$T = 2\pi \sqrt{\frac{mR^2}{2C}}$$

$$T^2 = 4\pi^2 \left(\frac{mR^2}{2C} \right)$$

$$C = \frac{2\pi^2 m R^2}{T^2}$$

Two small balls, each of mass m are connected by a light rigid rod of length L . The system is suspended from its center by a thin wire of torsional constant k . The rod is rotated about the wire through an angle θ_0 and released. Find the tension in the rod as the system passes through the mean position.



Solution 54

Torque when rod is at an angle θ about the wire.

$$\tau_{rest} = -k\theta$$

Work done in rotating from θ_0 to 0

$$= \int \tau_{rest} \cdot d\theta$$

$$= - \int k\theta \cdot d\theta$$

$$= \frac{k\theta_0^2}{2}$$

$$\text{Work} = \frac{k\theta_0^2}{2}$$

This work is in form of rotational kinetic energy.

$$\frac{1}{2} I \omega^2 = \frac{k\theta_0^2}{2}$$

$$\frac{1}{2} \left[m \left(\frac{L}{2} \right)^2 + m \left(\frac{L}{2} \right)^2 \right] \omega^2 = \frac{k\theta_0^2}{2}$$

$$\omega^2 \left(\frac{mL^2}{2} \right) = k\theta_0^2$$

$$\omega^2 = \frac{2k\theta_0^2}{mL^2}$$

Question 55

A particle is subjected to two simple harmonic motions of same time period in the same direction.

The amplitude of the first motion is 3.0cm and that of the second is 4.0cm. Find the resultant

amplitude if the phase difference between the motions is (a) 0° , (b) 60° , (c) 90° .

Solution 55

$$A_1 = 3 \text{ cm}; \quad A_2 = 4 \text{ cm}$$

$$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$

(a) $\phi = 0^\circ$

$$A_R = \sqrt{3^2 + 4^2 + 2(3)(4) \cos 0^\circ} = 7 \text{ cm}$$

(b) $\phi = 60^\circ$

$$A_R = \sqrt{3^2 + 4^2 + 2(3)(4)\cos 60^\circ}$$

$$= \sqrt{9 + 16 + 2 \times 3 \times 4 \times \frac{1}{2}}$$

$$A_R = \sqrt{37} = 6.1 \text{ cm}$$

(c) $\phi = 90^\circ$

$$A_R = \sqrt{3^2 + 4^2 + 2(3)(4)\cos 90^\circ}$$

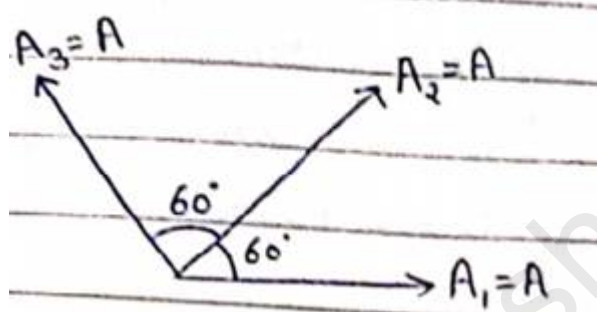
$$A_R = 5 \text{ cm}$$

Question 56

Three simple harmonic motions of equal amplitudes A and equal time periods in the same direction combine. The phase of the second motion is 60° ahead of the first. Find the amplitude of the resultant motion.

Solution 56

By superposition principle,



Resultant of A_1 and A_3 vector is $= \sqrt{A^2 + A^2 + 2AA\cos 120^\circ}$

$= A$ along A_2 vector

Now, resultant of A_2 vector and A_{13} vector $= A + A = 2A$

Question 57

A particle is subjected to two simple harmonic motions given by

$$x_1 = 2.0 \sin(100\pi t) \text{ and } x_2 = 2.0 \sin(120\pi t + \pi/3)$$

where x is in centimeter and t in second. Find the displacement of the particle at (a) $t=0.0125$, (b) $t=0.025$.

Solution 57

Resultant displacement

$$x_N = x_1 + x_2$$

$$x_N = 2 \sin 100\pi t + 2 \sin(120\pi t + \pi/3)$$

(a) $t=0.0125\text{sec}$

$$x_N = 2 \sin(100\pi \times 0.0125) + 2 \sin(120\pi \times 0.0125 + \pi/3)$$

$$x_N = 2 \sin\left(\frac{5\pi}{4}\right) + 2\left(\frac{-1}{2}\right) = -\sqrt{2} - 1$$

$$x_N = -2.41 \text{ cm}$$

(b) $t=0.025\text{sec}$

$$x_N = 2 \sin(100\pi \times 0.025) + 2 \sin(120\pi \times 0.025 + \pi/3)$$

$$x_N = 2 \sin\left(\frac{5\pi}{2}\right) + 2 \sin\left(3\pi + \frac{\pi}{3}\right)$$

$$= 2(1) + 2 \cdot \left(\frac{\sqrt{3}}{2}\right) = 0.27 \text{ cm}$$

Question 58

A particle is subjected to two simple harmonic motions, one along the X-axis and the other on a line making an angle of 45° with the X-axis. The two motions are given by

$$x = x_0 \sin \omega t \text{ and } s = s_0 \sin \omega t$$

Find the amplitude of the resultant motion.

Solution 58

Phase difference, $\phi = 45^\circ$

$$A_1 = x_0 ; A_2 = s_0$$

$$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$

$$= \sqrt{x_0^2 + s_0^2 + 2x_0s_0 \cos 45^\circ}$$

$$A_R = \sqrt{x_0^2 + s_0^2 + \sqrt{2}x_0s_0}$$