

HC VERMA Solutions for Class 11 Physics Chapter 11 Gravitation

Question 1

Two spherical balls of mass 10kg each are placed 10cm apart. Find the gravitational force of attraction between them.

Solution 1

$$F = \frac{Gm_1m_2}{r^2}$$

F = Gravitational force between two bodies

G = Gravitational constant

m_1, m_2 = Masses of the two bodies

r = Distance between the bodies

Here, $m_1 = m_2 = 10 \text{ kg}$

$$G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 10 \times 10}{0.1^2}$$

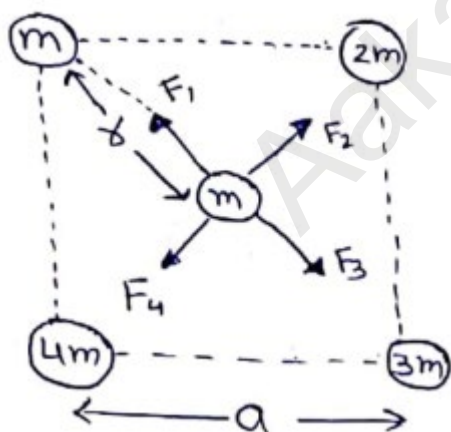
$$F = 6.67 \times 10^{-7} \text{ N}$$

Question 2

Four particles having masses m, 2m, 3m and 4m are placed at the four corners of a square of edge a. Find the gravitational force acting on a particle of mass m placed at the center.

Solution 2

$$r = \frac{a}{\sqrt{2}}$$



$$\text{Here } \vec{F}_1 = \frac{G \cdot m \cdot m}{r^2}$$

$$= \frac{G \cdot m \cdot m}{a^2} (2) = 2k$$

$$K = \frac{Gm^2}{a^2}$$

$$\vec{F}_2 = \frac{G \cdot 2m \cdot m}{r^2} = 4K$$

$$\vec{F}_3 = \frac{G \cdot 3m \cdot m}{r^2} = 6K$$

$$\vec{F}_4 = \frac{G \cdot 4m \cdot m}{r^2} = 8K$$

Here $\vec{F}_1$ and $\vec{F}_3$ are on the same straight line.

Thus, Resultant force = $\vec{F}_3 - \vec{F}_1$ [because $\vec{F}_3 > \vec{F}_1$]

$$\vec{F}_A = 6K - 2K = 4K$$

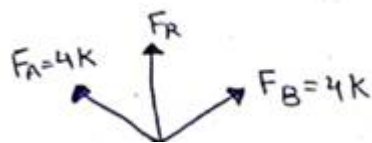
$\vec{F}_2$ and $\vec{F}_4$ are on the same straight line.

Resultant force = $\vec{F}_4 - \vec{F}_2$ [because $\vec{F}_4 > \vec{F}_2$]

$$\vec{F}_B = 8K - 4K = 4K$$

Total resultant force $\vec{F}_R = \sqrt{F_A^2 + F_B^2 + 2F_A F_B \cos \theta}$

Here $\cos \theta = 0$ because $\theta = 90^\circ$



$$\begin{aligned} \vec{F}_R &= \sqrt{F_A^2 + F_B^2} \\ &= \sqrt{(4K)^2 + (4K)^2} \\ &= 4\sqrt{2}K \\ &= 4\sqrt{2} \frac{Gm^2}{a^2} \end{aligned}$$

Resultant force on 'm' is = $4\sqrt{2} \frac{Gm^2}{a^2}$

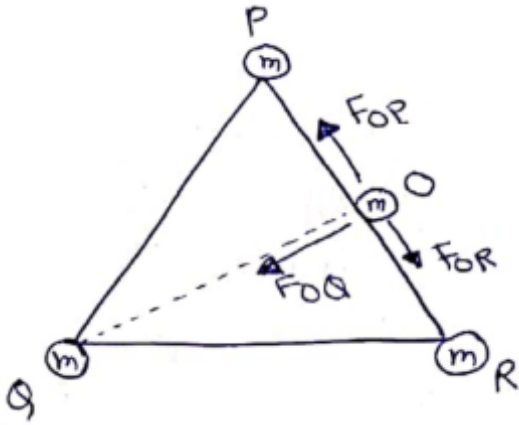
Chapter 11 - Gravitation Exercise 226

Question 3

Three equal masses m are placed at the three corners of an equilateral triangle of side a . Find the force exerted by this system on another particle of mass m placed at (a) the mid-point of a side, (b) at the center of the triangle.

Solution 3

Mass 'm' is at mid-point of side PR.



(a) $OP = OR = a/2$

$$\overrightarrow{F_{OP}} = \frac{G \times m \times m}{(a/2)^2} = \frac{Gm^2}{a^2}$$

$$\overrightarrow{F_{OP}} = \frac{4Gm^2}{a^2} = 4K$$

$$\overrightarrow{F_{OR}} = \frac{4Gm^2}{a^2} = 4K$$

$$\text{Altitude } OQ = \frac{\sqrt{3}}{2} a$$

$$\overrightarrow{F_{OQ}} = \frac{Gm^2}{(\frac{\sqrt{3}}{2} a)^2} = \frac{4Gm^2}{3a^2} = \frac{4}{3} K$$

Here, F_{OP} and F_{OR} are equal in magnitude and opposite in direction.

So, $F_{OP} + F_{OR} = 0$

Total resultant force (F)

$$= \overrightarrow{F_{OR}} + \overrightarrow{F_{OP}} + \overrightarrow{F_{OQ}}$$

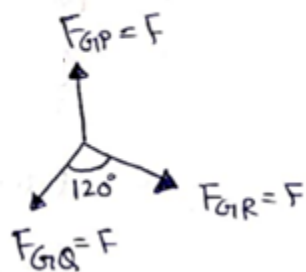
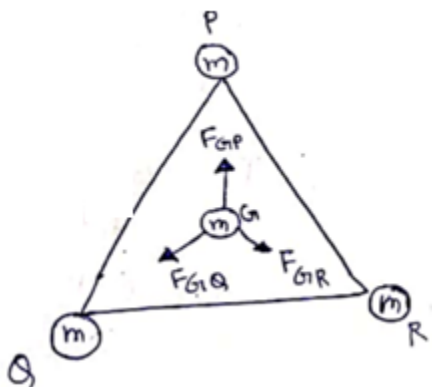
$$= 0 + \overrightarrow{F_{OQ}}$$

$$= \frac{4}{3} K$$

$$= \frac{4Gm^2}{3a^2}$$

$$= \frac{4Gm^2}{3a^2} \text{ (Along OQ)}$$

(b) If particle at centroid, All forces 'F' are equal in magnitude.



$$F = F_{GP} = F_{GQ} = F_{GR} = \frac{3Gm^2}{a^2}$$

Resultant of F_{GQ} and F_{GR}

$$F_R = \sqrt{F_{GQ}^2 + F_{GR}^2 - 2F_{GQ} \cdot F_{GR} \cdot \cos 120^\circ}$$

$$= \sqrt{F^2 + F^2 - 2F^2 \cdot \left(-\frac{1}{2}\right)}$$

$$F_R = F$$



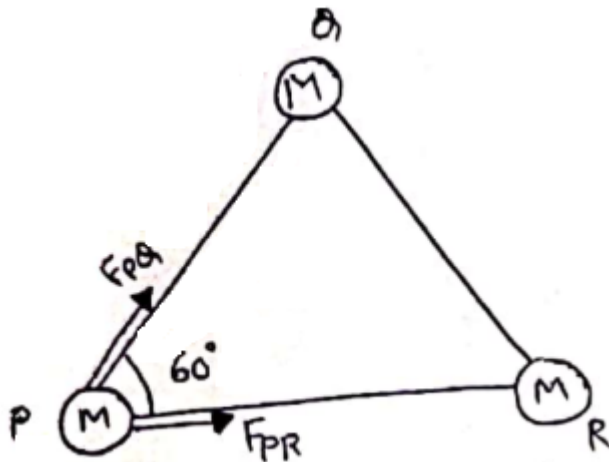
The resultant is 0, because they have equal magnitude and are in opposite direction.

Question 4

Three uniform spheres each having a mass M and radius a are kept in such a way that each touches the other two. Find the magnitude of the gravitational force on any of the spheres due to the other two.

Solution 4

Let us assume that the center of mass of spheres are located at point P, Q and R. So, re-drawing the figure, it is an equilateral triangle.



Force on mass M at point P

$$F_{PR} = F_{PQ} = \frac{GM^2}{(2a)^2} = \frac{GM^2}{4a^2} = k \text{ (let)}$$

Resultant

$$F_R = \sqrt{F_{PQ}^2 + F_{PR}^2 + 2F_{PQ} \cdot F_{PR} \cdot \cos 60^\circ}$$

$$F_R = \sqrt{k^2 + k^2 + 2k^2 \cdot \left(\frac{1}{2}\right)}$$

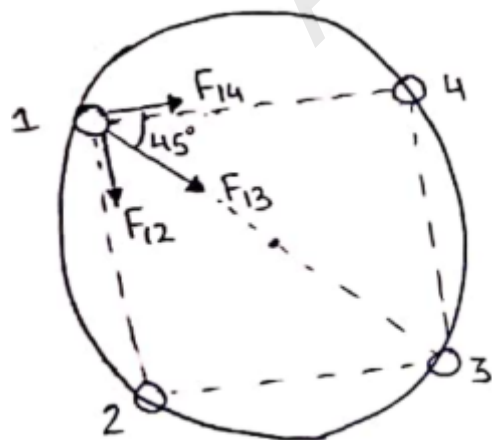
$$F_R = \sqrt{3}k$$

$$F_R = \sqrt{3} \frac{GM^2}{4a^2}$$

Question 5

Four particles of equal masses M move along a circle of radius R under the action of their mutual gravitational attraction. Find the speed of each particle.

Solution 5



Here, $r_{13} = 2R$

$$r_{14} = 2R \cos 45 = \sqrt{2}R$$

$$r_{12} = 2R \sin 45 = \sqrt{2}R$$

Now, force on particle 1

Due to particle 4: $F_{14} = \frac{GMM}{(\sqrt{2}R)^2} = \frac{GM^2}{2R^2}$

Due to particle 2: $F_{12} = \frac{GMM}{(\sqrt{2}R)^2} = \frac{GM^2}{2R^2}$

Due to particle 1: $F_{13} = \frac{GMM}{(2R)^2} = \frac{GM^2}{4R^2}$

Resultant of F_{14} & $F_{12} = \sqrt{F_{12}^2 + F_{14}^2 + 2(F_{12})(F_{14}) \cos 90^\circ}$

$$= \sqrt{\left(\frac{GM^2}{2R^2}\right)^2 + \left(\frac{GM^2}{2R^2}\right)^2}$$

$$F_R = \frac{GM^2}{\sqrt{2}R^2} \text{ (along } r_{13} \text{ direction)}$$

Net resultant force, $F_N = F_R + F_B$

$$= \frac{GM^2}{\sqrt{2}R^2} + \frac{GM^2}{4R^2}$$

$$F_N = \frac{GM^2}{4R^2} (2\sqrt{2} + 1)$$

Since particle is revolving in a circle. So, this is centripetal force.

$$F_N = \frac{Mv^2}{R}$$

$$\frac{GM^2}{4R^2} = \frac{Mv^2}{R}$$

$$v = \sqrt{\frac{GM}{R} \left(\frac{2\sqrt{2} + 1}{4} \right)}$$

Question 6

Find the acceleration due to gravity of the moon at a point 1000 km above the moon's surface. The mass of the moon is 7.4×10^{22} kg and its radius is 1740 km.

Solution 6

The acceleration due to gravity at a point at height 'h' from the surface of moon of radius R is

$$g = \frac{GM}{(R+h)^2}$$

$$= \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{(1740 + 1000)^2 \times 10^6}$$

$$= 0.65 \text{ m/s}^2$$

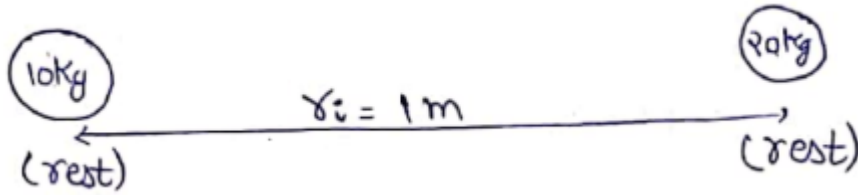
Question 7

Two small bodies of masses 10kg and 20kg are kept a distance 1.0 m apart and released, Assuming that only mutual gravitational forces are acting; find the speeds of the particles when the separation decreases to 0.5m.

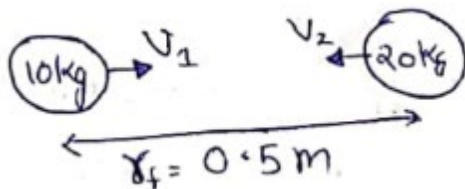
Solution 7

$$m_1 = 10\text{kg}, \quad m_2 = 20\text{kg}$$

Initially,



Finally,



If both bodies are taken as system.

$$\vec{F}_{ext} = 0$$

$$\vec{P}_i = \vec{P}_f$$

$$0 + 0 = 10v_1 - 20v_2$$

$$10v_1 = 20v_2$$

$$v_1 = 2v_2$$

Now, applying law of conservation of energy,

$$TE_i = TE_f$$

$$\begin{aligned} \frac{-Gm_1m_2}{r_i} + 0 + 0 &= \frac{-Gm_1m_2}{r_f} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \\ \frac{-6.67 \times 10^{-11} \times 10 \times 20}{1} &= \frac{-6.67 \times 10^{-11} \times 10 \times 20}{0.5} + \frac{1}{2}(10)(v_1)^2 + \frac{1}{2}(10)\left(\frac{v_1}{2}\right)^2 \end{aligned}$$

On solving,

$$v_1 = 4.2 \times 10^{-5} \text{ m/s}$$

and

$$v_2 = \frac{v_1}{2} = 2.1 \times 10^{-5} \text{ m/s}$$

Question 8

A semi-circular wire has a length L and mass M . A particle of mass m is placed at the centre of the circle.

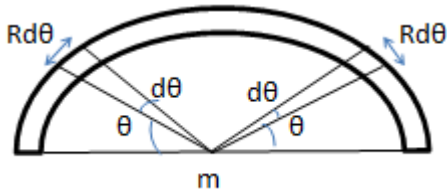
Find the gravitational attraction on the particle due to the wire.

Solution 8

Consider two small elements at an angle θ from horizontal length of the element $= R d\theta$

Mass of each element $dm = \frac{M}{L}(R d\theta)$

Force by elemental mass on particle



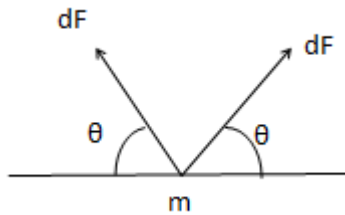
$$dF = \frac{Gm(dm)}{R^2}$$

$$dF = \frac{GmM(R d\theta)}{R^2 L}$$

$$dF = \frac{GmM}{RL} d\theta$$

Resultant of the both elemental forces $= 2dF \sin \theta$

Net Force



$$F_N = \int 2dF \sin \theta$$

$$= \int_0^{\pi/2} 2 \left(\frac{GmM}{RL} \right) d\theta \sin \theta$$

$$= \frac{2}{RL} \frac{GmM}{RL} [-\cos \theta]_0^{\pi/2}$$

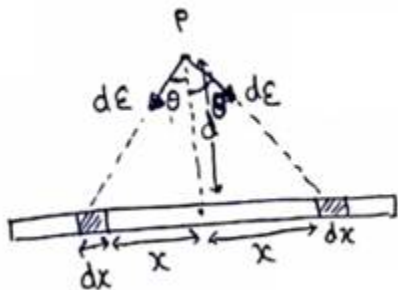
$$= \frac{2}{RL} \frac{GmM}{RL}$$

$$F_N = \frac{2\pi GmM}{L^2} \quad (\text{because } \pi R = L)$$

Question 9

Derive an expression for the gravitational field due to a uniform rod of length L and mass M at a point on its perpendicular bisector at a distance d from the centre.

Solution 9



Consider two small elements of length dx at a distance x from center on both sides of center.

Mass of each element, $dm = \frac{M}{L} dx$

Field due to each element at point P

$$= \frac{Gdm}{\sqrt{(d^2+x^2)^2}}$$

$$dE = \frac{Gdm}{d^2+x^2}$$

Resultant of the elemental field = $2dE \cos\theta$

$$\text{Net electric force} = \int \frac{Gdm}{d^2+x^2} \left(\frac{d}{\sqrt{d^2+x^2}} \right)$$

$$= \int \frac{2GdMdx}{L(d^2+x^2)^{3/2}}$$

$$= \frac{2GdM}{L} \int_0^L \left(\frac{dx}{(d^2+x^2)^{3/2}} \right)$$

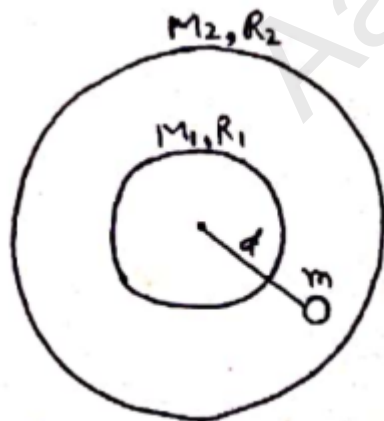
On integrating,

$$E_N = \frac{2GdM}{d\sqrt{L^2+4d^2}}$$

Question 10

Two concentric spherical shells have masses M_1 , M_2 and radii R_1 , R_2 ($R_1 < R_2$). What is the force exerted by this system on a particle of mass m if it is placed at a distance $(R_1 + R_2)/2$ from the centre?

Solution 10



$$\text{Here, } d = \frac{R_1 + R_2}{2}$$

The gravitational force of mass m due to shell of M_2 is zero as gravitational field inside shell is zero.

Gravitational field due to shell of mass M_1 outside the shell at mass $m = \frac{GM}{d^2}$

So, force $F = \frac{GMm}{d^2}$

$$F = \frac{GMm}{\left(\frac{R_1 + R_2}{2}\right)^2}$$

$$F = \frac{4GMm}{(R_1 + R_2)^2}$$

Question 11

A tunnel is dug along a diameter of the earth. Find the force on a particle of mass m placed in the tunnel at a distance x from the center.

Solution 11

Method 1:

The gravitational field inside a sphere of radius R at a distance x from center is $E = \frac{GM_s x}{R^3}$

So, force on particle of mass m is

$$F = mE$$

$$F = \frac{GM_s m}{R^3} x$$

Method 2:

Acceleration due to gravity at depth d from surface

$$g_d = g_s \left(1 - \frac{d}{R_s}\right)$$

Here, $d = R_s - x$

$$g = g_s \left[1 - \frac{(R_s - x)}{R_s}\right]$$

$$g = g_s \frac{x}{R_s}$$

So, force on particle of mass m is $= mg$

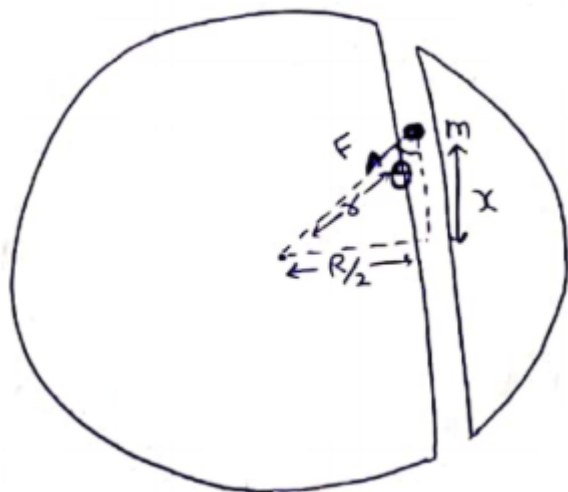
$$F = mg_s \frac{x}{R_s}$$

$$F = \frac{GM_s m}{R_s^3} x$$

Question 12

A tunnel is dug along a chord of the earth at a perpendicular distance $R/2$ from the earth's center. The wall of the tunnel may be assumed to be frictionless. Find the force exerted by the wall on a particle of mass m when it is at a distance x from the center of the tunnel.

Solution 12



Distance of the particle from earth's center

$$r = \sqrt{\left(\frac{R}{2}\right)^2 + x^2}$$

Force on the particle = mass $\times$ gravitational field intensity

$$F = m \left(\frac{GM}{R^3} \right)$$

$$G = \frac{GMm}{R^3} r$$

Force exerted by wall on particle = Force exerted by particle on wall

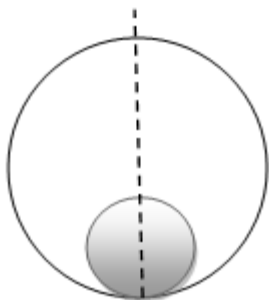
$$= F \sin \theta$$

$$= \frac{GMmr}{R^3} \left(\frac{R}{2r} \right)$$

$$= \frac{GMm}{2R^2}$$

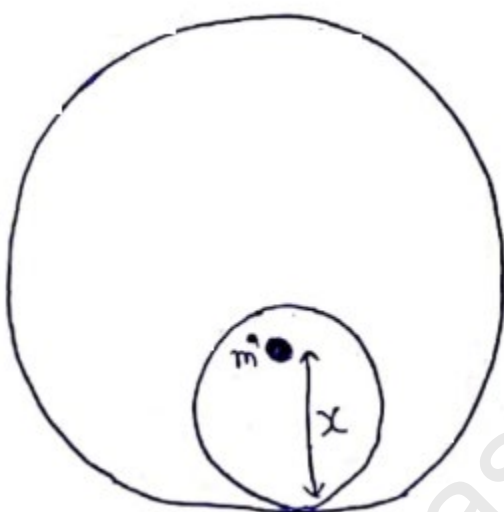
Question 13

A solid sphere of mass m and radius r is placed inside a hollow thin spherical shell of mass M and radius R as shown in figure (11-E1). A particle of mass m' is placed on the line joining the two centers at a distance x from the point of contact of the sphere and the shell. Find the magnitude of the resultant gravitational force on this particle due to the sphere and the shell if (a) $r < x < 2r$, (b) $2r < x < 2R$ and (c) $x > 2R$.



Solution 13

(a)



Gravitational field intensity inside shell is zero. So, force by shell on mass m' is zero.
Now, for solid sphere.

Gravitational field inside sphere at a distance $(x-r)$ from center is

$$E = \frac{Gm}{r^3}(x-r)$$

So, force by sphere on mass m' is

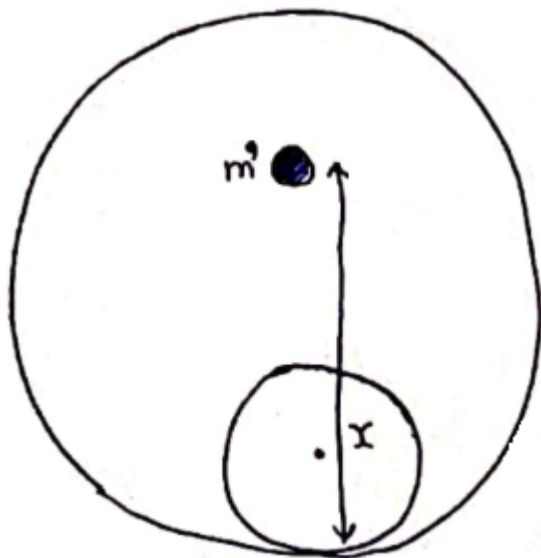
$$F = \frac{Gmm'}{r^3}(x-r)$$

Net force

$$\vec{F}_N = \vec{F}_{\text{sphere}} + \vec{F}_{\text{shell}}$$

$$F_N = \frac{Gmm'(r-x)}{r^3}$$

(b)



Gravitational field intensity inside shell is zero. So, force by shell on mass m' is zero.
Now, for solid sphere.

Gravitational field intensity outside sphere at distance $(x-r)$ is given by

$$E = \frac{Gm}{(x-r)^2}$$

So, force by sphere on mass m' is

$$F = m'E$$

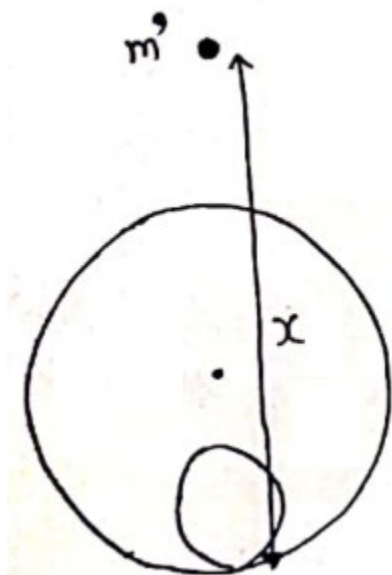
$$F = \frac{Gmm'}{(x-r)^2}$$

Net force

$$\vec{F}_N = \vec{F}_{\text{sphere}} + \vec{F}_{\text{shell}}$$

$$F_N = \frac{Gmm'}{(x-r)^2}$$

(c)



Force = mass $\times$ Gravitational field intensity

Force by shell = $m' \left(\frac{GM}{(x-R)^2} \right)$ (towards center)

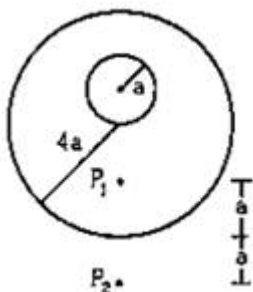
Force by sphere = $m' \left(\frac{Gm}{(x-R)^2} \right)$ (towards center)

$$\vec{F}_{Net} = \vec{F}_{sphere} + \vec{F}_{shell}$$

$$\vec{F}_{Net} = \frac{Gmm'}{(x-R)^2} + \frac{GMm'}{(x-R)^2}$$

Question 14

A uniform metal sphere of radius a and mass M is surrounded by a thin uniform spherical shell of equal mass and radius $4a$ (figure 11-E2). The center of the shell falls on the surface of the inner sphere. Find the gravitational field at the points P_1 and P_2 shown in the figure.



Solution 14

(a) At point P_1

Net gravitational field = field due to shell + field due to sphere

$$\vec{E}_N = 0 + \frac{GM}{(a + 4a - a)^2}$$

$$\vec{E}_N = \frac{GM}{16a^2}$$

(b) At point P_2

Net gravitational field = field due to shell + field due to sphere

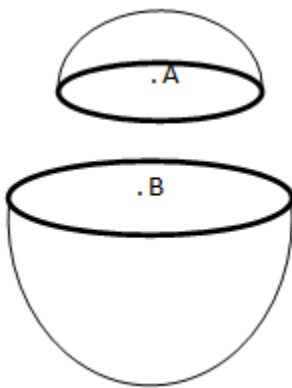
$$\vec{E}_N = \frac{GM}{(4a + a)^2} + \frac{GM}{(4a + a + a)^2}$$

$$\vec{E}_N = \frac{GM}{25a^2} + \frac{GM}{36a^2}$$

$$\vec{E}_N = \frac{61GM}{900a^2}$$

Question 15

A thin spherical shell having uniform density is cut in two parts by a plane and kept separated as shown in figure (11-E3). The point A is the centre of the plane section of the first part and B is the centre of the plane section of the second part. Show that the gravitational field at A due to the first part is equal in magnitude to the gravitational field at B due to the second part.



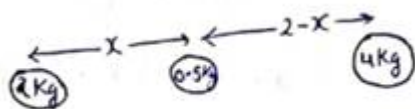
Solution 15

Resultant field inside any spherical shell is zero at all points. So, at point A and B, field is zero. The field due to upper part and lower part are equal and opposite.

Question 16

Two small bodies of masses 2.00 kg and 4.00 kg are kept at rest at a separation of 2.0 m. Where should a particle of mass 0.10 kg be placed to experience no net gravitational force from these bodies? The particle is placed at this point. What is the gravitational potential energy of the system of three particles with usual reference level?

Solution 16



Let mass 0.5 kg is placed at distance x from 2 kg between both masses. Since, particle is in equilibrium.

Force due to 2kg = Force due to 4kg

$$\frac{G(2)(0.5)}{x^2} = \frac{G(4)(0.5)}{(2-x)^2}$$

On solving,

$$x = 0.83\text{m}$$

Now, gravitational potential energy

$$U = \frac{-G(2)(4)}{2} - \frac{G(2)(0.5)}{(0.83)} - \frac{G(4)(0.5)}{(2-0.83)}$$

$$U = -3.06 \times 10^{-10} \text{ J}$$

Question 17

Three particles of mass m each are placed at the three corners of an equilateral triangle of side a . Find the work which should be done on this system to increase the sides of the triangle to $2a$.

Solution 17

Work done

$$= \Delta U = \text{final P.E} - \text{initial P.E}$$

$$= U_f - U_i$$

$$= 3\left(\frac{-Gmm}{2a}\right) - 3\left(\frac{-Gmm}{a}\right)$$

$$= \left(\frac{3Gm^2}{2a}\right)$$

Question 18

A particle of mass 100 g is kept on the surface of a uniform sphere of mass 10 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle away from the sphere.

Solution 18

Work done against gravitational force = change in potential energy

$$W = U_f - U_i$$

$$W = 0 - \left(-\frac{GMm}{r}\right)$$

$$W = \frac{GMm}{r} = \frac{6.67 \times 10^{-11} \times 0.1 \times 10}{0.1}$$

$$W = 6.67 \times 10^{-10} \text{ J}$$

Question 19

The gravitational field in a region is given by $E = (5 \text{ N/kg}) \hat{i} + (12 \text{ N/kg}) \hat{j}$. (a) Find the magnitude of the gravitational force acting on a particle of mass 2 kg placed at the origin. (b) Find the potential at the points (12 m, 0) and (0, 5 m) if the potential at the origin is taken to be zero. (c) Find the change in gravitational potential energy if a particle of mass 2 kg is taken from the origin to the point (12 m, 5 m). (d) Find the change in potential energy if the particle is taken from (12 m, 0) to (0, 5 m).

Solution 19

$$\vec{E} = 5\hat{i} + 12\hat{j}$$

$$(a) \text{ Force, } \vec{F} = m\vec{E}$$

$$\vec{F} = 2(5\hat{i} + 12\hat{j})$$

$$\vec{F} = 10\hat{i} + 24\hat{j}$$

$$|F| = \sqrt{(10)^2 + (24)^2} = 26 \text{ N}$$

$$\begin{aligned} \text{(b)} \quad \int dV &= - \int E \cdot dr \\ &= - \left[\int (5\hat{i} + 12\hat{j}) \cdot (dx\hat{i} + dy\hat{j}) \right] \\ &= - \left[\int 5 \cdot dx + \int 12 \cdot dy \right] \end{aligned}$$

$$V = -(5x + 12y) + C$$

at (0,0), potential = 0

$$0 = -[5(0) + 12(0)] + C$$

$$C = 0$$

So, $V = -5x - 12y$

At point (12,0)

$$V = -5 \times 12 - 12 \times 0 = -60 \text{ J/Kg}$$

At point (0,5)

$$V = -5 \times 0 - 12 \times 5 = -60 \text{ J/Kg}$$

(c) Potential at (12,5)

$$V = -5 \times 12 - 12 \times 5 = -120 \text{ J/kg}$$

and potential at (0, 0) is 0.

So, change in gravitational potential energy

$$\Delta U = m\Delta V$$

$$= 2(-120 - 0)$$

$$\Delta U = -240 \text{ J}$$

$$\text{(d)} \quad \Delta U = m\Delta V$$

$$= m(-60 - (-60)) = 0$$

Chapter 11 - Gravitation Exercise 227

Question 20

The gravitational potential in a region is given by $V = (20 \text{ N/kg})(x + y)$. (a) Show that the equation is dimensionally correct. (b) Find the gravitational field at the point (x, y). Leave your answer in terms of the unit vectors i, j, k. (c) Calculate the magnitude of the gravitational force on a particle of mass 500 g placed at the origin.

Solution 20

$$V = 20 \frac{\text{N}}{\text{kg}} (x + y)$$

Dimension of L.H.S = [V]

$$= \left[\frac{GM}{R} \right]$$

$$= \frac{[M^{-1}L^3T^{-2}][M]}{[L]}$$

$$= [M^0L^2T^{-2}]$$

$$\text{Dimension of RHS} = \frac{[M^4 L^4 T^{-2}]}{[M]} [L] = [M^3 L^2 T^{-2}]$$

$\therefore LHS = RHS$

Hence, dimensionally correct.

$$\vec{E} = -\left[\frac{\delta v}{\delta x} \hat{i} + \frac{\delta v}{\delta y} \hat{j}\right]$$

$$(b) \quad \vec{E} = -\left[\frac{\delta}{\delta x}(20x + 20y)\hat{i} + \frac{\delta}{\delta y}(20x + 20y)\hat{j}\right]$$

$$\vec{E} = -[20\hat{i} + 20\hat{j}]$$

$$\vec{E} = -20(\hat{i} + \hat{j})$$

$$(c) \quad \vec{F} = m\vec{E}$$

$$\vec{F} = (0.5)(-20\hat{i} - 20\hat{j})$$

$$\vec{F} = -10\hat{i} - 10\hat{j}$$

$$|\vec{F}| = \sqrt{(-10)^2 + (-10)^2} = 10\sqrt{2}$$

Question 21

The gravitational field in a region is given by $E = (2\hat{i} + 3\hat{j})$ N/kg. Show that no work is done by the gravitational field when a particle is moved on the line $3y + 2x = 5$
[Hint : If a line $y = mx + c$ makes angle θ with the X-axis, $m = \tan\theta$]

Solution 21

Gravitational field in the region is given by

$$E = 2\hat{i} + 3\hat{j}$$

Slope of gravitational field, $m_1 = \tan\theta_1 = 3/2$

The line $3y + 2x = 5$ can be written with the slope, $m_2 = \tan\theta_2 = -2/3$
 $m_1 \times m_2 = -1$

Thus, m_1, m_2 lines are perpendicular.

So, no work is done in by gravitational force when a particle is moved on the given line.

Question 22

Find the height over the earth's surface at which the weight of a body becomes half of its value at the surface.

Solution 22

Let height be h

$$m \frac{GM}{(R+h)^2} = \frac{1}{2} m \frac{GM}{R^2}$$

$$2R^2 = (R+h)^2$$

$$\sqrt{2}R = R+h$$

$$h = (\sqrt{2}-1)R$$

Question 23

What is the acceleration due to gravity on the top of Mount Everest? Mount Everest is the highest mountain peak of the world at the height of 8848 m. The value at sea level is 9.80 m/s^2 .

Solution 23

$$g' = g \left(1 - \frac{2h}{R}\right)$$

g' be the acceleration due to gravity at Mount Everest
 $h = 8848$

$$g' = 9.8 \times \left(1 - \frac{2 \times 8848}{6400 \times 10^3}\right)$$

$$= 9.77 \text{ m/s}^2$$

Question 24

Find the acceleration due to gravity in a mine of depth 640 m if the value at the surface is 9.800 m/s^2 . The radius of the earth is 6400 km.

Solution 24

$$g' = g \left(1 - \frac{d}{R}\right)$$

g' be the acceleration due to gravity at surface

$$= 9.8 \times \left(1 - \frac{640}{6400 \times 10^3}\right)$$

$$= 9.79 \text{ m/s}^2$$

Question 25

A body is weighed by a spring balance to be 1000 kg at the north pole. How much will it weigh at the equator? Account for the earth's rotation only.

Solution 25

Let g_p be the gravity at the poles, g_e be the gravity at the equator

$$g_e = g_p - \omega^2 R$$

$$= 9.81 - (7.3 \times 10^{-5})^2 \times (6400 \times 10^3)$$

$$= 9.766 \text{ N/m}^2$$

Now, $mg_e = 1 \text{ kg} \times 9.766 \text{ N/m}^2$
 $= 9.766 \text{ N}$

Thus the body weighs 9.766 N at the equator.

Question 26

A body stretches a spring by a particular length at the earth's surface at equator. At what height above the south pole will it stretch the same spring by the same length? Assume the earth to be spherical.

Solution 26

At equator, let stretch in spring be x

$$\text{Spring force} = mg_e$$

$$kx = m(g - R\omega^2) \quad (1)$$

Now, at height h above surface, stretch in spring is x

$$\text{Spring force} = mg_h$$

$$kx = mg \left(1 - \frac{2h}{R}\right) \quad (2)$$

From (1) and (2)

$$m(g - R\omega^2) = mg \left(1 - \frac{2h}{R}\right)$$

$$R\omega^2 = \frac{2h}{R} g$$

$$h = \frac{R^2 \omega^2}{2g} = \frac{R^2 \left(\frac{2\pi}{T}\right)^2}{2g}$$

$$h = \frac{(6400 \times 10^3)^2 \left(\frac{2 \times 3.14}{86400}\right)^2}{(2 \times 9.8)}$$

$$h \approx 10 \text{ Km}$$

Question 27

At what rate should the earth rotate so that the apparent g at the equator becomes zero? What will be the length of the day in this situation?

Solution 27

Apparent acceleration due to gravity at equator becomes 0.

$$G' = g - \omega^2 R = 0$$

$$g = \omega^2 R$$

$$\omega = \sqrt{\frac{g}{R}}$$

$$= \sqrt{\frac{9.8}{6400 \times 10^3}}$$

$$= 1.2 \times 10^{-3} \text{ rad/s}$$

$$\text{Time period} = T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{1.2 \times 10^{-3}} = 1.41 \text{ hrs}$$

Question 28

A pendulum having a bob of mass m is hanging in a ship sailing along the equator from east to west. When the ship is stationary with respect to water the tension in the string is T_0 . (a) Find the speed of the ship due to rotation of the earth about its axis. (b) Find the difference between T_0 and the earth's attraction on the bob. (c) If the ship sails at speed v , what is the tension in the string? Angular speed of earth's rotation is ω and radius of the earth is R .

Solution 28

(a) Since, ship is at rest with respect to water.

So, Angular speed of ship = Angular speed of earth

$$\omega_s = \omega$$

$$\frac{V_s}{R} = \omega$$

$$V_s = R\omega$$

(b) The tension in string is given by

$$T_0 = mg' - mR\omega^2$$

$$mg' - T_0 = mR\omega^2$$

(c) If ship sails at speed v , then tension

$$T = mg' - mR\omega'^2$$

$$T = mg' - mR\left(\frac{V - R\omega}{R}\right)^2$$

$$T = mg' - \frac{m(V^2 + R^2\omega^2 - 2VR\omega)}{R}$$

$$T = T_0 + mR\omega^2 - \frac{mV^2}{R} - \frac{mR^2\omega^2}{R} + \frac{2mVR\omega}{R}$$

$$T = T_0 - \frac{mV^2}{R} + 2mV(\omega) \quad \left[\text{Here } R \gg \frac{mV^2}{R} \approx 0 \right]$$

$$T \approx T_0 + 2mV\omega$$

Question 29

The time taken by Mars to revolve round the sun is 1.88 years. Find the ratio of average distance between Mars and the sun to that between the earth and the sun.

Solution 29

Kepler's law of planetary motion says ,

$$T^2 \propto R^3$$

$$\frac{T_m^2}{T_e^2} = \frac{R_m^3}{R_e^3}$$

$$\left(\frac{1.88}{1}\right)^2 = \left(\frac{R_m}{R_e}\right)^3$$

$$\frac{R_m}{R_e} = \left(\frac{1.88}{1}\right)^{\frac{2}{3}} = 1.52$$

Question 30

The moon takes about 27.3 days to revolve round the earth in a nearly circular orbit of radius 3.84×10^5 km. Calculate the mass of the earth from these data.

Solution 30

Time period of the moon around the earth is given by,

$$T = 2\pi \sqrt{\frac{r^3}{GM}}$$

m = mass of earth

r = Distance between center of the moon and earth

$$27.3 \times 86400 = 2 \times 3.14 \sqrt{\frac{(3.84 \times 10^5 \times 10^3)^3}{6.67 \times 10^{-11} \times M}}$$

$$M = 6.02 \times 10^{24} \text{ kg}$$

Question 31

A Mars satellite moving in an orbit of radius 9.4×10^3 km takes 27540 s to complete one evolution. Calculate the mass of Mars.

Solution 31

Time period of revolution of satellite around the mars is given by

$$T = 2\pi \sqrt{\frac{r^3}{GM}}$$

M = Mars mass

r = Distance of the satellite from center of the planet

$$27540 = 2 \times 3.14 \sqrt{\frac{(9.4 \times 10^3 \times 10^3)^3}{6.67 \times 10^{-11} \times M}}$$

$$M = 6.5 \times 10^{23} \text{ kgs}$$

Question 32

A satellite of mass 1000 kg is supposed to orbit the earth at a height of 2000 km above the earth's surface. Find (a) its speed in the orbit, (b) its kinetic energy, (c) the potential energy of the earth-satellite system and (d) its time period. Mass of the earth = 6×10^{24} kg.

Solution 32

(a) Speed of satellite in its orbit

$$V_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}}$$

$$V_0 = \sqrt{\frac{9.8 \times (6400)^2}{1000 \times (6400 + 2000)}} = 6.9 \text{ km/sec}$$

$$\begin{aligned} \text{(b) Kinetic energy of satellite} &= \left(\frac{1}{2}\right) \times 1000 \times (6.9 \times 10^3)^2 \\ &= 2.38 \times 10^{10} \text{ J} \end{aligned}$$

$$\begin{aligned} \text{(c) Potential energy of satellite} &= P.E = \frac{-GMm}{R+h} \\ &= \frac{6.67 \times 10^{-12} \times 6 \times 10^{24} \times 10^3}{(6400 + 2000) \times 10^3} \\ &= -4.76 \times 10^{10} \text{ J} \end{aligned}$$

(d) Time period of the satellite

$$T = \frac{2\pi(R+h)}{V_0}$$

$$T = \frac{2 \times 3.14 \times (6400 + 2000) \times 10^3}{6.9 \times 10^3}$$

$$T = 7645.2 \text{ sec}$$

$$T = \frac{7645.2}{3600} \text{ hrs} = 2.1 \text{ hrs}$$

Question 33

(a) Find the radius of the circular orbit of a satellite moving with an angular speed equal to the angular speed of earth's rotation. (b) If the satellite is directly above the north pole at some instant, find the time it takes to come over the equatorial plane. Mass of the earth = 6×10^{24} kg.

Solution 33

(a) Since, angular speed of satellite is equal to angular speed of earth.

So, time period of satellite = 24 hrs

$$T = 24 \text{ hrs}$$

$$2\pi \sqrt{\frac{r^3}{GM}} = 24$$

$$2\pi \sqrt{\frac{(R+h)^3}{gR^2}} = 24$$

$$\frac{(2 \times 3.14)^2 (6300 + h)^3}{\left(\frac{9.81}{1000} \times 6400\right)} = (24 \times 86400)^2$$

$$h = 42300 \text{ Km}$$

(b) Time taken from North pole to equatorial plane

$$\frac{T}{4} = \frac{24}{4} = 6 \text{ hrs}$$

Question 34

What is the true weight of an object in a geostationary satellite that weighed exactly 10'0 N at the north pole?

Solution 34

Weight at North pole = 10 N

$$W = mg$$

$$10 = mg$$

Weight of body in a satellite at height 'h' is

$$W_s = mg'$$

$$= m \left[g \left(\frac{R}{R+h} \right)^2 \right]$$

For a geo-stationary satellite, $R = 6.4 \times 10^3 \text{ km}$

$$h = 3.6 \times 10^3 \text{ km}$$

$$h = 10 \left(\frac{6400}{6400+3600} \right)^2 = 0.23 \text{ N}$$

Question 35

The radius of a planet is R, and a satellite revolves round it in a circle of radius R₂. The time period of revolution is T. Find the acceleration due to the gravitation of the planet at its surface.

Solution 35

Time period of the revolution of the satellite around a planet in terms of the radius of the orbit of the satellite is given by:

$$T = 2\pi \sqrt{\frac{R_2^3}{gR_1^2}}$$

$$T^2 = 4\pi^2 \left(\frac{R_2^3}{gR_1^2} \right)$$

$$g = \frac{4\pi^2}{T^2} \times \frac{R_2^3}{R_1^2}$$

Where 'g' is acceleration due to gravity.

Question 36

Find the minimum colatitude which can directly receive a signal from a geostationary satellite.

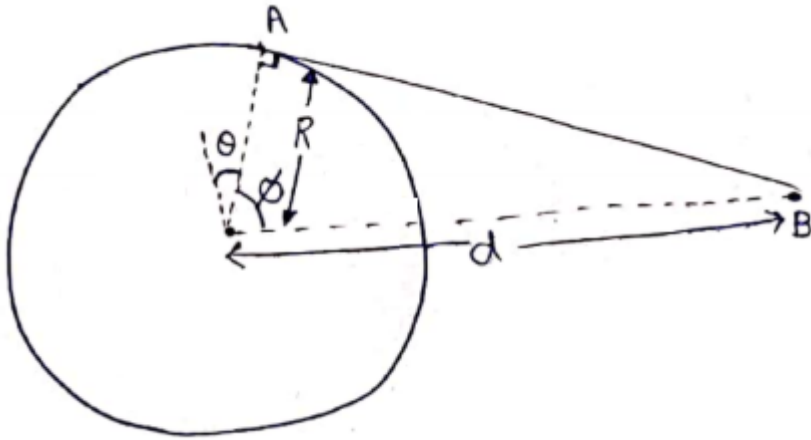
Solution 36

Consider that 'B' is the position of the geo- stationary satellite.

Here R= 6400 kms (Radius of earth)

d = altitude

θ = colatitude of the plane which can directly receive a signal from the satellite.



Take $\triangle OAB$,

$$\cos \phi = \frac{6400}{42000} = \frac{8}{53}$$

$$\phi = \cos^{-1} \frac{8}{53}$$

$$= \cos^{-1}(0.15)$$

$$\text{Now } \theta + \phi = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2} - \cos^{-1}(0.15)$$

$$\theta = \sin^{-1}(0.15)$$

Question 37

A particle is fired vertically upward from earth's surface and it goes up to a maximum height of 6400 km. Find the initial speed of the particle.

Solution 37

Upon Earth's surface, its total energy is

$$E = \text{K.E} + \text{P.E}$$

$$= \frac{1}{2} mv^2 + \left(-\frac{GMm}{R} \right) \quad (1)$$

In space at maximum height, its K.E = 0 and P.E = $-\frac{GMm}{R+h}$

$$E_s = 0 + \frac{-GMm}{R+h} \quad (2)$$

Total energy must be conserved, therefore (1) = (2)

$$\frac{1}{2} mv^2 + \left(-\frac{GMm}{R} \right) = \frac{-GMm}{R+h}$$

Here $h = R$, so

$$\frac{1}{2} mv^2 = \frac{-GMm}{R} \left(\frac{1}{2} - 1 \right)$$

$$\frac{1}{2} mv^2 = \frac{1}{2} \frac{GMm}{R}$$

$$v = \sqrt{\frac{GM}{R}}$$

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6400 \times 10^3}}$$

$$= 7.9 \text{ km/s}$$

Question 38

A particle is fired vertically upward with a speed of 15 km/s. With what speed will it move in Interstellar space. Assume only earth's gravitational field.

Solution 38

Velocity of particle on the Earth surface = $V_e = 15 \text{ km/s}$

Let V_s be the particle velocity in space.

Total Energy must be conserved.

Change in K.E. = Change in P.E.

$\Delta \text{K.E.} = \Delta \text{P.E.}$

$$\frac{1}{2} m (v_s^2 - v_e^2) = \frac{GMm}{R}$$

$$\frac{1}{2} [(225) - v_s^2] = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6400 \times 10^3}$$

On solving

$$V_s = 10 \text{ km/s}$$

Question 39

A mass of $6 \times 10^{24} \text{ kg}$ (equal to the mass of the earth) is to be compressed in a sphere in such a way that the escape velocity from its surface is $3 \times 10^8 \text{ m/s}$. What should be the radius of the sphere?

Solution 39

$$\text{Escape velocity} = V_e = \sqrt{\frac{2GM}{R}}$$

Mass = $6 \times 10^{24} \text{ kgs.}$

$V_e = 3 \times 10^8 \text{ m/s.}$

$$\text{i.e., } 3 \times 10^8 = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{R}}$$

On solving,

$$R = 8.89 \times 10^{-3}$$

$$= 9 \times 10^{-3}$$

$$= 9 \text{ mm.}$$