

HC VERMA Solutions for Class 11 Physics Chapter 10 Rotational Mechanics

Question 1

A wheel is making revolutions about its axis with uniform angular acceleration. Starting from rest, it reaches 100 rev/sec in 4 seconds. Find the angular acceleration. Find the angle rotated during these four seconds.

Solution 1

$$\omega_0 = 0; \quad t = 4 \text{ sec};$$

$$\omega = 100 \frac{\text{rev}}{\text{sec}} = 100 \times 2\pi \frac{\text{rad}}{\text{sec}}$$

$$\text{Using, } \omega = \omega_0 + \alpha t$$

$$\alpha = 50\pi \frac{\text{rad}}{\text{sec}^2} \text{ or}$$

$$\alpha = 25\pi \frac{\text{rad}}{\text{sec}^2}$$

$$\text{Using, } \theta = \omega_0 t + \frac{1}{2} \alpha t^2 = 400\pi \text{ rad}$$

Question 2

A wheel rotating with uniform angular acceleration covers 50 revolutions in the first five seconds after the start. Find the angular acceleration and the angular velocity at the end of five seconds.

Solution 2

$$\omega_0 = 0; \quad t = 5 \text{ sec}; \quad \theta = 50 \text{ rev} = 100\pi \frac{\text{rad}}{\text{sec}}$$

$$\text{Using, } \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\alpha = 8\pi \frac{\text{rad}}{\text{sec}^2} \text{ or } \alpha = 4\pi \frac{\text{rev}}{\text{sec}^2}$$

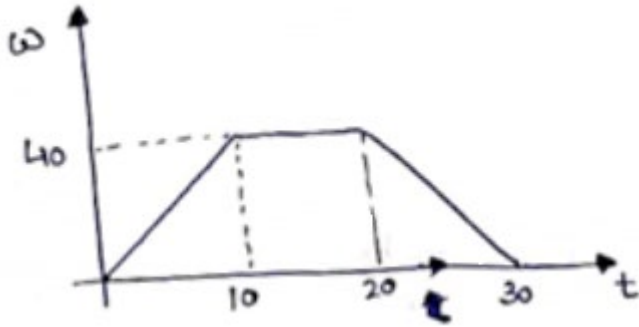
$$\text{Using, } \omega = \omega_0 + \alpha t$$

$$\omega = 40\pi \frac{\text{rad}}{\text{sec}} \text{ or } \omega = 20 \frac{\text{rev}}{\text{sec}}$$

Question 3

A wheel starting from rest is uniformly accelerated at 4 rad/s^2 for 10 seconds. It is allowed to rotate uniformly for the next 10 seconds and is finally brought to rest in the next 10 seconds. Find the total angle rotated by the wheel.

Solution 3



Angular displacement=Area under curve

$$= \frac{1}{2} (30 + 10)(40)$$

$$\theta = 800 \text{ rad.}$$

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Question 4

A body rotates about a fixed axis with an angular acceleration of one radian/second/second. Through what angle does it rotate during the time in which its angular velocity increases from 5 rad/s to 15 rad/s.

Solution 4

$$\omega_0 = 5 \text{ rad/s}; \omega = 15 \text{ rad/sec}; \alpha = 1 \text{ rad/sec}^2$$

Substituting values in,

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = 100 \text{ rad}$$

Question 5

Find the angular velocity of a body rotating with an acceleration of 2 rev/s^2 as it completes the 5th revolution after the start.

Solution 5

$$\omega_0 = 0; \alpha = 2 \frac{\text{rev}}{\text{s}^2} = 2 \times 2\pi = 4\pi \frac{\text{rad}}{\text{s}^2};$$

$$\theta = 5 \text{ rev} = 5 \times 2\pi \text{ rad} = 10\pi \text{ rad}$$

Substituting values in,

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\omega = 4\pi\sqrt{5} \text{ rad/sec}$$

$$\omega = 2\sqrt{5} \text{ rev/sec}$$

Question 6

A disc of radius 10cm is rotating about its axis at an angular speed of 20 rad/s. Find the linear speed of

- (a) a point on the rim.
- (b) the middle point of a radius.

Solution 6

$$(a) v = r\omega = (0.1)(20)$$

$$v = 2 \text{ m/s}$$

$$(b) v = r\omega = \left(\frac{0.1}{2}\right)(20)$$

$$v = 1 \text{ m/s}$$

Question 7

A disc rotates about its axis with a constant angular acceleration of 4 rad/s^2 . Find the radial and tangential accelerations of a particular at a distance of 1 cm from the axis end of the first second after the disc starts rotating.

Solution 7

$$\omega_0 = 0, \alpha = 4 \text{ rad/s}^2; t = 1 \text{ sec}$$

$$\omega = \omega_0 + \alpha t$$

$$\omega = 4 \text{ rad/sec}$$

$$\text{Radial acceleration} = R\omega^2$$

$$= (0.01)(4)^2$$

$$= 0.16 \text{ m/s}^2$$

$$= 16 \text{ cm/s}^2$$

Tangential acceleration

$$a_T = r\alpha$$

$$= (0.01)(4)$$

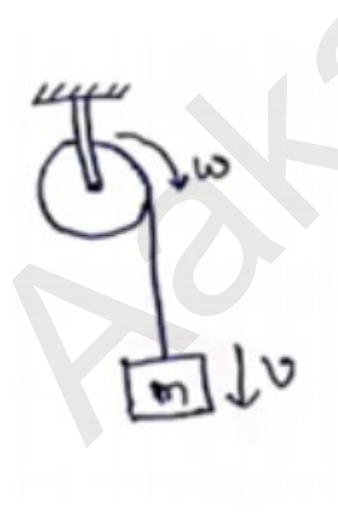
$$= 0.04 \text{ m/s}^2$$

$$= 4 \text{ cm/s}^2$$

Question 8

A block hangs from a string wrapped on a disc of radius 20 cm free to rotate about its axis which is fixed in a horizontal position. If the angular speed of the disc is 10 rad/s at some instant, with what speed is the block going down at the instant?

Solution 8



Block speed = Rim speed of the disc

$$v = R\omega = (0.2)(10)$$

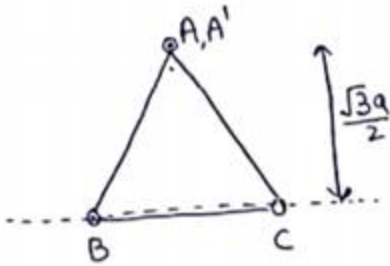
$$v = 2 \text{ m/s}$$

Question 9

Three particles, each of mass 200 g, are kept at the corners of an equilateral triangle of side 10cm. Find the moment of inertia of the system about an axis

- (a) joining two of the particles and
- (b) passing through one of the particles and perpendicular to the plane of the particles.

Solution 9



$$(a) I_{BC} = m \left(\frac{\sqrt{3}}{2} a \right)^2 + m(0)^2 + m(0)^2$$

$$= 0.2 \left(\frac{\sqrt{3}}{2} \times 0.1 \right)^2$$

$$= 1.5 \times 10^{-3} \text{ Kg-m}^2$$

$$(b) I_{AA'} = m(0)^2 + m(a)^2 + m(0)^2$$

$$= 2 \times 0.2 (0.1)^2$$

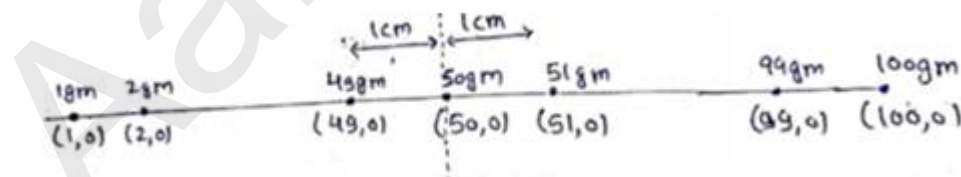
$$= 4 \times 10^{-3} \text{ Kg-m}^2$$

Question 10

Particles of masses 1 g, 2 g, 3 g, ..., 100 g are kept at the marks 1 cm, 2 cm, 3 cm, ..., 100 cm respectively on a metrescale. Find the moment of inertia of the system of particles about a perpendicular bisector of the metre scale.

Solution 10

The perpendicular bisector passes from 50cm mark. So, there will be 49 particles on LHS and 50 particles on RHS.



$$I = [49(1)^2 + 51(1)^2] + [48(2)^2 + 52(2)^2] \dots \dots + [1(49)^2 + 99(49)^2] + 100(50)^2$$

$$= (100) [1^2 + 2^2 + \dots \dots + 50^2]$$

$$= (100) \left[\frac{50 \times (50+1) (2 \times 50 + 1)}{6} \right] \text{ gm-cm}^2$$

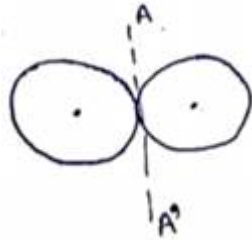
$$I \approx 0.43 \text{ kg cm}^2$$

$$[\because \text{Sum of square of } n \text{ natural numbers, } \Delta = \frac{n(n+1)(2n+1)}{6}]$$

Question 11

Find the moment of inertia of a pair of spheres, each having a mass m and radius r , kept in contact about the tangent passing through the point of contact.

Solution 11



By parallel axis theorem,

$$I_{axis} = I_{COM} + mr^2$$

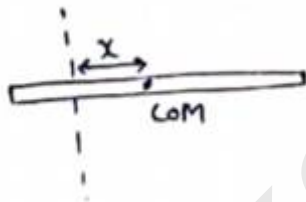
$$I_{AA'} = \left(\frac{2}{5}mr^2 + mr^2\right)2$$

$$I_{AA'} = \frac{14}{5}mr^2$$

Question 12

The moment of inertia of a uniform rod of mass 0.50 kg and length 1 m is 0.10 kg m^2 about a line perpendicular to the rod. Find the distance of this line from the middle point of the rod.

Solution 12



By parallel axis theorem,

$$I_{axis} = I_{COM} + mx^2$$

$$I_{axis} = \frac{mL^2}{12} + mx^2$$

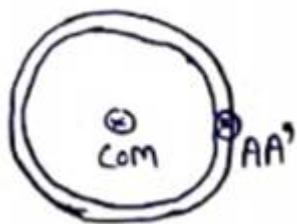
$$0.1 = \frac{(0.5)(1)^2}{12} + (0.5)x^2$$

$$X = 0.34 \text{ m}$$

Question 13

Find the radius of gyration of a circular ring of radius r about a line perpendicular to the plane of the ring passing through one of its particles.

Solution 13



$$I_{axis} = I_{COM} + mx^2$$

$$I_{AA'} = mr^2 + mr^2$$

$$I_{AA'} = 2mr^2$$

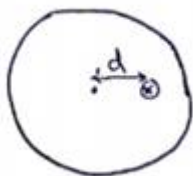
$$mk^2 = 2mr^2$$

$$\therefore k = \sqrt{2}r$$

Question 14

The radius of gyration of a uniform disc about a line perpendicular to the disc equals its radius. Find the distance of the line from the centre.

Solution 14



$$I_{axis} = I_{COM} + md^2$$

$$mk^2 = \frac{mr^2}{2} + md^2$$

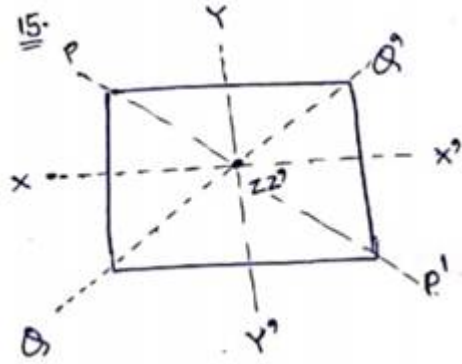
$$\Rightarrow mr^2 = \frac{mr^2}{2} + md^2$$

$$\Rightarrow d = \frac{r}{\sqrt{2}}$$

Question 15

Find the moment of inertia of a uniform square plate of mass m and edge a about one of its diagonals.

Solution 15



If we squeeze rod along XX' and YY' the square becomes rod of side 'a'
So,

$$I_{XX'} = I_{YY'} = \frac{ma^2}{12}$$

By perpendicular axis theorem,

$$I_{ZZ'} = I_{XX'} + I_{YY'} = I_{PP'} + I_{QQ'}$$

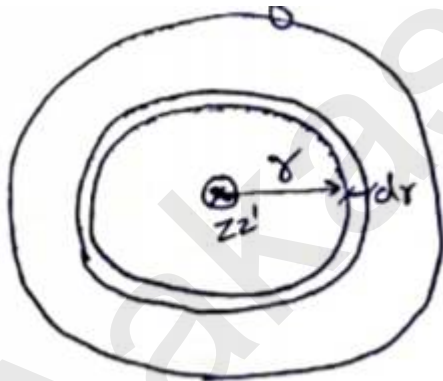
$$\frac{ma^2}{12} + \frac{ma^2}{12} = I_{PP'} + I_{QQ'}$$

$$I_{PP'} = \frac{ma^2}{12} \quad [\because I_{PP'} = I_{QQ'} \text{ As square is symmetric}]$$

Question 16

The surface density (mass/area) of a circular disc of radius a depends on the distance from the centre as $p(r)=A+Br$. Find its moment of inertia about the line perpendicular to the plane of the disc through its centre.

Solution 16



Surface density of disc is varying radially so considering an element of thickness 'dr' at a distance 'r' from center of disc.

$$\begin{aligned} \sigma &= A + Br = \frac{dm}{dA} \\ dm &= (A + Br)(2\pi r dr) \\ \int dI_{ZZ'} &= \int dm (r)^2 \end{aligned}$$

$$\begin{aligned}
 I_{zz'} &= \int_0^a (A + Br)(2\pi r dr) (r)^2 \\
 &= 2\pi \left[A \int_0^a r^3 + B \int_0^a r^4 \right] \\
 I_{zz'} &= 2\pi \left[\frac{Aa^4}{4} + \frac{Ba^5}{5} \right]
 \end{aligned}$$

Question 17

A particle of mass m is projected with the speed u at an angle θ with the horizontal. Find the torque of the weight of the particle about the point of projection when the particle is at the highest point.

Solution 17

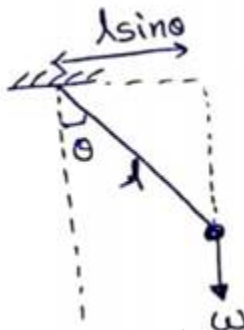
$$\begin{aligned}
 \tau &= F \times d_{\perp} \\
 &= mg \left(\frac{\text{Range}}{u^2 \sin 2\theta} \right) \\
 &= mg \left(\frac{2g}{u^2 \sin 2\theta} \right) \\
 \tau &= mu^2 \sin \theta \cos \theta
 \end{aligned}$$

Torque acts perpendicular to plane of the motion.

Question 18

A simple pendulum of length l is pulled aside to make an angle θ with the vertical. Find the magnitude of the torque of the weight w of the bob about the point of suspension. When is the torque zero?

Solution 18

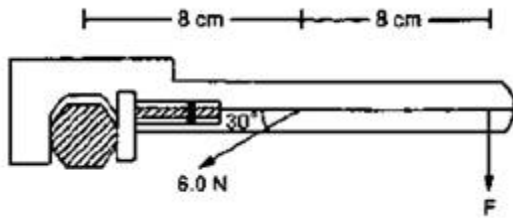


$$\begin{aligned}
 \text{torque} &= F \times d_{\perp} \\
 \tau &= wl \sin \theta
 \end{aligned}$$

Torque will be zero at the lowest point of suspension as θ will be zero.

Question 19

When a force of 6.0 N is exerted at 30° to a wrench at a distance of 8cm from the nut, it is just able to loosen the nut. What force F would be sufficient to loosen it if it acts perpendicularly to the wrench at 16cm from the nut?



Solution 19

Torque in both the cases will be equal

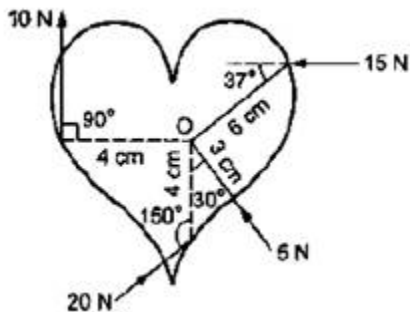
$$\tau_1 = \tau_2$$

$$(6 \sin 30^\circ)(8) = (F)(16)$$

$$F = 1.6 \text{ N}$$

Question 20

Calculate the total torque acting on the body shown in figure about the point O.



Solution 20

$$\text{Torque} = F \times d_{\perp}$$

Consider anticlockwise torque as negative and clockwise torque as positive.

$$\tau_N = \tau_{10} + \tau_{15} + \tau_5 + \tau_{20}$$

$$= (10)(4) - (15)(6 \sin 37^\circ) + 5(0) - 20(4 \cos 60^\circ)$$

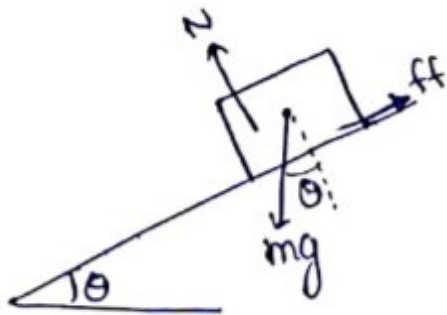
$$\tau_{Net} = -0.54$$

$$\tau = 0.54 \text{ N-m (anticlockwise direction)}$$

Question 21

A cubical block of mass m and edge a slides down a rough inclined plane of inclination θ with a uniform speed. Find the torque of the normal force acting on the block about its centre.

Solution 21



Since,
acceleration=0

$$ff = mg \sin \theta$$

The normal contact will shift as block is not toppling.

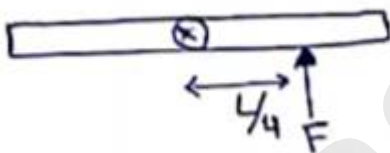
$$\text{So, } \tau_{\text{Normal}} = \tau_{ff}$$

$$= mg \sin \theta \left(\frac{a}{2} \right)$$

Question 22

A rod of mass m and length L , lying horizontally, is free to rotate about a vertical axis through its centre. A horizontal force of constant magnitude F acts on the rod at a distance of $L/4$ from the centre. The force is always perpendicular to the rod. Find the angle rotated by the rod during the time t after the motion starts.

Solution 22



$$\text{Torque} = \vec{r} \times \vec{F}$$

$$\tau = F \left(\frac{L}{4} \right) \sin 90^\circ$$

$$\tau = F \frac{FL}{4}$$

$$I \propto L^4$$

$$\frac{mL^2}{12}, \alpha = \frac{FL}{4}$$

$$\alpha = \frac{3F}{mL}$$

$$\text{Now, } \omega_0 = 0; \text{ time} = t$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

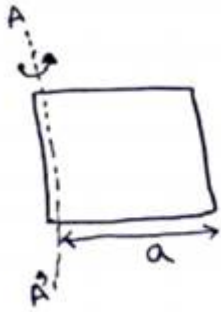
$$\theta = \frac{3Ft^2}{2mL}$$

$$\theta = \frac{3Ft^2}{2mL}$$

Question 23

A square plate of mass 120g and edge 5.0 cm rotates about one of the edges. If it has a uniform angular acceleration of 0.2 rad/s^2 , what torque acts on the plate?

Solution 23



Squeeze rod along axis $A-A'$ so it will behave as rod

$$I = \frac{ma^2}{3}$$

$$\tau = I\alpha$$

$$= \frac{(0.120)(0.05)^2}{3} (0.2)$$

$$\tau = 2 \times 10^{-5} \text{ N-m}$$

Question 24

Calculate the torque on the square plate of the previous problem if it rotates about a diagonal with the same angular acceleration.

Solution 24

M.I. of square plate about its diagonal

$$I = \frac{ma^2}{12}$$

$$\tau = I\alpha$$

$$= \frac{(0.12)(0.05)^2}{12} (0.2)$$

$$= 0.5 \times 10^{-5} \text{ N-m}$$

Question 25

A flywheel of moment of inertia 5.0 kg-m^2 is rotated at a speed of 60 rad/s. Because of the friction at the axle, it comes to rest in 5.0 minutes. Find (a) the average torque of the friction, (b) the total work done by the friction and (c) the angular momentum of the wheel 1 minute before it stops rotating.

Solution 25

$$\omega_0 = 60 \frac{\text{rad}}{\text{s}}; \omega = 0; t = 5 \text{ min} = 300 \text{ sec}$$

$$\omega = \omega_0 + \alpha t$$

$$\alpha = -\frac{1}{2} \text{ rad/sec}^2$$

$$(a) \tau = I\alpha$$

$$\begin{aligned}
 &= 5 \left(\frac{-1}{5} \right) \\
 &= -1 \text{ N-m} \\
 \text{(b) } W_{\text{all}} &= \Delta KE \\
 W_{\text{ff}} &= \frac{1}{2} I \omega^2 \\
 &= \frac{1}{2} (5) (60)^2 \\
 &= 9 \text{ KJ} \\
 \text{(c) } \omega_0 &= 60 \frac{\text{rad}}{\text{sec}}; t = 4 \text{ min} = 240 \text{ sec}; \alpha = \frac{-1 \text{ rad}}{5 \text{ s}^2} \\
 \omega &= \omega_0 + \alpha t \\
 \omega &= 12 \text{ rad/sec}
 \end{aligned}$$

Angular momentum

$$\begin{aligned}
 L &= I \omega \\
 &= 5(12) \\
 &= 60 \frac{\text{Kg-m}^2}{\text{sec}}
 \end{aligned}$$

Question 26

Because of the friction between the water in oceans with the earth's surface, the rotational kinetic energy of the earth is continuously decreasing. If the earth's angular speed decreases by 0.0016 rad/day in 100 years, find the average torque of the friction on the earth. Radius of the earth is 6400 km and its mass is 6.0×10^{24} kg.

Solution 26

$$\begin{aligned}
 \alpha &= \frac{\Delta \omega}{t} \\
 &= \frac{0.0016}{(86400 \times 100 \times 365 \times 86400)} \\
 \tau &= I \alpha \\
 &= \left(\frac{2}{5} MR^2 \right) \alpha \\
 &= \frac{2}{5} (6 \times 10^{24}) (6400 \times 1000)^2 \left(\frac{0.0016}{(86400)^2 \times 100 \times 365} \right) \\
 \tau &= 5.8 \times 10^{20} \text{ N-m}
 \end{aligned}$$

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Question 27

A wheel rotating at a speed of 600 rpm (revolutions per minute) about its axis is brought to rest by applying a constant torque for 10 seconds. Find the angular deceleration and the angular velocity 5 seconds after the application of the torque.

Solution 27

$$\begin{aligned}
 \omega_0 &= 600 \text{ rpm} = \frac{600 \text{ rev}}{60 \text{ sec}} \\
 \omega_0 &= 10 \frac{\text{rev}}{\text{sec}}; \omega = 0; t = 10 \text{ sec} \\
 \omega &= \omega_0 + \alpha t \\
 \alpha &= -1 \text{ rev/sec}^2
 \end{aligned}$$

For $t=5\text{sec}$,

$$\omega = \omega_0 + \alpha t$$

$$= 10 + (-1)(5)$$

$$\omega = 5 \text{ rev/sec}$$

Question 28

A wheel of mass 10 kg and radius 20cm is rotating at an angular speed of 100 rev/min when the motor is turned off. Neglecting the friction at the axle, calculate the force that must be applied tangentially to the wheel to bring it to rest in 10 revolutions.

Solution 28

$$\omega_0 = 100 \frac{\text{rev}}{\text{min}} = 100 \times \frac{2\pi}{60}$$

$$\omega_0 = \frac{10\pi \text{ rad}}{3 \text{ sec}}$$

$$\theta = 10 \text{ rev} = 20\pi \text{ rad}$$

$$\omega = 0$$

$$\text{Using, } \omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\alpha = \frac{-\omega_0^2}{2\theta}$$

$$\tau = Fr = I\alpha$$

$$F = \left(\frac{mr^2}{2} \right) \left(\frac{\alpha}{r} \right)$$
$$= \frac{10 \times 0.2}{2} \times \left(\frac{10\pi}{3} \right)^2 \times \frac{1}{2 \times 20\pi}$$

$$F = 0.87 \text{ N}$$

Question 29

A cylinder rotating at an angular speed of 50 rev/s is brought in contact with an identical stationary cylinder. Because of the kinetic friction, torques act on the two cylinders, accelerating the stationary one and decelerating the moving one. If the common magnitude of the acceleration and deceleration be one revolution per second square, how long will it take before the two cylinders have equal angular speed?

Solution 29

Let after time 't' their angular velocity be ' ω '

$$\text{I}^{\text{st}} \text{ cylinder :- } \omega_0 = 50 \frac{\text{rev}}{\text{sec}}; \alpha = -1 \frac{\text{rev}}{\text{sec}^2};$$

$$\omega = \omega_0 + \alpha t$$

$$\omega = 50 - t \text{ -(i)}$$

$$\text{II}^{\text{nd}} \text{ cylinder :- } \omega_0 = 0; \alpha = 1 \frac{\text{rev}}{\text{sec}^2}$$

$$\omega = \omega_0 + \alpha t$$

$$\omega = t \text{ -(ii)}$$

From (i) and (ii)

$$\omega = 25 \text{ rev/sec}$$

$$t = 25 \text{ sec}$$

Question 30

A body rotating at 20 rad/s is acted upon by a constant torque providing it a deceleration of 2 rad/s^2 . At what time will the body have kinetic energy same as the initial value if the torque continues to act?

Solution 30

$$\omega_0 = 20 \frac{\text{rad}}{\text{sec}}; \alpha = -\frac{2\text{rad}}{\text{s}^2}; \omega = 0$$

$$\omega = \omega_0 + \alpha t_1$$

$$t_1 = 10 \text{ sec}$$

Now, wheel accelerates and when angular velocity becomes 20 rad/sec it will have same KE.

$$\omega = \omega_0 + \alpha t_2$$

$$20 = 0 + 2t_2$$

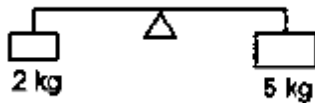
$$t_2 = 10 \text{ sec}$$

$$T = t_1 + t_2$$

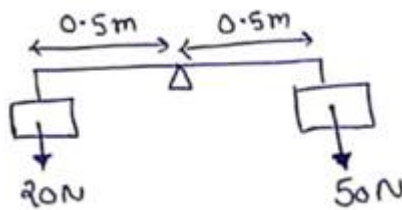
$$T = 20 \text{ sec}$$

Question 31

A light rod of length 1 m is pivoted at its centre and two masses of 5 kg and 2 kg are hung from the ends as shown in figure. Find the initial angular acceleration of the rod assuming that it was horizontal in the beginning.



Solution 31



$$\tau_{\text{Net}} = 50 \times 0.5 - 20 \times 0.5$$

$$\tau_{\text{Net}} = 15 \text{ N-m}$$

$$\tau = I\alpha$$

$$15 = [(2)(0.5)^2 + 5(0.5)^2]\alpha$$

$$\alpha = 8.57 \frac{\text{rad}}{\text{sec}^2}$$

Question 32

Suppose the rod in the previous problem has a mass of 1 kg distributed uniformly over its length.

(a) Find the initial angular acceleration of the rod.

(b) Find the tension in the supports to the blocks of mass 2 kg and 5 kg.

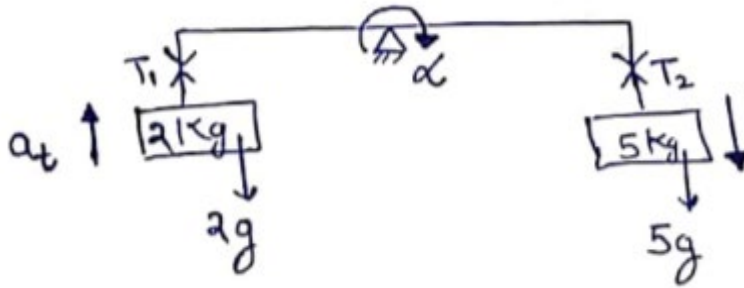
Solution 32

$$(a) \tau = I\alpha$$

$$(50 \times 0.5 - 20 \times 0.5) = [(2)(0.5)^2 + 5(0.5)^2 + \frac{(1)(1)^2}{12}]\alpha$$

$$\alpha = 8.1 \frac{\text{rad}}{\text{sec}^2}$$

(b)



$$a_t = R\alpha = (0.5)(8) \approx 4$$

$$T_1 - 2g = 2a_t$$

$$T_1 = 2(9.8) + 2(4)$$

$$T_1 = 27.6 \text{ N}$$

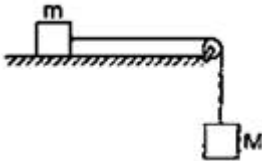
$$5g - T_2 = 5a_t$$

$$T_2 = 5(9.8) - 5(4)$$

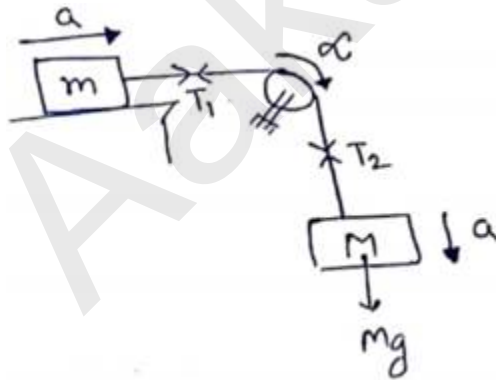
$$T_2 = 29 \text{ N}$$

Question 33

Figure shows two blocks of masses m and M connected by a string passing over a pulley. The horizontal table over which the mass m slides is smooth. The pulley has a radius r and moment of inertia I about its axis and it can freely rotate about the axis. Find the acceleration of the mass M assuming that the string does not slip on the pulley.



Solution 33



$$Mg - T_2 = Ma \quad \text{-(i)}$$

$$T_2 r - T_1 r = I\alpha \quad \text{-(ii)}$$

$$T_1 = ma \text{---(iii)}$$

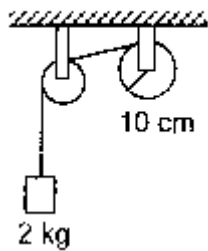
$$a = r\alpha \text{---(iv)}$$

Solving , (i),(ii),(iii) and (iv)

$$a = \frac{Mg}{M + m + \frac{I}{r^2}}$$

Question 34

A string is wrapped on a wheel of moment of inertia $I = 0.20 \text{ kgm}^2$ and radius 10cm and goes through a light pulley to support a block of mass 2.0kg as shown in the figure. Find the acceleration of the block



Solution 34

Moment of inertia of the pulley, $I = 0.20 \text{ kgm}^2$

From the diagram we get

$$mg - T = ma \text{---(i)}$$

$$Tr = I\alpha$$

$$a = \alpha r$$

$$T = \frac{I\alpha}{r^2} \text{---(ii)}$$

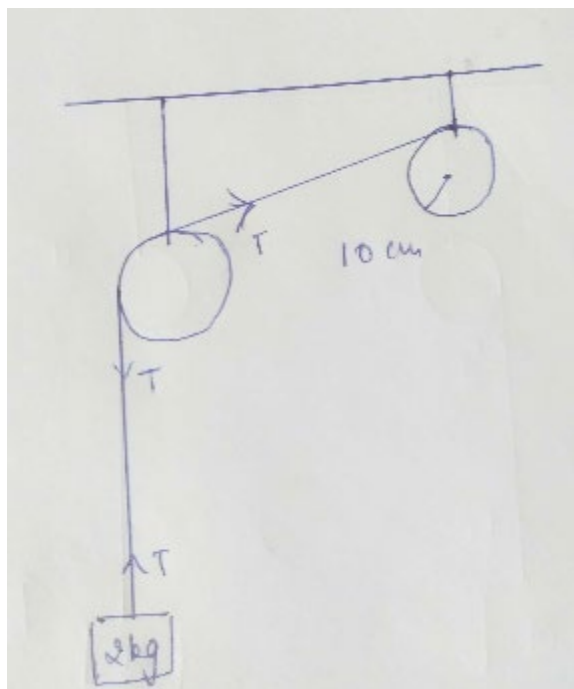
Using equation (i) and (ii)

$$mg = \left(m + \frac{I}{r^2} \right) a$$

$$a = \frac{mg}{m + \frac{I}{r^2}}$$

$$a = \frac{2 \times 9.8}{2 + \frac{0.2}{0.01}}$$

$$a = \frac{19.6}{22} = 0.89 \text{ ms}^{-2}$$



Moment of inertia of the pulley, $I = 0.20 \text{ kgm}^2$

From the diagram we get

$$mg - T = ma \text{ ---- (i)}$$

$$Tr = I\alpha$$

$$a = \alpha r$$

$$T = \frac{I\alpha}{r^2} \text{ ---- (ii)}$$

Using equation (i) and (ii)

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$$a = \frac{mg}{m + \frac{I}{r^2}}$$

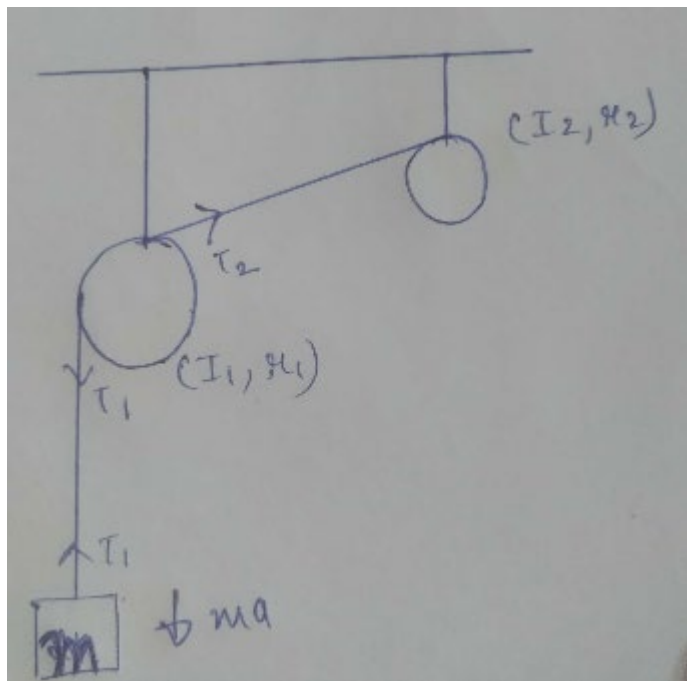
$$a = \frac{2 \times 9.8}{2 + \frac{0.2}{0.01}}$$

$$a = \frac{19.6}{22} = 0.89 \text{ ms}^{-2}$$

Question 35

Suppose the smaller pulley of the previous problem has its radius 5.0cm and moment of inertia 0.1 kgm^2 . Find the tension in the part of the string joining the pulleys.

Solution 35



$$mg - T_1 = ma \text{ ----- (i)}$$

$$(T_1 - T_2)r_1 = I_1\alpha \text{ ----- (ii)}$$

$$T_2r_2 = I_2\alpha \text{ ----- (iii)}$$

substituting the value of T_2 in equation (ii)

$$\left(T_1 - \frac{I_2\alpha}{r_2}\right)r_1 = I_1\alpha$$

$$T_1 - \frac{I_2\alpha}{r_2} = \frac{I_1\alpha}{r_1}$$

as

$$a = \alpha r$$

$$T_1 - \frac{I_2a}{r_2^2} = \frac{I_1a}{r_1^2}$$

$$T_1 = \left(\frac{I_2}{r_2^2} + \frac{I_1}{r_1^2}\right)a$$

Substituting the value of T_1 in equation 1

we get

$$mg - \left(\frac{I_2}{r_2^2} + \frac{I_1}{r_1^2}\right)a = ma$$

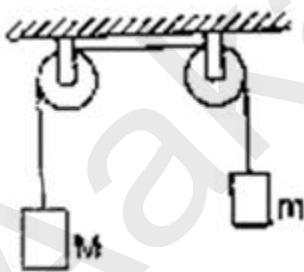
On substituting the values

$$a = 0.316 \text{ ms}^{-2}$$

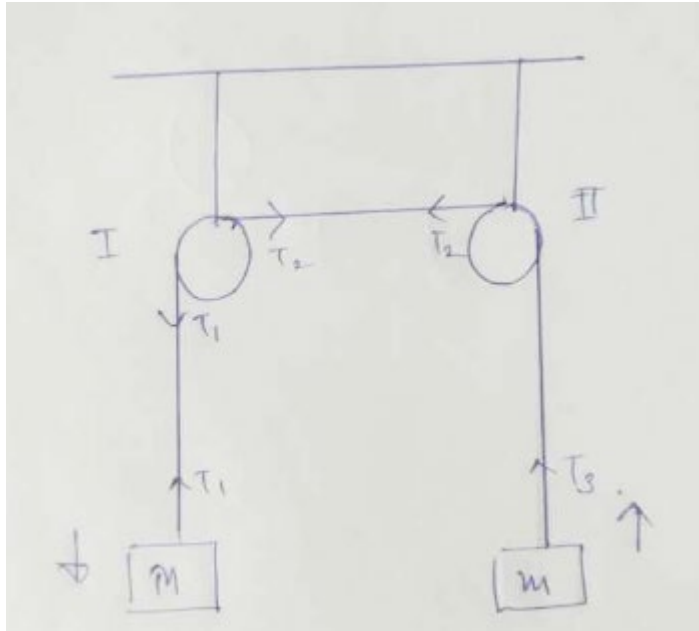
$$T_2 = 6.32 \text{ N}$$

Question 36

The pulleys in figure are identical each having a radius R and moment of inertia I . Find the acceleration of block M .



Solution 36



$$mg - T_1 = ma \text{ --- (i)}$$

$$(T_1 - T_2)r_1 = I\alpha$$

$$a = \alpha r$$

$$(T_1 - T_2) = \frac{I\alpha}{R^2} \text{ For pulley 1 --- (ii)}$$

$$(T_2 - T_3) = \frac{I\alpha}{R^2} \text{ For pulley 2 --- (iii)}$$

For block of mass m

$$T_3 - mg = ma \text{ --- (iv)}$$

$$T_1 - T_3 = \frac{2I\alpha}{R^2} \text{ --- (v)}$$

Adding equation (i) and (v)

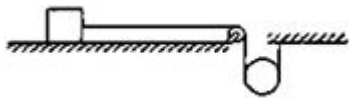
$$-mg + Mg + (T_3 - T_1) = Ma + ma$$

$$Mg - mg = Ma + ma + \frac{I\alpha}{R^2}$$

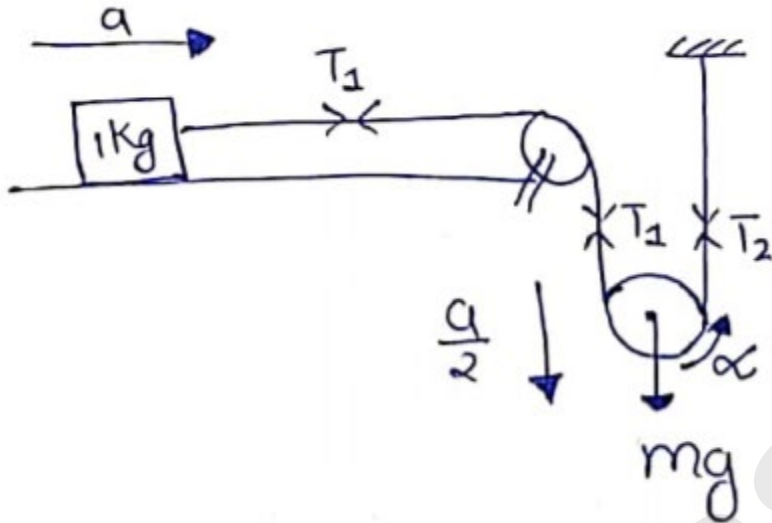
$$a = \frac{(M - m)g}{Ma + ma + \frac{I\alpha}{R^2}}$$

Question 37

The descending pulley shown in figure has a radius 20 cm and moment of inertia $0.20 \text{ kg}\cdot\text{m}^2$. The fixed pulley is light and the horizontal plane frictionless. Find the acceleration of the block if its mass is 1.0 kg.



Solution 37



Translatory Motion Equation

$$T_1 = 1 \cdot a \quad \text{-(i)}$$

$$mg - (T_1 + T_2) = m \frac{a}{2} \quad \text{-(ii)}$$

Rotational Motion Equation

$$\tau = I\alpha$$

$$T_2 R - T_1 R = I\alpha$$

$$T_2 - T_1 = \frac{I}{R} \left(\frac{a}{2R} \right) \quad \text{-(iii)}$$

For mass m

$$I = \frac{mR^2}{2}$$

$$(0.2) = \frac{m}{2} (0.2)^2$$

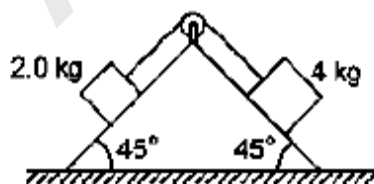
$$m = 10 \text{ kg} \quad \text{-(iv)}$$

Using, (i), (ii), (iii) and (iv)

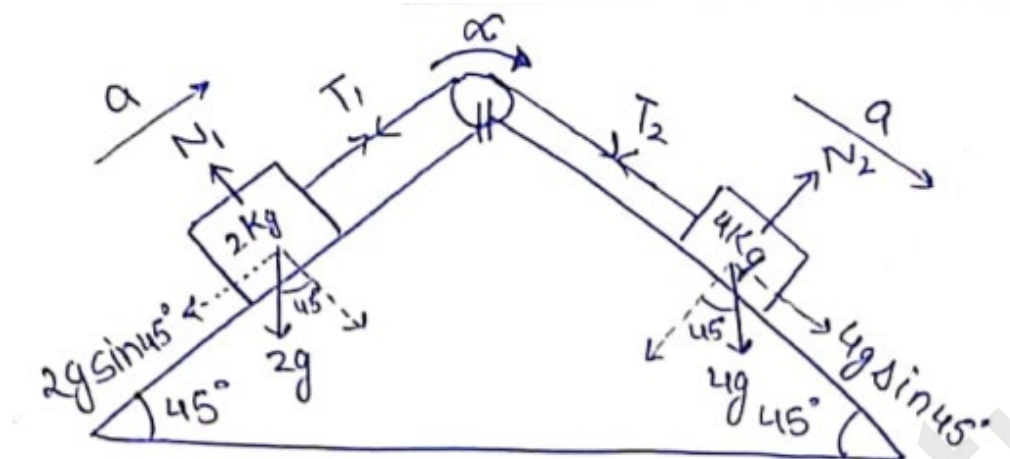
$$a = 10 \text{ m/s}^2$$

Question 38

The pulley shown in figure has a radius 10 cm and moment of inertia 0.5 kg-m^2 about its axis. Assuming the inclined planes to be frictionless, calculate the acceleration of the 4.0 kg block.



Solution 38



Here,

$$I = 0.5 \text{ kg} \cdot \text{m}^2$$

$$R = 0.1 \text{ m}$$

Translatory Motion Equation

$$4g \sin 45 - T_2 = 4a \quad \text{-(i)}$$

$$T_1 - 2g \sin 45 = 2a \quad \text{-(ii)}$$

Rotational Motion Equation

$$\tau = I\alpha$$

$$T_2 R - T_1 R = I \left(\frac{a}{R} \right) \quad \text{-(iii)}$$

Using (i), (ii) and (iii)

$$a = 0.25 \text{ m/s}^2$$

Question 39

Solve the previous problem if the friction coefficient between the 2.0 kg block and the plane below it is 0.5 and the plane below the 4.0 kg block is frictionless.

Solution 39

$$\text{Here, } I = 0.5 \text{ Kg} \cdot \text{m}^2; R = 0.1 \text{ m}; \text{ ff} = \mu N_1 = (0.5)(2g \cos 45)$$

Translatory Motion Equation

$$4g \sin 45 - T_2 = 4a \quad \text{-(i)}$$

$$T_1 - 2g \sin 45 - \text{ff} = 2a \quad \text{-(ii)}$$

Rotational Motion Equation

$$\tau = I\alpha$$

$$T_2 R - T_1 R = I \left(\frac{a}{R} \right) \quad \text{(iii)}$$

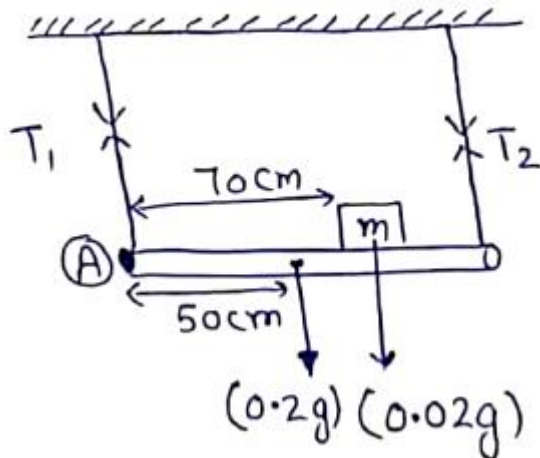
Using (i), (ii) and (iii)

$$a = 0.125 \text{ m/s}^2$$

Question 40

A uniform metre stick of mass 200 g is suspended from the ceiling through two vertical strings of equal lengths fixed at the ends. A small object of mass 20 g is placed on the stick at a distance of 70 cm from the left end. Find the tensions in the two strings.

Solution 40



Translatory Equilibrium Equation

$$T_1 + T_2 = (0.2g) + (0.02g)$$

$$T_1 + T_2 = 0.22g \quad \text{---(i)}$$

Rotational Equilibrium Equation at point A
clockwise Torque = anticlockwise Torque

$$(0.2g)(0.5) + (0.02g)(0.7) = T_2(1)$$

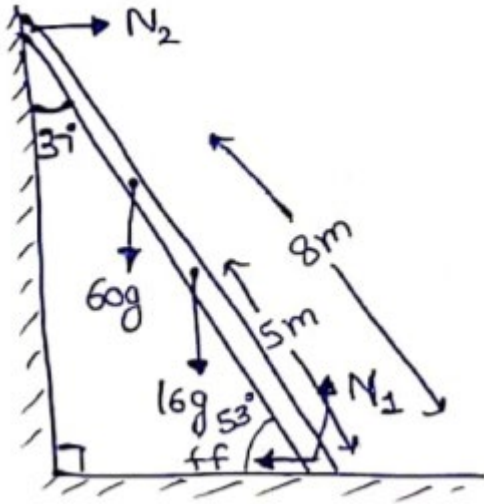
$$T_2 = 1.12 \text{ N}$$

From (i) $T_1 = 1.04 \text{ N}$

Question 41

A uniform ladder of length 10.0 m and mass 16.0 kg is resting against a vertical wall making an angle of 37° with it. The vertical wall is frictionless but the ground is rough. An electrician weighing 60.0 kg climbs up the ladder. If he stays on the ladder at a point 8.00 m from the lower end, what will be the normal force and the force of friction on the ladder by the ground? What should be the minimum coefficient of friction for the electrician to work safely?

Solution 41



Translatory Equilibrium Equation

$$N_2 = ff \text{ (Horizontal direction)}$$

$$N_1 = 60g + 16g \text{ (Vertical direction)}$$

$$N_1 = 76g = 745 \text{ N}$$

Rotational Equilibrium at bottom of ladder

Clockwise Torque = Anticlockwise Torque

$$N_2 (10 \cos 37^\circ) = 16g (5 \sin 53^\circ) + 60g (8 \sin 53^\circ)$$

$$\text{Solving, } N_2 = 412 \text{ N}$$

$$ff = N_2 = 412 = \mu N_1$$

$$412 = \mu (76g)$$

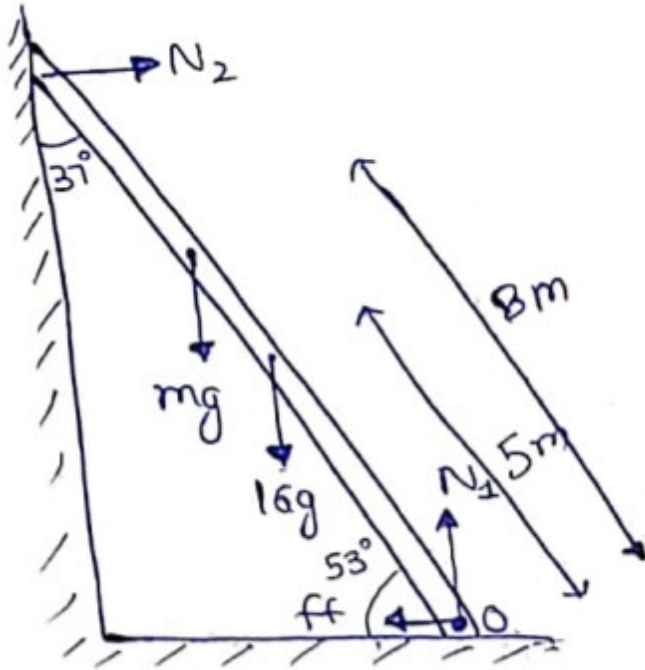
$$\mu = 0.553$$

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Question 42

Suppose the friction coefficient between the ground and the ladder of the previous problem is 0.540. Find the maximum weight of a mechanic who could go up and do the work from the same position of the ladder.

Solution 42



Here, $\mu = 0.54$

Translatory Equilibrium

$$N_1 = mg + 16g \quad \text{-(i)}$$

$$N_2 = ff = \mu N_1 \quad \text{-(ii)}$$

Rotational Equilibrium about point 'O'

$$N_2 (10 \sin 53^\circ) = mg(8 \cos 53^\circ) + 16g \cos 53^\circ \quad \text{-(iii)}$$

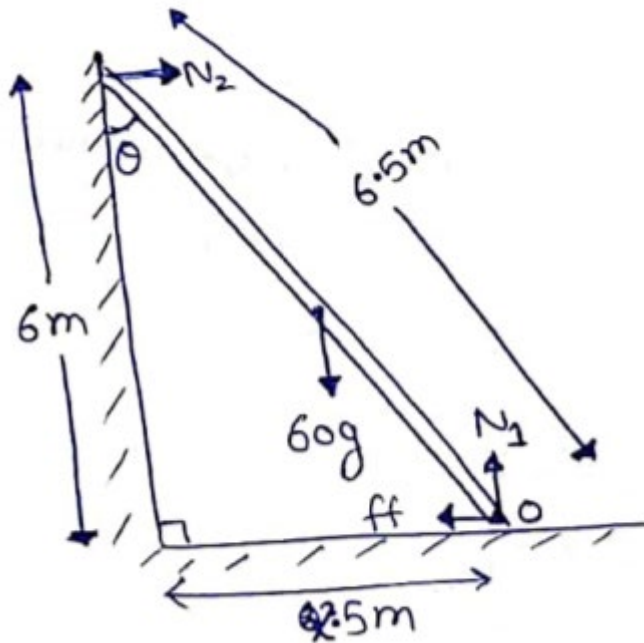
Solving (i), (ii) and (iii)

$$m = 44 \text{ kg}$$

Question 43

A 6.5 m long ladder rests against a vertical wall reaching a height of 6.0 m. A 60 kg man stands half way up the ladder. (a) Find the torque of the force exerted by the man on the ladder about the upper end of the ladder. (b) Assuming the weight of the ladder to be negligible as compared to the man and assuming the wall to be smooth, find the force exerted by the ground on the ladder.

Solution 43



By Pythagoras theorem,

$$(6.5)^2 = (6)^2 + B^2$$

$$B = 2.5 \text{ m}$$

$$(a) \tau = F \times d_{\perp}$$

$$\tau = 60g \left(\frac{6.5}{2} \sin \theta \right)$$

$$= 60g \left(\frac{6.5}{2} \times \frac{2.5}{6.5} \right)$$

$$\tau = 740 \text{ N-m}$$

$$(b) N_1 = 60g \text{ (Vertical Translatory Equation)}$$

$$N_1 = 590 \text{ N}$$

Rotational Equilibrium at O

$$N_2(6.5 \cos \theta) = 60g \left(\frac{6.5}{2} \sin \theta \right)$$

$$N_2 = 30g \tan \theta$$

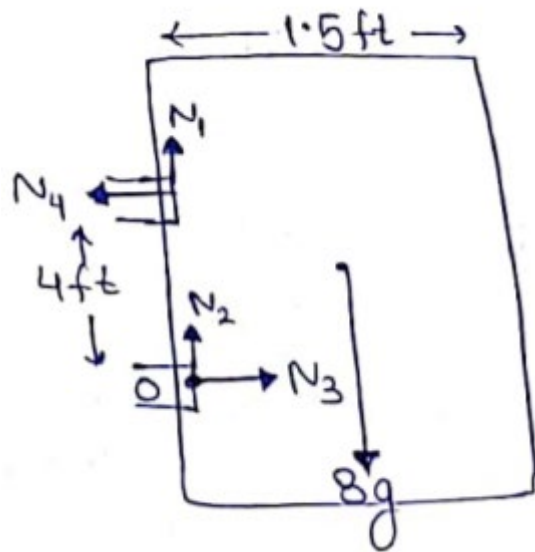
$$N_2 = 30g \left(\frac{2.5}{6} \right)$$

$$N_2 = 120 \text{ N}$$

Question 44

The door of an almirah is 6 ft high, 1.5 ft wide and weighs 8 kg. The door is supported by two hinges situated at a distance of 1 ft from the ends. If the magnitudes of the force exerted by the hinges on the door are equal, find this magnitude.

Solution 44



Magnitude of forces by hinges are equal.

$$\sqrt{N_1^2 + N_4^2} = \sqrt{N_2^2 + N_3^2}$$

$$N_1^2 + N_4^2 = N_2^2 + N_3^2 \quad \text{-(i)}$$

Rotational Equilibrium at point O

$$N_4(4) = 8g\left(\frac{1.5}{2}\right)$$

$$N_4 = 1.5g$$

By translational Equilibrium

$$N_3 = N_4 = 1.5g \quad \text{-(ii)}$$

$$N_1 + N_2 = 8g \quad \text{-(iii)}$$

Solving (i),(ii),(iii)

$$N_1 = N_2 = 4g$$

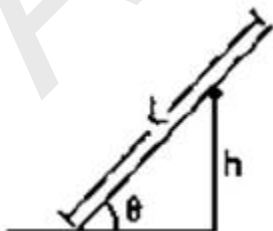
$$\text{Magnitude of force} = \sqrt{N_1^2 + N_4^2}$$

$$= \sqrt{(4g)^2 + (1.5g)^2}$$

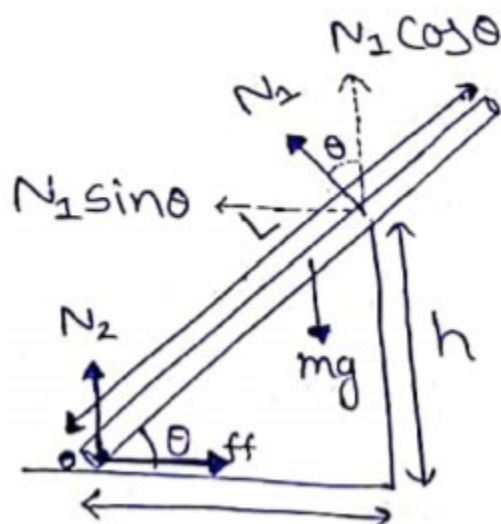
$$= 43 \text{ N}$$

Question 45

A uniform rod of length L sets against a smooth roller as shown in figure. Find the friction coefficient between the ground and the lower end if the minimum angle that the rod can make with the horizontal is θ .



Solution 45



Translatory equilibrium

$$N_1 \cos \theta + N_2 = mg \quad \text{-(i)}$$

$$N_1 \sin \theta = ff \quad \text{-(ii)}$$

Rotational Equilibrium about bottom

$$N_1 \left(\frac{h}{\sin \theta} \right) = mg \left(\frac{L}{2} \cos \theta \right) \quad \text{-(iii)}$$

$$ff = \mu N_2 \quad \text{-(iv)}$$

Solving (i), (ii), (iii) and (iv)

$$\mu = \frac{L \cos \theta \sin^2 \theta}{2h - L \cos^2 \theta \sin^2 \theta}$$

Question 46

A uniform rod of mass 300 g and length 50 cm rotates at a uniform angular speed of 2 rad/s about an axis perpendicular to the rod through an end. Calculate (a) the angular momentum of the rod about the axis of the rotation, (b) the speed of the centre of the rod and (c) its kinetic energy.

Solution 46



(a) $L = I\omega$

$$= \frac{mL^2}{3} (\omega)$$

$$= \frac{(0.3)(0.5)^2}{3} (2)$$

$$= \frac{m^2}{3}$$

$$= 0.05 \text{ kg-} \frac{m^2}{s}$$

(b) $v = R\omega$

$$= (0.25)(2)$$

$$v = 0.5 \text{ m/s}$$

$$\begin{aligned}
 \text{(c) } KE &= \frac{1}{2} I \omega^2 \\
 &= \frac{1}{2} \left(\frac{mL^2}{3} \right) (\omega^2) \\
 &= \frac{1}{2} \frac{(0.3)(0.5)^2}{3} (2^2) \\
 KE &= 0.05 \text{ J}
 \end{aligned}$$

Question 47

A uniform square plate of mass 2.0 kg and edge 10 cm rotates about one of its diagonals under the action of a constant torque of 0.10 N-m. Calculate the angular momentum and kinetic energy of the plate at the end of the fifth second after the start.

Solution 47

$$\begin{aligned}
 \tau &= I \alpha \\
 0.1 &= \frac{M a^2}{12} (\alpha) \\
 0.1 &= \frac{(2)(0.1)^2}{12} (\alpha) \\
 \alpha &= 60 \text{ rad/sec} \\
 \omega_0 &= 0, t = 5 \text{ sec} \\
 \omega &= \omega_0 + \alpha t \Rightarrow \omega = 30 \text{ rad/sec} \\
 \text{Angular momentum, } L &= I \alpha \\
 &= \left[\frac{(2)(0.1)^2}{12} (300) \right] \\
 &= 0.5 \text{ kg-m}^2 \text{ sec} \\
 KE &= \frac{1}{2} I \omega^2 \\
 &= \frac{1}{2} \left[\frac{(2)(0.1)^2}{12} \right] (300)^2 \\
 KE &= 75 \text{ J}
 \end{aligned}$$

Question 48

Calculate the ratio of the angular momentum of the earth about its axis due to its spinning motion to that about the sun due to its orbital motion. Radius of the earth = 6400 km and radius of the orbit of the earth about the sun = 1.5×10^8 km.

Solution 48

$$\begin{aligned}
 L_{\text{spinning}} &= I \omega \\
 &= \left(\frac{2}{5} M_e R_e^2 \right) \left(\frac{2\pi}{T_e} \right) \\
 &= \left(\frac{2}{5} \times M_e \times 6400 \right)^2 \left(\frac{2\pi}{24 \times 60 \times 60} \right) \text{---(i)} \\
 L_{\text{orbital}} &= I \omega \\
 &= (M_e R_0^2) \left(\frac{2\pi}{T} \right) \\
 &= (M_e \times 1.5 \times 10^8)^2 \left(\frac{2\pi}{365 \times 24 \times 60 \times 60} \right) \text{---(ii)}
 \end{aligned}$$

Divide (i) by (ii)

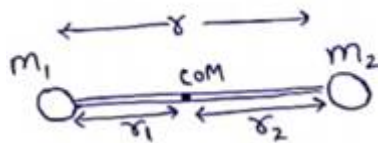
$$\frac{L_S}{L_0} = 2.65 \times 10^7$$

Question 49

Two particles of masses m_1 and m_2 are joined by a light rigid rod of length r . The system rotates at an angular speed ω about an axis through the centre of mass of the system and perpendicular to the rod. Show that the angular momentum of the system is $L = \mu r^2 \omega$ where μ is the reduced mass of the system defined as

$$\mu = \frac{m_1 m_2}{m_1 + m_2}.$$

Solution 49



Distance of COM from m_1 will be $r_1 = \frac{m_2 r}{(m_1 + m_2)}$

Distance of COM from m_2 will be $r_2 = \frac{m_1 r}{(m_1 + m_2)}$

$$L = I\omega$$

$$L = (m_1 r_1^2 + m_2 r_2^2) \omega$$

$$L = \left[m_1 \left(\frac{m_2 r}{(m_1 + m_2)} \right)^2 + m_2 \left(\frac{m_1 r}{(m_1 + m_2)} \right)^2 \right] \omega$$

$$L = \mu r^2 \omega \text{ where } \mu = \frac{m_1 m_2}{m_1 + m_2}$$

Question 50

A dumb-bell consists of two identical small balls of mass $\frac{1}{2}$ kg each connected to the two ends of a 50 cm long light rod. The dumb-bell is rotating about a fixed axis through the centre of the rod and perpendicular to it at an angular speed of 10 rad/s. An impulsive force of average magnitude 5.0 N acts on one of the masses in the direction of its velocity for 0.10 s. Find the new angular velocity of the system.

Solution 50



$$\tau = I\alpha$$

$$F \times r = (mr^2 + mr^2) \alpha$$

$$5(0.25) = 2 \left(\frac{1}{2} \right) (0.5)^2 \alpha$$

$$\alpha = 20 \text{ rad/s}^2$$

$$\omega_0 = 10 \text{ rad/sec}$$

$$\tau = 0.1$$

$$\omega = \omega_0 + \alpha \tau$$

$$\omega = 12 \text{ rad/sec}$$

Question 51

A wheel of moment of inertia 0.500 kg-m^2 and radius 20.0 cm is rotating about its axis at an angular speed of 20.0 rad/s . It picks up a stationary particle of mass 200 g at its edge. Find the new angular speed of the wheel.

Solution 51

$$\text{Since, } \tau_{\text{ext}} = 0$$

$$\therefore L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

$$(0.5)(20) = [0.5 + (0.2)^2] \omega_f$$

$$\omega_f = 19.7 \frac{\text{rad}}{\text{sec}}$$

Question 52

A driver having a moment of inertia of 6.0 kg-m^2 about an axis through its centre of mass rotates at an angular speed of 2 rad/s about the axis. If he folds his hands and feet to decrease the moment of inertia to 5.0 kg-m^2 , what will be the new angular speed?

Solution 52

$$\text{Since, } \tau_{\text{ext}} = 0$$

$$\therefore L_i = L_f$$

$$6(2) = 5 \omega_f$$

$$\omega_f = 2.4 \text{ rad/sec}$$

Question 53

A boy is seated in a revolving chair revolving at an angular speed of $120 \text{ revolutions per minute}$. Two heavy balls form part of the revolving system and the boy can pull the balls closer to himself or may push them apart. If by pulling the balls closer, the boy decreases the moment of inertia of the system by 6 kg-m^2 to 2 kg-m^2 , what will be the new angular speed?

Solution 53

$$\text{Since, } \tau_{\text{ext}} = 0$$

$$\therefore L_i = L_f$$

$$6(120) = 2 (\omega_f)$$

$$\omega_f = 360 \text{ rev/sec}$$

Question 54

A boy is standing on a platform which is free to rotate about its axis. The boy holds an open umbrella coincides with that of the platform. The moment of inertia of " the platform plus the boy system" is 3.0

$\times 10^{-3} \text{ kg}\cdot\text{m}^2$ and that of the umbrella is $2.0 \times 10^{-3} \text{ kg}\cdot\text{m}^2$. The boy starts spinning the umbrella about the axis at an angular speed of 2.0 rev/s with respect to himself. Find the angular velocity imparted to the platform.

Solution 54

Since, $\tau_{\text{ext}} = 0$

$$\therefore L_i = L_f$$

$$0 + 0 = L_{\text{boy-final}} + L_{\text{umb-final}} \quad \text{---(i)}$$

So, Angular speed of umbrella with respect to boy will be in the opposite direction.

Let angular speed of boy be ω

$$\text{Now, } \omega_{\text{umb-disc}} = \omega_{\text{umb}} - \omega_{\text{disc}}$$

$$-2 = \omega_{\text{umb}} - \omega$$

$$\omega_{\text{umb}} = \omega - 2$$

In equation (i),

$$0 = (3 \times 10^{-3})(\omega) + (2 \times 10^{-3})(\omega - 2)$$

$$\omega = 0.8 \text{ rev/sec}$$

Question 55

A wheel of moment of inertia $0.10 \text{ kg}\cdot\text{m}^2$ is rotating about a shaft at an angular speed of 160 rev/minute . A second wheel is set into rotation at 300 rev/minute and is coupled to the same shaft so that both the wheels finally rotate with a common angular speed of 200 rev/minute . Find the moment of inertia of the second wheel.

Solution 55

Taking wheels as a system

$$\tau_{\text{ext}} = 0$$

$$L_i = L_f$$

$$I_1\omega_1 = I_2\omega_2 = (I_1 + I_2)\omega$$

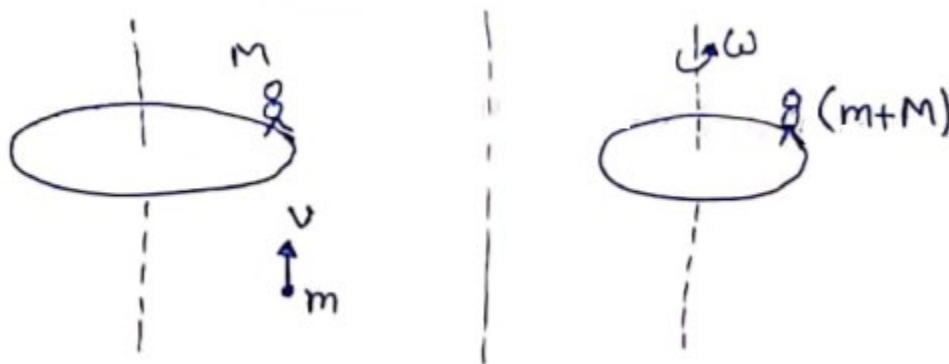
$$(0.1)(160) + I_2(300) = (0.1 + I_2)(200)$$

$$I_2 = 0.04 \text{ kg}\cdot\text{m}^2$$

Question 56

A kid of mass M stands at the edge of a platform of radius R which can be freely rotated about its axis. The moment of inertia of the platform is I . The system is at rest when a friend throws a ball of mass m and the kid catches it. If the velocity of the ball is v horizontally along the tangent to the edge of the platform when it was caught by the kid, find the angular speed of the platform after the event.

Solution 56



Considering ball, boy and platform as system

$$L_i = L_f$$

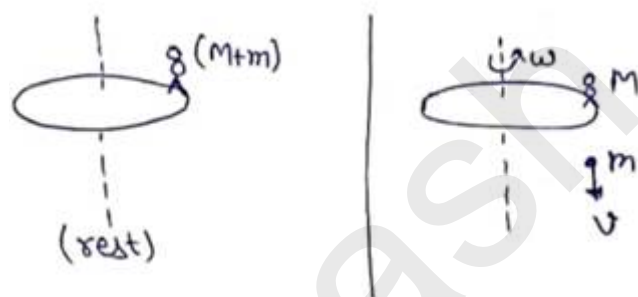
$$mvR = [I + (M+m)R^2]\omega$$

$$\omega = \frac{mvR}{I + (M+m)R^2}$$

Question 57

Suppose the platform of the previous problem is brought to rest with the ball in hand of the kid standing on the rim. The kid throws the ball horizontally to his friend in a direction tangential to the rim with a speed v as seen by friend. Find the angular velocity with which the platform will start rotating.

Solution 57



Considering ball, boy and platform as system

$$L_i = L_f$$

$$0 + 0 + 0 = mvR + I(-\omega) + (M+m)R^2(-\omega)$$

$$\omega = \frac{mvR}{I + MR^2}$$

Question 58

Suppose the platform with the kid in the previous problem is rotating in anticlockwise direction at an angular speed ω . The kid starts walking along the rim with the speed v relative to the platform also in the anticlockwise direction. Find the new angular speed of the platform.

Solution 58

Let angular velocity of platform after kid start running ω'

So, angular velocity of kid with respect to earth $= (\omega' + \frac{v}{R})$

$$\therefore \tau_{ext} = 0$$

$$\therefore L_i = L_{\omega}$$

$$(I + MR^2) \omega = I \omega' + MR^2 \left(\omega' + \frac{v}{R} \right)$$

$$\omega' = \omega - \frac{MvR}{I + MR^2}$$

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Question 59

A uniform rod of mass m and length l is struck at an end by a force F perpendicular to the rod for short time interval t . Calculate

(a) the speed of the centre of mass, (b) the angular speed of the rod about the centre of mass, (c) the kinetic energy of the rod and (d) the angular momentum of the rod about the centre of mass after the force has stopped to act. Assume that t is so small that the rod does not appreciably change its direction while the force acts.

Solution 59

(a) $F = ma$

acceleration, $a = \frac{F}{m}$; $u = 0$; t

$$v = u + at$$

$$v = \frac{F}{m} t$$

(b) $\tau = I\alpha$

$$\frac{L}{2} = \frac{ML^2}{12}$$

$$F \left(\frac{L}{2} \right) = \frac{ML^2}{12} \cdot \alpha$$

$$\alpha = \frac{6F}{ML}; \omega_0 = 0; t = t$$

$$\omega = \omega_0 + \alpha t$$

$$\alpha = \frac{6Ft}{ML}$$

$$(c) KE = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} m \left(\frac{Ft}{m} \right)^2 + \frac{1}{2} \left(\frac{ML^2}{12} \right) \left(\frac{6Ft}{ML} \right)^2$$

$$= \frac{2F^2 t^2}{2m}$$

$$= \frac{2F^2 t^2}{2m}$$

(d) $L = I \omega$

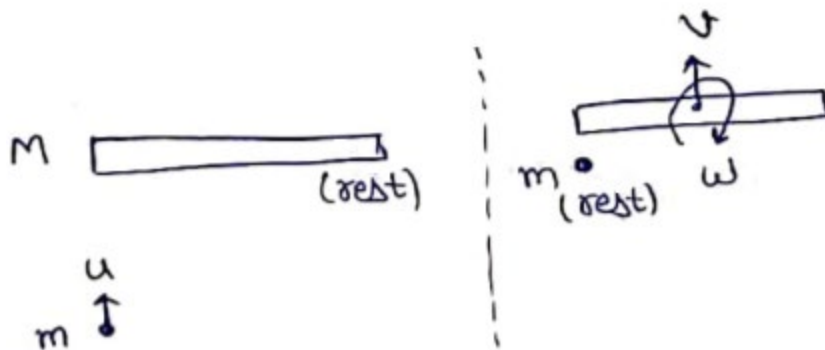
$$= \left(\frac{ML^2}{12} \right) \left(\frac{6Ft}{ML} \right)$$

$$L = \frac{FLt}{2m}$$

Question 60

A uniform rod of length L lies on a smooth horizontal table. A particle moving on a table strikes the rod perpendicularly at an end and stops. Find the distance travelled by the centre of the rod by the time it turns through a right angle. Show that if the mass of the rod is four times that of the particle, the collision is elastic.

Solution 60



Momentum conservation

$$P_i = P_f$$

$$mu + 0 = m(0) + M(v)$$

$$v = \frac{mu}{M}$$

Angular Momentum conservation

$$L_i = L_f$$

$$0 + mu\left(\frac{L}{12}\right) = \left(\frac{ML^2}{12}\right)\omega + 0$$

$$\omega = \frac{12mu}{ML}$$

Time to rotate by 90°

$$\omega = \frac{\theta}{t}$$

$$t = \frac{\frac{\pi/2}{\left(\frac{12mu}{ML}\right)}}{\frac{\pi/2}{\left(\frac{12mu}{ML}\right)}}$$

$$t = \frac{\pi ML}{12mu}$$

distance travelled by rod

distance = speed $\times$ time

$$= \left(\frac{mu}{M}\right) \times \left(\frac{\pi ML}{12mu}\right)$$

$$s = \frac{\pi L}{12}$$

Now, if $M = 4m$ then

$$KE_i = \frac{1}{2}mu^2$$

$$KE_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}(4m)\left(\frac{mu}{4m}\right)^2 + \frac{1}{2}\left(\frac{ML^2}{12}\right)\left(\frac{6mu}{ML}\right)^2$$

$$KE_f = \frac{1}{2}mu^2$$

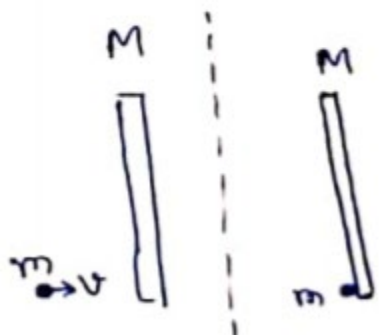
Hence proved.

Question 61

Suppose the particle of the previous problem has a mass m and speed v before the collision and sticks to the rod after the collision. The rod has a mass M . (a) Find the velocity of the centre of mass C of the system constituting "the rod plus the particle". (b) Find the velocity of the particle with respect to C before the collision. (c) Find the velocity of the rod with respect to C before the collision.

(d) Find the angular momentum of the particle and of the rod about the centre of mass C before the collision. (e) Find the moment of inertia of the system about the vertical axis through the centre of mass C after the collision. (f) Find the velocity of the centre of mass C and the angular velocity of the system about the centre of mass after the collision.

Solution 61



(a) $F_{ext} = 0$

$P_i = P_f$

$mv = (M+m)v'$

$v' = \frac{mv}{M+m}$

(b) velocity of the particle with respect to COM

$v_c = v - v'$

$= v - \frac{mv}{M+m}$

$= \frac{Mv}{M+m}$

$= \frac{Mv}{M+m}$

(c) velocity of the rod wrt the centre of mass = $\frac{-mv}{M+m}$

(d) Distance of COM from the particle

$x_{cm} = \frac{M \times \frac{L}{2} + m \times 0}{M+m} \Rightarrow x_{cm} = \frac{ML}{2(M+m)}$

Angular momentum of body about COM = mvr

$= m \times \frac{Mv}{M+m} \times \frac{ML}{2(M+m)}$

$= \frac{mM^2vL}{2(M+m)^2}$

$= \frac{mM^2vL}{2(M+m)^2}$

Angular momentum of rod about COM

$= M \times \frac{Mv}{M+m} \times \frac{1}{2} \frac{ML}{M+m}$

$= \frac{Mm^2vL}{2(M+m)^2}$

$= \frac{Mm^2vL}{2(M+m)^2}$

(e) M.I. about COM = $I_1 + I_2$

$I = m \left[\frac{ML}{2(M+m)} \right]^2 + \frac{ML^2}{12} + M \left[\frac{ML}{2(M+m)} \right]^2$

$= \frac{M(N+4m)L^2}{12(M+m)}$

$I = \frac{M(N+4m)L^2}{12(M+m)}$

(f) About COM

$$v_{CM} = \frac{mv}{(M+m)}$$

$$\therefore l\omega = mvr \Rightarrow \omega = \frac{6mv}{(M+4m)L}$$

Question 62

Two small balls A and B, each of mass m , are joined rigidly by a light horizontal rod of length L . The rod is clamped at the centre in such a way that it can rotate freely about a vertical axis through its centre. The system is rotated with an angular speed ω about the axis. A particle P of mass m kept at rest sticks to the ball A as the ball collides with it. Find the new angular speed of the rod.

Solution 62

$$\therefore \tau_{ext} = 0$$

$$L_i = L_f$$

$$\left[M \left(\frac{L}{2} \right)^2 + m \left(\frac{L}{2} \right)^2 \right] \omega = \left[3m \left(\frac{L}{2} \right)^2 \right] \omega'$$

$$\omega' = \frac{2}{3} \omega$$

Question 63

Two small balls A and B, each of mass m , are joined rigidly by a light horizontal rod of length L . The system translates on a frictionless horizontal surface with a velocity v_0 in a direction perpendicular to the rod. A particle P of mass m kept at rest on the surface sticks to the ball A as the ball collides with it. Find

(a) the linear speeds of the balls A and B after the collision, (b) the velocity of the centre of mass C of the system A+B+P and (c) the system about C after the collision.



[Hint: The light rod will exert a force on the ball B only along its length.]

Solution 63

(a) The light rod will exert a force on the ball B only along its length. So collision will not effect its velocity.

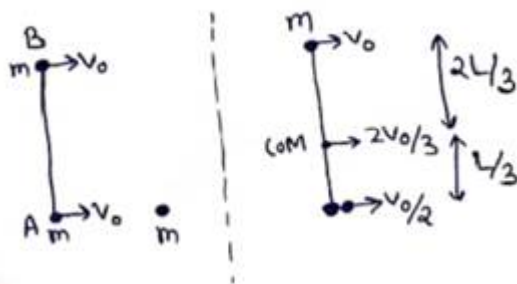
$$v_B = v_0$$

Momentum conservation for ball A

$$m v_0 = 2m v_A$$

$$v_A = \frac{v_0}{2}$$

(b)



$$\vec{v}_{CM} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

$$= \frac{mv_0 + 2m(\frac{v_0}{2})}{m + 2m}$$

$$\vec{v}_{CM} = \frac{2v_0}{3}$$

(c) velocity of A with respect to centre of mass = $\frac{2v_0}{3} - \frac{v_0}{2} = \frac{v_0}{6}$

velocity of B with respect to centre of mass = $v_0 - \frac{2v_0}{3} = \frac{v_0}{3}$

Applying, $m v_{COM} r = I_{COM} \omega$

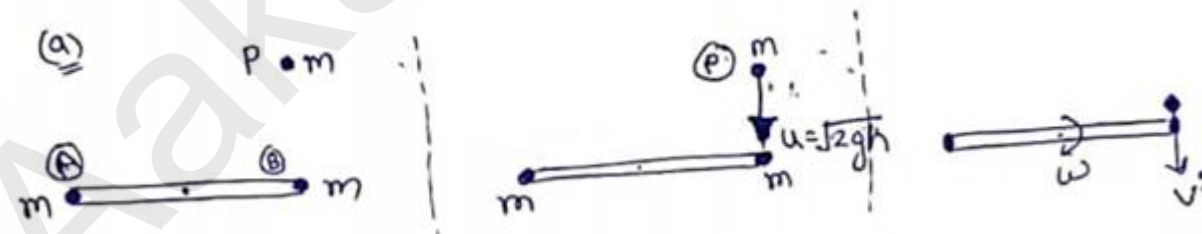
$$(2m \times \frac{v_0}{6} \times \frac{l}{3}) + (m \times \frac{v_0}{3} \times \frac{2l}{3}) = [2m(\frac{l}{3})^2 + m(\frac{2l}{3})^2] \omega$$

$$\omega = \frac{v_0}{2l}$$

Question 64

Suppose the rod with the balls A and B of the previous problem is clamped at the centre in such a way that it can rotate freely about a horizontal axis through the clamp. The system is kept at rest in the horizontal position. A particle P of the same mass m is dropped from a height h on the ball B. The particle collides with B and sticks to it. (a) Find the angular momentum and the angular speed of the system just after the collision. (b) What should be the minimum value of h so that the system makes a full rotation after the collision.

Solution 64



(a) If we consider two bodies P and B as system

then $F_{ext} = 0$ and $\tau_{ext} = 0$

$P_i = P_f$

$m(\sqrt{2gh}) = 2mv'$

$v' = \sqrt{\frac{gh}{2}}$

$$L_i = L_f$$

$$m(\sqrt{2gh})\left(\frac{L}{2}\right) = I\omega$$

$$mL\sqrt{\frac{gh}{2}} = \left(\frac{2mL^2}{4} + \frac{mL^2}{4}\right)\omega$$

$$\omega = \frac{\sqrt{8gh}}{3L}$$

(b) When the mass $2m$ and m are at the top most position and at the lowest point respectively, they will automatically rotate.

Change in Potential Energy = Change in kinetic Energy

$$2mg\left(\frac{L}{2}\right) - mg\left(\frac{L}{2}\right) = \frac{1}{2}I\omega^2$$

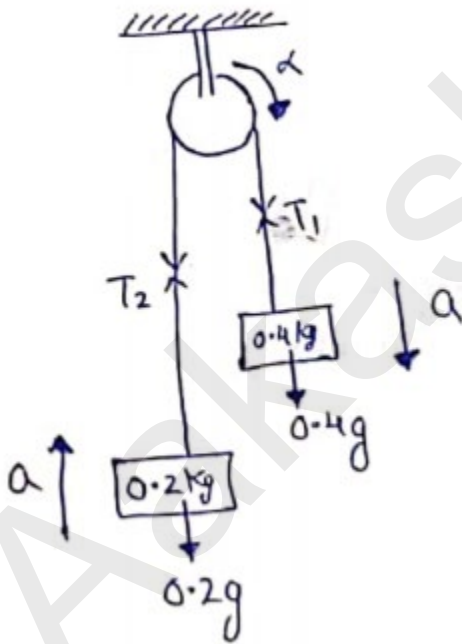
$$mg\left(\frac{L}{2}\right) = \frac{1}{2} \times \frac{3}{4}ML^2 \times \frac{8gh}{9L^2}$$

$$h = \frac{3L}{2}$$

Question 65

Two blocks of masses 400 g and 200 g are connected through a light string going over a pulley which is free to rotate about its axis. The pulley has a moment of inertia $1.6 \times 10^{-4}\text{ kg-m}^2$ and a radius 2.0 cm . Find (a) the kinetic energy of the system as the 400 g block falls through 50 cm , (b) the speed of the blocks at this instant.

Solution 65



Translatory Motion Equation

$$0.4g - T_1 = 0.4a \quad \text{---(i)}$$

$$T_2 - 0.2g = 0.2a \quad \text{---(ii)}$$

Rotational Motion Equation, $\tau = I\alpha$

$$T_1 R - T_2 R = (1.6 \times 10^{-4}) \left(\frac{a}{R} \right) \quad \text{---(iii)}$$

Solving (i), (ii) and (iii)

$$a = g/5$$

$$\text{Speed of block} = \sqrt{2 \times \text{acc.} \times \text{height}}$$

$$= \sqrt{2 \times \frac{g}{5} \times \frac{1}{2}}$$

$$= 1.4 \text{ m/s}$$

Total kinetic energy of system

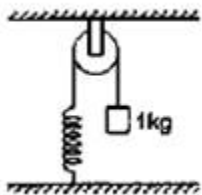
$$= \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} (0.4) (1.4)^2 + \frac{1}{2} (0.2) (1.4)^2 + \frac{1}{2} (1.6 \times 10^{-4}) \left(\frac{1.4}{0.02} \right)^2$$

$$= 0.98 \text{ J}$$

Question 66

The pulley shown in figure has a radius of 20 cm and moment of inertia 0.2 kg-m^2 . The string going over it is attached at one end to a vertical spring of spring constant 50 N/m fixed from below, and supports a 1 kg mass at the other end. The system is released from rest with the spring at its natural length. Find the speed of the block when it has descended through 10 cm. Take $g = 10 \text{ m/s}^2$.



Solution 66

By energy conservation

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 + \frac{1}{2} I \omega^2$$

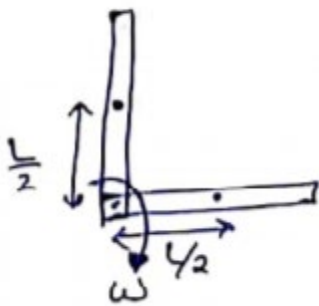
$$1 \times g \times (0.1) = \frac{1}{2} (1) (v)^2 + \frac{1}{2} (50) (0.1)^2 + \frac{1}{2} (0.2) \frac{v^2}{(0.2)^2}$$

$$v = 0.5 \text{ m/s}$$

Question 67

A metric stick is held vertically with one end on a rough horizontal floor. It is gently allowed to fall on the floor. Assuming that the end at the floor does not slip, find the angular speed of the rod when it hits the floor.

Solution 67



By energy conservation

$$mg \frac{L}{2} = \frac{1}{2} I \omega^2$$

$$mg \frac{L}{2} = \frac{1}{2} \left(\frac{mL^2}{3} \right) \omega^2$$

$$\omega = \sqrt{\frac{3g}{L}} = \sqrt{3 \times \frac{9.8}{1}}$$

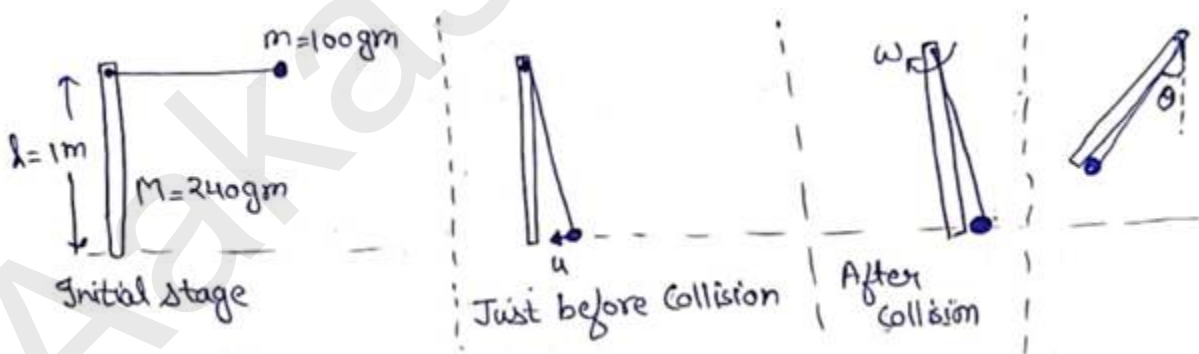
$$\omega = 5.42 \text{ rad/sec}$$

Question 68

A metre stick weighing 240 g is pivoted at its upper end in such a way that it can freely rotate in a vertical plane through the end. A particle of mass 100 g is attached to the upper end of the stick through a light string of length 1 m. Initially, the rod is kept vertical and the string horizontal when the system is released from rest. The particle collides with the lower end of the stick there. Find the maximum angle through which the stick will rise.



Solution 68



Velocity of ball just before the collision:-

$$\frac{1}{2} mu^2 = mgh$$

$$u = \sqrt{2gh}$$

$$u = \sqrt{2 \times 10 \times 1}$$

$$u = \sqrt{20} \text{ m/sec}$$

Now, Torque about hinge is zero

$$L_i = L_f$$

$$mvl + 0 = I\omega$$

$$(0.1 \times \sqrt{20} \times 1) = \left(ml^2 + \frac{ML^2}{3} \right) \omega$$

$$(0.1 \times \sqrt{20} \times 1) = \left[(0.1) \frac{(1)^2}{\sqrt{20}} + \frac{(0.24)(1)^2}{3} \right] \omega$$

$$\omega = \frac{(10)(0.18)}{(10)(0.18)}$$

Now, By energy conservation

$$\frac{1}{2} I \omega^2 = mg(1 - \cos \theta) + Mg \frac{l}{2} (1 - \cos \theta)$$

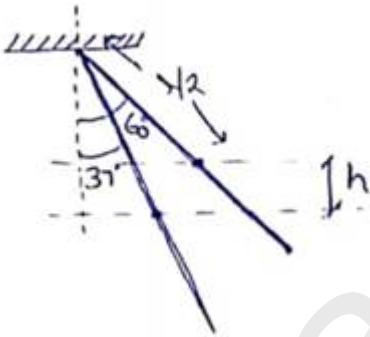
$$\frac{1}{2} \left[(0.1)(1)^2 + \frac{(0.24)(1)^2}{3} \right] \left[\frac{\sqrt{20}}{(10)(0.18)} \right]^2 = 0.1 \times g \times 1(1 - \cos \theta) + \frac{0.24 \times 10 \times 1}{2} [1 - \cos \theta]$$

$$\theta = 41^\circ$$

Question 69

A uniform rod pivoted at its upper end hangs vertically. It is displaced through an angle of 60° and then released. Find the magnitude of the force acting on a particle of mass dm at the tip of the rod when the rod makes an angle of 37° with the vertical.

Solution 69



By energy conservation

$$\frac{1}{2} I \omega^2 = mgh$$

$$\frac{1}{2} \left(\frac{mL^2}{3} \right) \omega^2 = mg \frac{l}{2} (\cos 37^\circ - \cos 60^\circ)$$

$$\omega^2 = \frac{9g}{10l}$$

So, centrifugal force

$$F_c = (dm)l\omega^2$$

$$= (dm)l \left(\frac{9g}{10l} \right)$$

$$F_c = 0.9g(dm)$$

Let angular acceleration of the rod be α when it makes an angle of 37° with the vertical.

$$\tau = I\alpha$$

$$mg \left(\frac{l}{2} \sin 37^\circ \right) = \left(\frac{mL^2}{3} \right) \alpha$$

$$\alpha = 0.9 \left(\frac{g}{l} \right)$$

So, tangential force

$$F_t = (dm)(a_t)$$

$$= (dm)(l\alpha)$$

$$F_t = 0.9g(dm)$$

So,

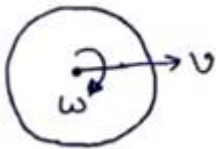
$$F_N = \sqrt{F_t^2 + F_c^2}$$

$$F_N = 0.9\sqrt{2g}(dm)$$

Question 70

A cylinder rolls on a horizontal plane surface. If the speed of the centre is 25 m/s, what is the speed of the highest point?

Solution 70



For pure rolling

$$v_c = R\omega$$

$$\begin{aligned} \text{Speed at highest point} &= v + R\omega \\ &= 2v \\ &= 2(25) \\ &= 50 \text{ m/s} \end{aligned}$$

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Question 71

A sphere of mass m rolls on a plane surface. Find its kinetic energy at an instant when its centre moves with speed v .

Solution 71

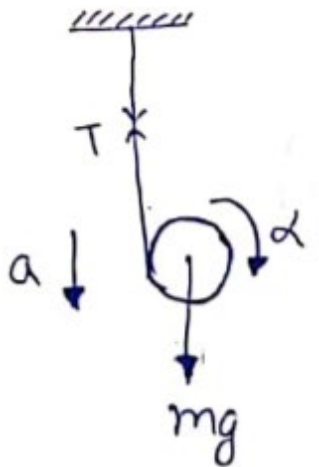
$$v = R\omega \text{ (Pure rolling)}$$

$$\begin{aligned} \text{K.E} &= \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2 \\ &= \frac{1}{2} \left(\frac{2}{5} m R^2 \right) \left(\frac{v^2}{R^2} \right) + \frac{1}{2} m v^2 \\ &= \frac{7}{10} m v^2 \end{aligned}$$

Question 72

A string is wrapped over the edge of a uniform disc and the free end is fixed with the ceiling. The disc moves down, unwinding the string. Find the downward acceleration of the disc.

Solution 72



$$mg - T = ma \quad \text{---(i)}$$

$$\tau = I\alpha$$

$$T \cdot R = \frac{mR^2}{2} \left(\frac{a}{R} \right) \quad \text{---(ii)}$$

From (i) and (ii)

$$a = \frac{2g}{3}$$

Question 73

A small spherical ball is released from a point at a height h on a rough track shown in figure. Assuming that it does not slip anywhere, find its linear speed when it rolls on the horizontal part of the track.



Solution 73

By energy conservation,

$$mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} mv^2$$

$$mgh = \frac{1}{2} \left(\frac{2}{5} mR^2 \right) \left(\frac{v^2}{R^2} \right) + \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{10gh}{7}}$$

Question 74

A small disc is set rolling with a speed v on the horizontal part of the track of the previous problem from right to left. To what height will it climb up the curved part?

Solution 74

By energy conservation,

$$\frac{1}{2} I \omega^2 + \frac{1}{2} mv^2 = mgh$$

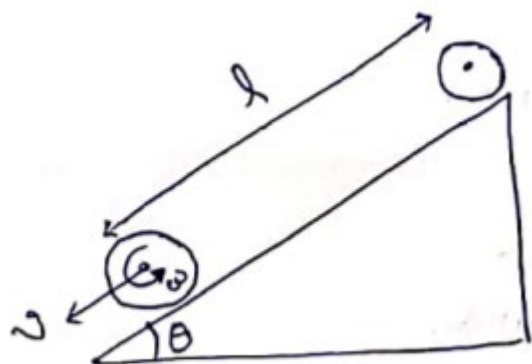
$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v^2}{R^2}\right) = mgh$$

$$h = \frac{3v^2}{4g}$$

Question 75

A sphere starts rolling down an incline of inclination θ . Find the speed of its centre when it has covered a distance l .

Solution 75



By energy conservation,

$$mg(l \sin \theta) = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

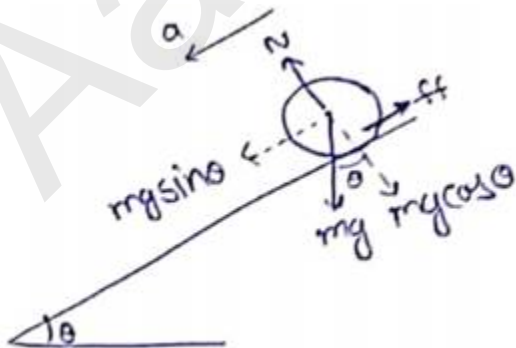
$$mgl \sin \theta = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{10}{7}gl \sin \theta}$$

Question 76

A hollow sphere is released from the top of an inclined plane of inclination θ . (a) What should be the minimum coefficient of friction between the sphere and the plane to prevent sliding? (b) Find the kinetic energy of the ball as it moves a length l on the incline if the friction coefficient is half the value calculated in part (a).

Solution 76



$$(a) \quad mg \sin \theta - f = ma \quad \text{---(i)}$$

$$f = \mu N = \mu mg \cos \theta \quad \text{---(ii)}$$

$$\tau = I\alpha$$

$$fR = \left(\frac{2}{5}mR^2\right)\alpha \quad \text{---(iii)}$$

Solving (i), (ii) and (iii)

$$\mu = \frac{2}{5} \tan \theta$$

$$(b) \quad \mu = \frac{\tan \theta}{5}$$

$$f = \mu N = \frac{\tan \theta}{5} (mg \sin \theta)$$

$$f = \frac{mg \sin \theta}{5}$$

Translatory Motion Equation

$$mg \sin \theta - f = ma$$

$$mg \sin \theta - \frac{mg \sin \theta}{5} = ma$$

$$a = \frac{4g \sin \theta}{5}$$

$$u = 0, s = l$$

$$v^2 = u^2 + 2as$$

$$v^2 = 2\left(\frac{4g \sin \theta}{5}\right)(l)$$

$$\text{Time to travel, } s = ut + \frac{1}{2}at^2$$

$$t^2 = \frac{2l}{a}$$

Rotational Motion Equation

$$\tau = I\alpha$$

$$fR = \left(\frac{2}{5}mR^2\right)\alpha$$

$$\frac{mg \sin \theta}{5} = \frac{2}{5}mR^2\alpha$$

$$\alpha = \frac{3g \sin \theta}{10R}$$

$$\omega = \alpha t$$

$$\text{K.E.} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

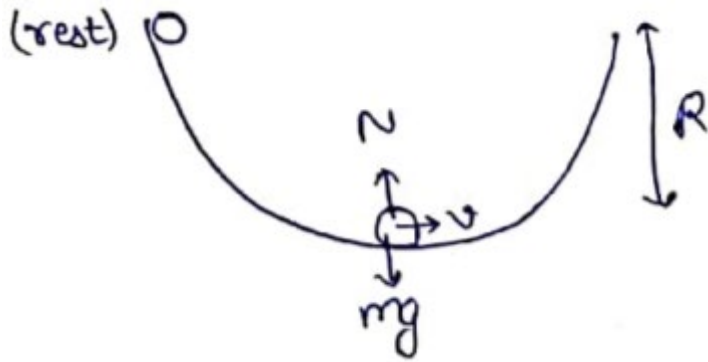
$$= \frac{1}{2}\left(\frac{2}{5}mR^2\right)(\alpha t)^2 + \frac{1}{2}m\left(\frac{8g \sin \theta l}{5}\right)$$

$$\text{K.E.} = \frac{7}{10}mgl \sin \theta$$

Question 77

A solid sphere of mass m is released from rest from the rim of a hemispherical cup so that it rolls along the surface. If the rim of the hemisphere is kept horizontal, find the normal force exerted by the cup on the ball when the ball reaches the bottom of the cup.

Solution 77



By conservation of energy

$$mgR = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$mgR = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2$$

$$v^2 = \frac{10}{7}gR$$

Normal contact at bottom

$$N - mg = \frac{mv^2}{R}$$

$$N = mg + \frac{m}{R}\left(\frac{10}{7}gR\right)$$

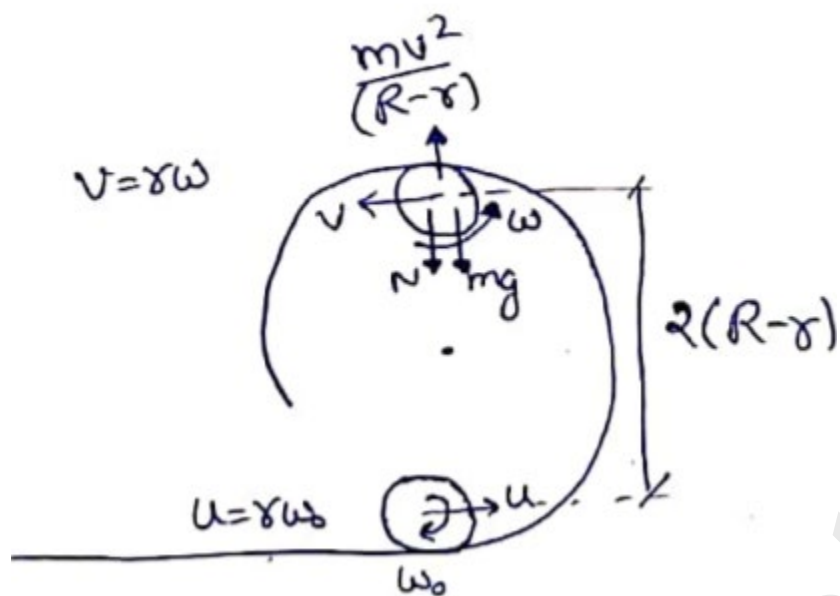
$$N = \frac{17}{7}mg$$

Question 78

Figure shows a rough track, a portion of which is in the form of a cylinder of radius R . With what minimum linear speed should a sphere of radius r be set rolling on the horizontal part so that it completely goes round the circle on the cylindrical part.



Solution 78



For minimum u , the ball will just be in contact with surface at top. So, $N=0$

$$mg = \frac{mv^2}{(R-r)}$$

$$v^2 = g(R-r)$$

Now, by energy conservation

$$\frac{1}{2}mu^2 + \frac{1}{2}I\omega_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mg \cdot 2(R-r)$$

$$\frac{1}{2}mu^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{u^2}{r^2}\right) = \frac{1}{2}mg(R-r) + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{g(R-r)}{r^2}\right) + 2mg(R-r)$$

$$u = \sqrt{\frac{27}{7}g(R-r)}$$

Question 79

Figure shows a small spherical ball of mass m rolling down the loop track. The ball is released on the linear portion at a vertical height H from the lowest point. The circular part shown has a radius R .
 (a) Find the kinetic energy of the ball when it is at a point A where the radius makes an angle θ with the horizontal.

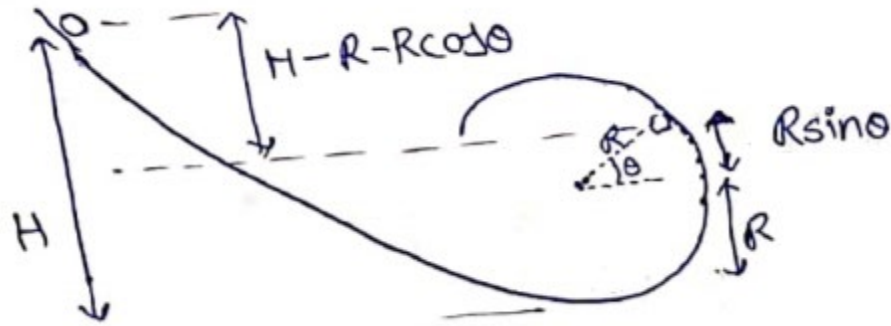
(b) Find the radial and the tangential accelerations of the centre when the ball is at A.

(c) Find the normal force and the frictional force acting on the ball if $H=60$ cm, $R=10$ cm, $\theta=0$ and $m=70$ g.



Solution 79

(a)



Energy conservation

$$K.E_i + P.E_i = K.E_f + P.E_f$$

$$0 + mg(H - R - R \sin \theta) = K.E_A$$

$$(b) \quad K.E_A = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$$

$$= \frac{1}{2} \left(\frac{2}{5} m r^2 \right) \left(\frac{v^2}{r^2} \right) + \frac{1}{2} m v^2$$

$$K.E_A = \frac{7}{10} m v^2 = mg(H - R - R \sin \theta)$$

$$v^2 = \frac{10}{7} g(H - R - R \sin \theta) \quad \text{---(i)}$$

Radial acceleration

$$a_r = \frac{v^2}{R} = \frac{10g}{7R} (H - R - R \sin \theta)$$

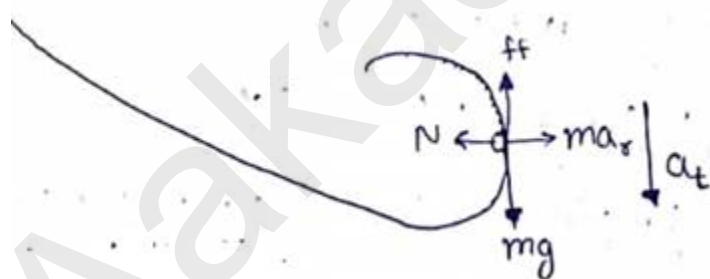
Differentiate equation (i) for tangential acceleration

$$2v \frac{dv}{dt} = \frac{-10g}{7} R \cos \theta \cdot \frac{d\theta}{dt}$$

$$\frac{dv}{dt} = \frac{-5}{7} g \cos \theta \cdot \frac{R\omega}{v} \quad \because v = R\omega$$

$$a_t = \frac{-5}{7} g \cos \theta$$

(c)



Horizontal direction

$$N = ma_r = m \times \frac{10g}{7} \frac{(H - R - R \sin \theta)}{R}$$

$$= \left(\frac{70}{1000} \right) \left(\frac{10}{7} \times 10 \right) \left(\frac{0.6 - 0.1 - 0.1 \times \sin 0^\circ}{0.1} \right)$$

$$N = 5$$

Vertical direction

$$mg - ff = ma_t$$

$$ff = m(g - a_t)$$

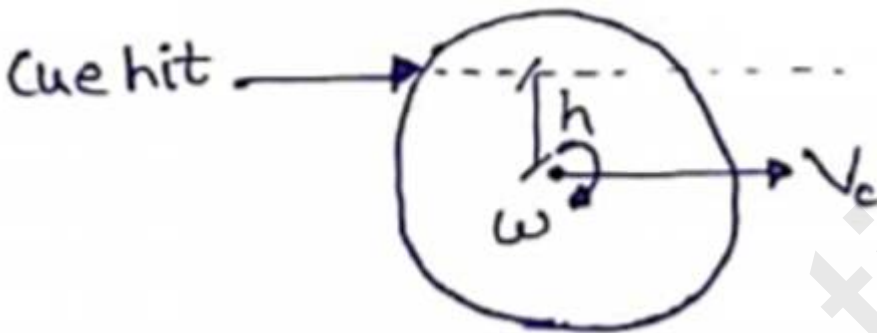
$$\frac{70}{1000} \left(10 - \frac{5}{7} \times 10 \right)$$

$$ff = 0.2 N$$

Question 80

A thin spherical shell of radius R lying on a rough horizontal surface is hit sharply and horizontally by a cue. Where should it be hit so that the shell does not slip on the surface?

Solution 80



$$v_c = R\omega \quad (\text{Pure Rolling})$$

By angular momentum conservation

$$mv_ch = I\omega$$

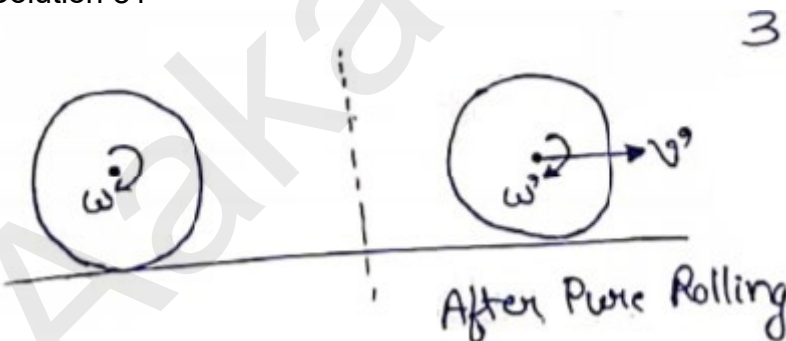
$$mv_ch = \left(\frac{2}{3}mR^2 \right) \left(\frac{v_c}{R} \right)$$

$$h = \frac{2R}{3}$$

Question 81

A uniform wheel of radius R is set into rotation about its axis at an angular speed ω . This rotating wheel is now placed on a rough horizontal surface with its axis horizontal. Because of friction at the contact, the wheel accelerates forward and its rotation decelerates till the wheel starts pure rolling on the surface. Find the linear speed of the wheel after it starts pure rolling.

Solution 81



By angular momentum conservation at bottom point since $\tau_{ext} = 0$

$$L_i = L_f$$

$$I\omega = mv'R + I\omega'$$

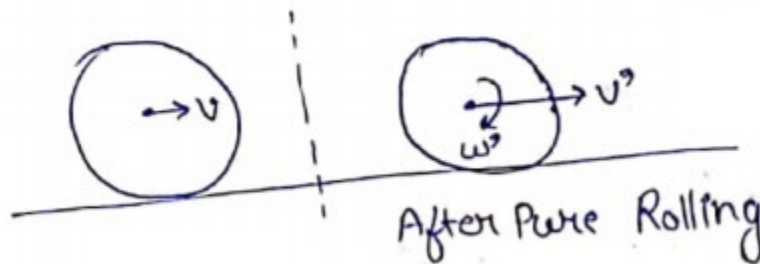
$$\left(\frac{mR^2}{2} \right) \omega = mv'R + \left(\frac{mR^2}{2} \right) \left(\frac{v'}{R} \right)$$

$$v' = \frac{\omega R}{3}$$

Question 82

A thin spherical shell lying on a rough horizontal surface is hit by cue in such a way that the line of action passes through the centre of the shell. As a result the shell starts moving with the linear speed v without any initial angular velocity. Find the linear speed of the shell after it starts pure rolling on the surface.

Solution 82



By angular momentum conservation at bottom most point

$$L_i = L_f$$

$$mvR = mv'R + I\omega'$$

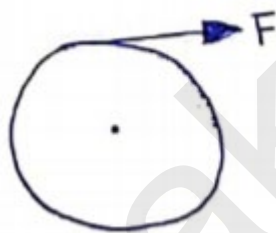
$$mvR = \left(\frac{2}{3}mR^2\right)\left(\frac{v'}{R}\right) + mv'R$$

$$v' = \frac{3v}{5}$$

Question 83

A hollow sphere of radius R lies on smooth horizontal surface. It is pulled by a horizontal force acting tangentially from the highest point. Find the distance travelled by the sphere during the time it makes one full rotation.

Solution 83



$$\tau = I\alpha$$

$$FR = \left(\frac{2}{3}MR^2\right)\alpha$$

$$\alpha = \frac{3F}{2MR}$$

Time to complete one rotation

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

By,

$$2\pi = \frac{1}{2}\left(\frac{3F}{2MR}\right)t^2$$

$$t = \sqrt{\frac{8\pi MR}{2F}}$$

For translatory motion

$$u = 0; t = \sqrt{\frac{8\pi MR}{2F}}; a = \frac{F}{m}$$

$$s = ut + \frac{1}{2}at^2$$

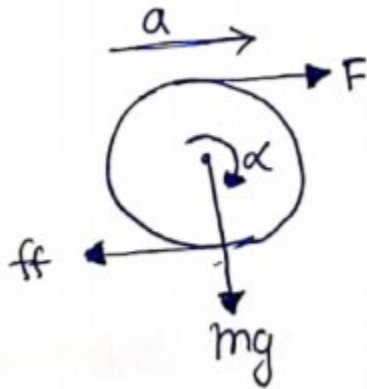
$$s = 0 + \frac{1}{2}\left(\frac{F}{m}\right)\left(\sqrt{\frac{8\pi MR}{2F}}\right)^2$$

$$s = \frac{4\pi R}{3}$$

Question 84

A solid sphere of mass 0.50 kg is kept on a horizontal surface. The coefficient of static friction between the surfaces in contact is $\frac{2}{7}$. What maximum force can be applied at the highest point in the horizontal direction so that the sphere does not slip on the surface?

Solution 84



$$F - ff = ma \quad \text{---(i)}$$

$$\tau = I\alpha$$

$$(FR + ff R) = \left(\frac{2}{5}mR^2\right)\left(\frac{a}{R}\right)$$

$$F + ff = \frac{2}{5}ma \quad \text{---(ii)}$$

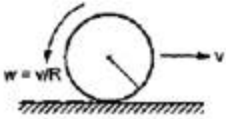
$$ff = \mu N = \frac{2}{7}mg \quad \text{---(iii)}$$

Solving (i), (ii) and (iii)

$$F = 3.3 \text{ N}$$

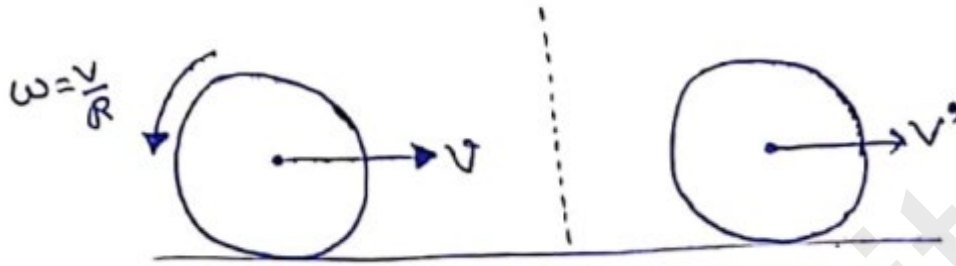
Question 85

A solid sphere is set into motion on a rough horizontal surface with a linear speed v in the forward direction and an angular speed v/R in the anticlockwise direction as shown in figure. Find the linear speed of the sphere (a) when it stops rotating and (b) when slipping finally ceases and pure rolling starts.



Solution 85

(a)



Conserving angular momentum about bottom point $L_i = L_f$

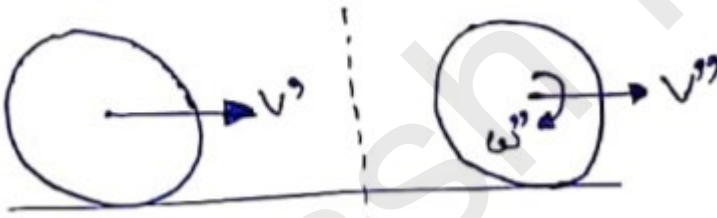
$$mvR - I\omega = mv'R$$

$$mvR - \left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right) = mv'R$$

$$\frac{3}{5}mvR = mv'R$$

$$v' = \frac{3v}{5}$$

(b)



Conserving angular momentum

$$mv'R = mv'''R + I\omega'''$$

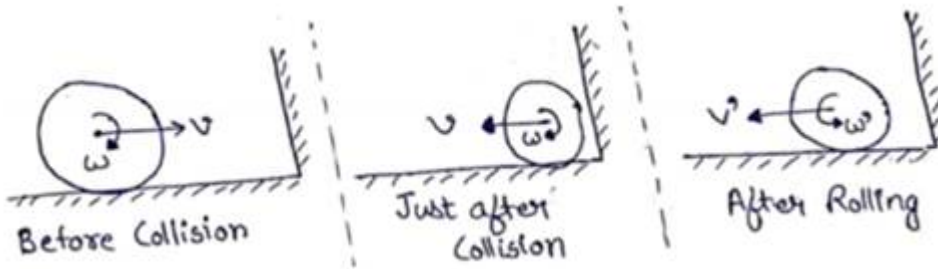
$$m\left(\frac{3v}{5}\right)R = mv'''R + \left(\frac{2}{5}mR^2\right)\left(\frac{v'''}{R}\right)$$

$$v''' = \frac{3v}{7}$$

Question 86

A solid sphere rolling on a rough horizontal surface with a linear speed v collides elastically with a fixed, smooth, vertical wall. Find the speed of the sphere after it has started pure rolling in the backward direction.

Solution 86



$$L_i = L_f$$

$$mvR - I\omega = mv'R + I\omega'$$

$$mvR - \left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right) = mv'R + \left(\frac{2}{5}mR^2\right)\left(\frac{v'}{R}\right)$$

$$v' = \frac{3v}{7}$$