Aakash
Medical | IIT-JEE | Foundations

Answers & Solutions
for
RMO 2026-27

Gold Medalists





Aarav Gupta
International Mathematical Olympiad (IMO) 2026 & 2025



Yug Gandhi
Singapore Math Olympiad 2025



Arjun Tyagi
International Olympiad in Artificial Intelligence (IOAI) 2025



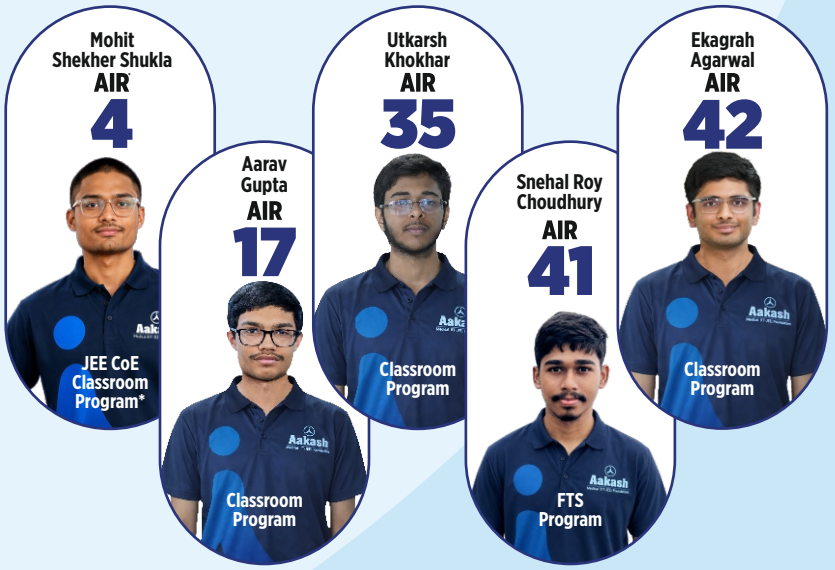
Dhruv Advani
34th International Biology Olympiad (IBO) 2023



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International Olympiad on Astronomy & Astrophysics 2019



*Fully sponsored student from March 2022 to June 2025

Top Ranks in
JEE (Advanced) 2026

11	26	57
Ranks in Top 100 AIR*	Ranks in Top 200 AIR*	Ranks in Top 500 AIR*

*Includes students of classroom, digital & distance across all categories.

Trust Aakash

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **All questions carry equal marks. Maximum marks : 102.**
- **No marks will be awarded for stating an answer without justification.**
- **Answer all the questions.**
- **All questions carry equal marks.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. Let ABC be a triangle with $AB = AC$. Let D be the midpoint of BC and P be a point on the segment AB satisfying $\angle CPA < 90°$. Suppose Q ($\neq P$) is a point on the circumcircle of triangle BPC such that $\angle AQC = 90°$. Prove that $\angle DQP = 90°$.

Sol. $\angle ABC = \angle BCA = \alpha$

$\Rightarrow$ $AD \perp BC$, $D \in BC$

$\Rightarrow$ $\angle ADC = 90°$

$\Rightarrow$ A, D, Q, C are cyclic as $\angle CQA = \angle CDA$ on the same chord AC also AC is diameter of circumcircle of $AQDC$

$\Rightarrow$ $\angle DAC = \angle DQC = 90° - \alpha$

Also, B, P, Q and C are cyclic

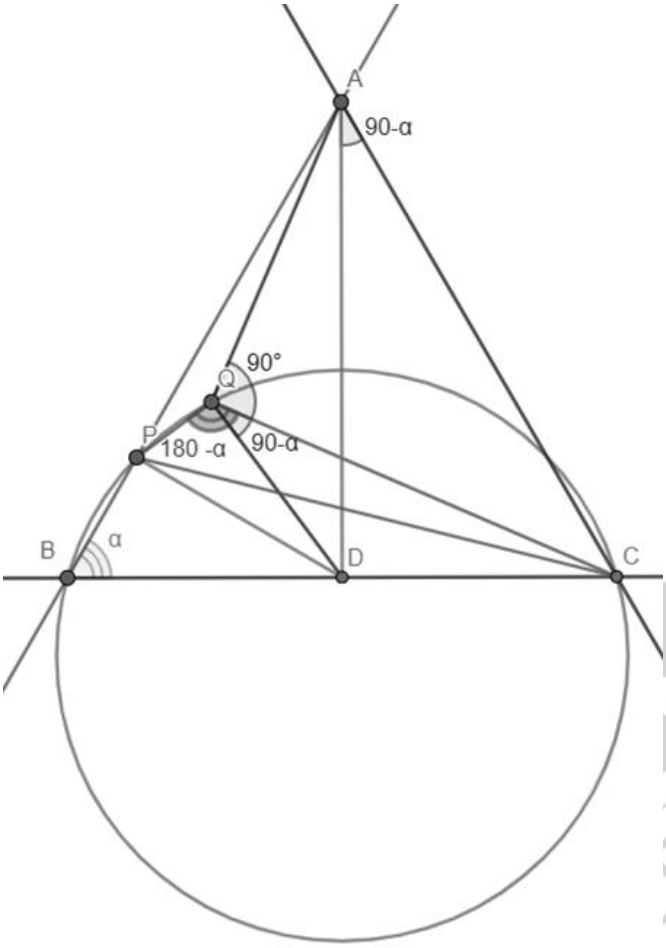
$\angle CQP = \angle CBP$ on the chord PC

$\Rightarrow$ $\angle CQP = \alpha$

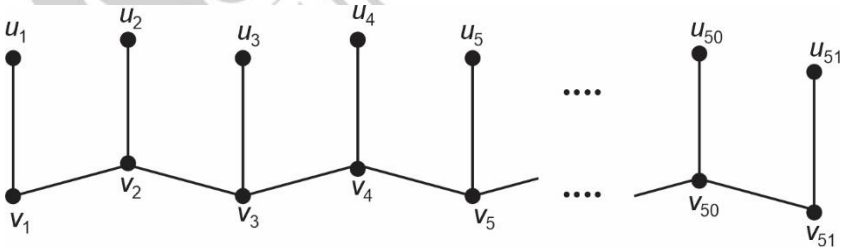
Also, $\angle DQP = \angle CQP - \angle DQC = (180° - (\alpha)) - (90° - \alpha))$

$= 90°$ hence proved.

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2. Consider the following diagram with 102 vertices :



Show that one can assign the numbers 1, 2, …, 102 to the 102 vertices $u_1, u_2, \ldots, u_{51}, v_1, v_2, \ldots, v_{51}$ such that no two vertices have the same number and the absolute differences of the numbers on adjacent vertices is a rearrangement of 1, 2, …, 101. (Two points are adjacent if they have a line segment joining them in the above diagram)

Sol. Construction :

For $j = 0, 1, \ldots, 25$ assign

$v_{2j+1} = 2j + 1$, $u_{2j+1} = 102 - 2j$

For $j = 1, 2, \ldots, 25$ assign

$v_{2j} = 103 - 2j$, $u_{2j} = 2j$

Olympiads Results

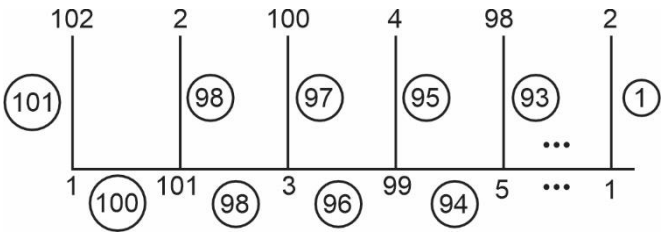
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v_i are odd number and u_i are even numbers

Verticle edge :

$|u_{2j+1} - v_{2j+1}| = |(102 - 2j) - (2j + 1)|$

$= 101 - 4j$

$\Rightarrow$ 101, 97, 93, …, 1.

$|v_{2j} - u_{2j}| = |103 - 2j - (2j)|$

$= 103 - 4j$

$\Rightarrow$ 99, 95, 91, … 3

$\Rightarrow$ Verticle edge has all odd differences.

Similarly horizontal edges

$|v_{2j+1} - v_{2j+2}| = |(2j + 1) - (101 - 2j)|$

$= 100 - 4j$

$\Rightarrow$ 100, 96, 92, … 4

And for $|v_{2j} - v_{2j+1}| = |(103 - 2j)|$

$= 102 - 4j$

$\Rightarrow$ 98, 94, 90, … 2

$\Rightarrow$ Horizontal edge contains all even differences.

$\Rightarrow$ Combining both differences we exhaust the differences

1, 2, …, 99, 100, 101. Hence shown.

3. Prove that for any two real numbers x and y we have $(x^6 - x^2 + 3)(y^6 - y^2 + 3) > 2(x^2 + y^2)^2$.

Sol. Let $a = x^2 \geq 0$, $b = y^2 \geq 0$

Consider $t \in R^+ \cup \{0\}$

$(t^3 - t + 3) - (t^2 + 2) = (t - 1)^2(t + 1) \geq 0$

$\Rightarrow$ $(t^3 - t + 3) \geq t^2 + 2$

Equality holds its $t = 1$

$\Rightarrow$ $(a^3 - a + 3)(b^3 - b + 3) \geq (a^2 + 2)(b^2 + 2)$

Using Cauchy – Schwarz

$$(a^2 + 2)(b^2 + 2) = \left(a^2 + \left(\sqrt{2}\right)^2\right)\left(b^2 + \left(\sqrt{2}\right)^2\right) \geq \left(a\sqrt{2} + b\sqrt{2}\right)^2 = 2(a+b)^2$$

$\Rightarrow$ $(a^3 - a + 3)(b^2 - b + 3) \geq 2(a + b)^2$

For equality of Cauchy – Schwarz

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$\frac{a}{\sqrt{2}} = \frac{\sqrt{2}}{b} \Rightarrow ab = 2$

$\Rightarrow \quad x^2y^2 = 2$

But $t^3 - t + 3 = t^2 + 2$ holds for $t = 1$

$\Rightarrow \quad x^2y^2 \neq 2$ if $x = y = 1$.

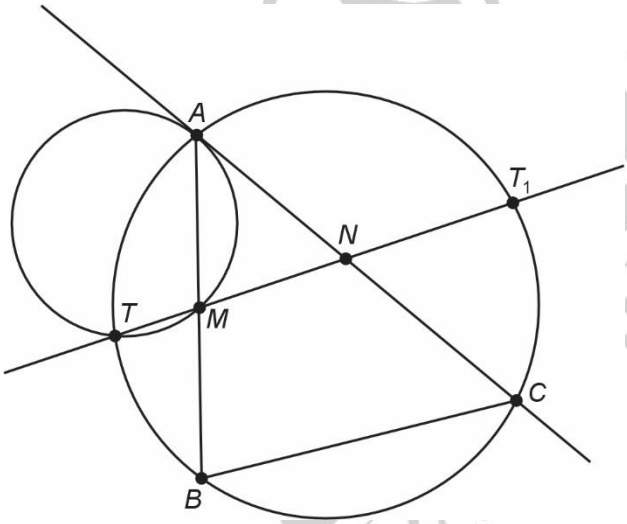
$\Rightarrow$ Equality doesn't hold

$\Rightarrow \quad (a^3 - a + 3)(b^3 - b + 3) > 2(a + b)^2$

Substituting back

$(x^6 - x^2 + 3)(y^6 - y^2 + 3) > 2(x^2 + y^2)^2$.

4. Let ABC be an acute triangle and let M and N be the midpoints of AB and AC, respectively. Let T be the point of intersection of ray NM with the circumcircle of ABC. Suppose that the circumcircle of ATM is tangent to AC at A. Let S be the foot of perpendicular from C on AB. Determine the value of $\frac{AS}{SB}$.



Sol.

$\frac{AS}{SB}$. Here (ABC) denotes circumcircle of ΔABC

Let $a = BC$, $b = CA$, $C = AB$

Since M, N are mid-point

$\Rightarrow \quad AM = MB = \frac{C}{2}$

$AN = NC = \frac{b}{2}$

Also, $MN = \frac{a}{2}$ and $MN \parallel BC$

Using mid-point theorem

$AN^2 = NM \cdot NT$ (using tangent at A for (AMT))

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$\Rightarrow \quad NT = \frac{b^2}{2a}$

$MT = NT - NM = \frac{b^2 - a^2}{2a}$

Also, power of point about point N on (ABC) we get

$MT \cdot NT_1 = NA \cdot NC = \left(\frac{b^2}{2}\right) = \frac{b^2}{4}$

$\Rightarrow \quad NT_1 = \frac{a}{2}$

$\Rightarrow \quad MT_1 = MN + NT_1 = a$

Now, power of point about M on (ABC) we get

$MT \cdot MT_1 = MA \cdot MB = \left(\frac{c}{2}\right)^2 = \frac{c^2}{4}$

$\Rightarrow \quad \left(\frac{b^2 - a^2}{2a}\right).(a) = \frac{c^2}{4}$

$\Rightarrow \quad c^2 = 2(b^2 - a^2) \qquad \ldots$ (i)

Let $AS = x$

$\Rightarrow \quad SB = c - x$

Using Pythagoras theorem in the triangles ASC, BSC.

$b^2 - x^2 = a^2 - (c - x)^2$

$\Rightarrow \quad b^2 - x^2 = a^2 - c^2 + 2cx - x^2$

$\Rightarrow \quad 2cx = b^2 + c^2 - a^2$

$\Rightarrow \quad AS = x = \frac{b^2 + c^2 - a^2}{2c}$

$SB = c - x = \frac{a^2 + c^2 - b^2}{2c}$

$\Rightarrow \quad \frac{AS}{SB} = \frac{b^2 + c^2 - a^2}{a^2 + c^2 - b^2}$

Using (i)

$\frac{AS}{SB} = \frac{3\left(b^2 - a^2\right)}{\left(b^2 - a^2\right)} = 3$

$\Rightarrow \quad \frac{AS}{SB} == 3$

5. Let a be an integer greater than 1. Determine all positive integers b which have the property that $a^b + 1$ divides a number of the form $a^n + b$ for some positive integer n.

Sol. All positive integers b of the form

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$b = a^k$ for some integer $k \geq 0$

Proof :

Let $m = a^b + 1$

$\Rightarrow \quad a^b \equiv -1 \text{ mod } (m)$

$\Rightarrow \quad a^{2b} \equiv 1 \text{ mod } (m)$

Consider T be minimum positive number such that $a^T \equiv 1 \text{ mod } (m)$

$\Rightarrow \quad T \mid 2b$

Suppose $n \in I^+$ such that

$M \mid (a^n + b)$

Let $n = q(2b) + r$ such that

$r \in \{0, 1, \ldots (2b - 1)\}$

$\Rightarrow \quad a^n = a^{q(2b)} \cdot a^r \equiv (1)\, a^r \text{ mod } (m)$

$\Rightarrow \quad a^r + b \equiv 0 \text{ mod } (m)$

Therefore,

If $r = 0 \Rightarrow m \mid (a^0 + b) = b + 1$

Which is impossible as $a^b + 1 > b + 1$

If $1 \leq r < b$, then

$0 < a^r + b \leq a^{b-1} + b < a^b + 1 = m$

$\Rightarrow \quad m \nmid a^r + b$

If $r = b$

$\Rightarrow \quad m \mid a^b + b \equiv (b - 1) \text{ mod } (m)$

But $0 \leq b - 1 < m$

$\Rightarrow \quad b = 1$

Hence for $b > 1$

$\Rightarrow \quad r \in (b, 2b)$

$\Rightarrow \quad$ Let $r = b + S$, $1 \leq S \leq b - 1$

$\Rightarrow \quad a^r + b = a^{b+S} + b \equiv -a^S + b (\text{mod } m)$

$\Rightarrow \quad m \mid b - a^S$

But $|b - a^S| < m$ as

$0 < a^S \leq a^b - 1 < a^b + 1 = m$, $b \in (0, m)$

$\Rightarrow \quad b = a^S$

$\Rightarrow \quad b$ is positive power of a

$\Rightarrow \quad b = 1 = a^0$ or $b = a^S$ for some $S \geq 1$

$\Rightarrow \quad b = a^k$, $(k \geq 0)$

Conversely

Let $b = a^k$ for some $k \geq 0$, $K \in I$

$\Rightarrow \quad n = b + k$

Then $a^n + b = a^{b+k} + a^k = a^k(a^b + 1)$

$\Rightarrow \quad (a^b + 1) \mid (a^b + 1)$

$\Rightarrow \quad (a^b + 1) \mid a^k(a^b + 1) = a^n + b$

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$\Rightarrow$ $(a^b + 1) \mid a^n + b$ for

Some $n = b + k$

$\Rightarrow$ $b = a^k$, $k = 0, 1, 2, \ldots$ is

Indeed a solution.

6. Let n be a fixed positive integer and S be a fixed positive real number. Find the minimum constant c (in terms of S, n) such that :

Given any positive real numbers $a_1, a_2, \ldots, a_n$ with $a_1 + a_2 + \ldots + a_n = S$, there is a rearrangement $b_1, b_2, \ldots, b_n$ of $a_1, a_2, \ldots$, an such that $(a_1 + b_1) + (a_2^2 + b_2^2) + \ldots + (a_n^n + b_n^n) \le c$.

Sol. For a rearrangement b

Let $E = \sum_{i=1}^{n} \left(a_i^i + b_i^i\right)$

Let A = the minimum of E over all arrangement of b.

We need the smallest C such that $A \le C$ for every choice of a.

Lemma :

For x > 0 and $i \in \{1, 2, \ldots., n\}$ $x^i + x^{n+i-i} \le x + x^n$

Proof: Let $B = x + x^n - x^i - x^{n+1-i}$

$B = (x^n - x^i)(1 - x^{1-i})$

If $x \ge 1$: $x^n \ge x^i$ and $x^{1-i} \le 1$, so $B \ge 0$

If $x < 1$: $x^n \le x^i$ and $x^{1-i} \ge 1$ so $B \ge 0$

Now Choose $b_i = a^{n+1-i}$

$\Rightarrow \sum b_i^i = \sum a_i^{n+1-i}$

$\because$ A is minimum

$A \le \sum (a_i + a_i^n) = S + \sum a_i^n$

Apply Lemma

$A \le \sum (a_i + a_i^n) = S + \sum a_i^n$

$\because \quad \sum a_i^n \le S^n$

$\Rightarrow A \le S + S^n$

Now Proof that no smaller C works.

Let $\in > 0$ be small and M = S – (n – 1) $\in > 0$

So, M $\to$ S as $\in \to 0$, In every rearrangement b, the value of M sits at some position *i.*

Case 1: $S > 1$: Take a_n = M and all other $a_i = \in$

$\Rightarrow$ a-part $\ge M^n$

b-part $\ge M^j \ge M$ $(\because M \ge 1)$

So, $E \ge M + M^n$

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Case 2: $S \leq 1$ Take $a_i = M$ and all other $a_i = \in$, Then $M \leq 1$

$\Rightarrow$ a – part $\geq M$

b-part $\geq M^j \geq M^n$ $(\because M \leq 1$ and $j \leq n)$

So $E \geq M + M^n$

In both cases this holds for every rearrangement

So, $A \geq M + M^n$ and $M + M^n \to S + S^n$ as $\varepsilon \to 0$

Hence for any $C < S + S^n$ as small enough $\in$ gives $M + M^n > C$

So, $A > C$ and C fails

$\Rightarrow$ Minimum $C = S + S^n$

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