All India Aakash Test Series for JEE (Main)-2024

TEST - 6

Test Date : 21/01/2024

ANSWERS

MATHEMATICS	PHYSICS	CHEMISTRY
1. (3)	31. (2)	61. (3)
2. (2)	32. (4)	62. (4)
3. (3)	33. (3)	63. (1)
4. (3)	34. (4)	64. (3)
5. (2)	35. (1)	65. (4)
6. (4)	36. (4)	66. (3)
7. (3)	37. (3)	67. (3)
8. (1)	38. (1)	68. (2)
9. (2)	39. (3)	69. (1)
10. (2)	40. (2)	70. (3)
11. (3)	41. (1)	71. (2)
12. (2)	42. (3)	72. (2)
13. (3)	43. (4)	73. (3)
14. (4)	44. (3)	74. (2)
15. (2)	45. (1)	75. (2)
16. (3)	46. (1)	76. (4)
17. (2)	47. (3)	77. (3)
18. (3)	48. (2)	78. (4)
19. (2)	49. (4)	79. (3)
20. (2)	50. (2)	80. (4)
21. (05.00)	51. (01.50)	81. (23.00)
22. (05.00)	52. (37.00)	82. (00.00)
23. (05.00)	53. (50.00)	83. (10.00)
24. (12.00)	54. (00.50)	84. (04.00)
25. (03.00)	55. (03.00)	85. (03.00)
26. (07.00)	56. (18.00)	86. (13.00)
27. (23.00)	57. (02.00)	87. (15.00)
28. (18.00)	58. (03.00)	88. (03.00)
29. (99.00)	59. (00.00)	89. (01.00)
30. (04.00)	60. (02.00)	90. (06.00)

Hints & Solutions

PART - A (MATHEMATICS)

1. Answer (3)

Hint :

R is not reflexive as $(2, 2) \notin R$, $(3, 3) \notin R$.

Sol. :

R is not reflexive as $(2, 2) \notin R$, $(3, 3) \notin R$. Also R is not transitive as $(2, 1) \in R$ and $(1, 2) \in R$. But $(2, 2) \notin R$. However R is symmetric.

2. Answer (2)

Hint :

$|\omega| = 2 \Rightarrow \omega 2e^{i\theta} = 2(\cos\theta + \sin\theta)$

Sol. :

$|\omega| = 2 \Rightarrow \omega = 2e^{i\theta} = 2(\cos\theta + \sin\theta)$

$\Rightarrow \quad x + iy = \omega - \frac{1}{\omega} = 2e^{i\theta} - \frac{1}{2}e^{-i\theta}$

$\Rightarrow \quad x = \frac{3}{2}\cos\theta,\ y = \frac{5}{2}\sin\theta$

$\Rightarrow \quad \frac{x^2}{9/4} + \frac{y^2}{25/4} = 1$ will represents an ellipse.

3. Answer (3)

Hint :

We have, $2(ab) = a + 3c$

Sol. :

We have, $2(ab) = a + 3c$

$b^2 = ac$

$\therefore \quad \frac{1}{16}(a + 3c)^2 = ac$

$\Rightarrow \quad a^2 + 9c^2 + 6ac = 16ac$

$\Rightarrow \quad a^2 - 10ac + 9c^2 = 0$

$\Rightarrow \quad a = 9c$

$\Rightarrow \quad \frac{c}{a} = \frac{1}{9}$ or $r^2 = \frac{1}{9} \Rightarrow r = \frac{1}{3}$ $[\because a, b, c > 0]$

4. Answer (3)

Hint :

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = (\lambda - 3)$$

Sol. :

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = (\lambda - 3)$$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix} = 2\lambda - 16 + \mu$$

$$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & \mu & \lambda \end{vmatrix} = 2(2\lambda - \mu + 4)$$

$$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & \mu \end{vmatrix} = \mu - 10$$

For $\lambda = 3$, $\mu = 10 \rightarrow$ option (3)

$D = 0$ & $D_1 = D_2 = D_3 = 0$

Hence given system has infinite solution

5. Answer (2)

Hint :

$\cos\sqrt{2}x + \cos x = 2$

Sol. :

$\cos\sqrt{2}\,x + \cos x = 2$

$\Rightarrow \quad \cos\sqrt{2}x = 1,\ \cos x = 1$

$x = 0$ is the only solution

6. Answer (4)

Hint :

$y = x^2 - 5x + 5$

$\frac{dy}{dx} = 2x - 5 = 2$

Sol. :

$y = x^2 - 5x + 5$

$\frac{dy}{dx} = 2x - 5 = 2 \Rightarrow x = \frac{7}{2}$

at $x = \frac{7}{2}, y = \frac{-1}{4}$

Equation of tangent at $\left(\frac{7}{2}, \frac{-1}{4}\right)$ is

$2x - y - \frac{29}{4} = 0$

Now check options

$x = \frac{1}{8}, y = -7$

7. Answer (3)

Hint :

$2I = \int_{-1/8}^{1/8} \pi dx$

Sol. :

$I = \int_{-1/8}^{1/8} \cos^{-1}(x^5 + 5x)dx$

$2I = \int_{-1/8}^{1/8} \pi dx \quad \therefore I = \frac{\pi}{8}$

8. Answer (1)

Hint :

Draw perpendiculars *PA* and *PB* to *x* and *y* axes respectively. Clearly *PA* = *PB* = *PS*

Sol. :

S(3, 10) suggests that parabolas are opening up and opening right respectively. Let they intersect at *P* and *Q*. Draw perpendiculars *PA* and *PB* to *x* and *y* axes respectively. Clearly *PA* = *PB* = *PS*

$\Rightarrow$ *P* lies on *y* = *x*

Similarly *Q* lies on *y* = *x*

i.e., *y* = *x* is the common chord

9. Answer (2)

Hint :

$\sum_{i=1}^{n}(x_i + 1)^2 = 9n \Rightarrow \sum_{i=1}^{n} x_i^2 + 2\sum_{i=1}^{n} x_i + n = 9n$

Sol. :

$\sum_{i=1}^{n}(x_i + 1)^2 = 9n \Rightarrow \sum_{i=1}^{n} x_i^2 + 2\sum_{i=1}^{n} x_i + n = 9n \ldots(i)$

$\sum_{i=1}^{n}(x_i - 1)^2 = 5n \Rightarrow \sum_{i=1}^{n} x_i^2 - 2\sum_{i=1}^{n} x_i + n = 5n \ldots(ii)$

Subtract (ii) from (i)

$\sum_{i=1}^{n} x_i = n$

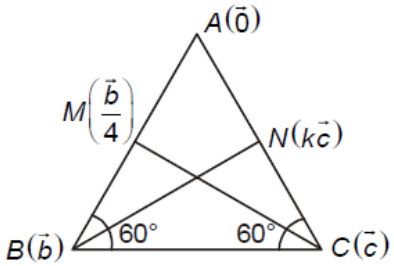
$SD = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2} = \sqrt{\frac{6n}{n} - \left(\frac{n}{n}\right)^2} = \sqrt{6-1} = \sqrt{5}$

10. Answer (2)

Hint :

Use dot product.

Sol. :



$\overrightarrow{AN} = k\overrightarrow{AC}$, let side = *a*

$\left(k\vec{c} - \vec{b}\right).\left(\frac{\vec{b}}{4} - \vec{c}\right) = 0$

where $\vec{b}.\vec{c} = \frac{a^2}{2}$

$\Rightarrow \quad \frac{k}{8} - k + \frac{1}{4} = 0$

$\therefore \quad k = \frac{2}{7}$

11. Answer (3)

Hint :

There are 3 points in sample space either both are red or both are yellow or one is red other is yellow.

Sol. :

5 red, 3 yellow

There are 3 points in sample space either both are red or both are yellow or one is red other is yellow.

$\Rightarrow \quad P(E) = \frac{3.5 + 5.3}{3.5 + 5.3 + 5.4 + 3.2} = \frac{15}{28}$

12. Answer (2)

Hint :

Replace $x \to \frac{x}{3} \Rightarrow f(x) - f\left(\frac{x}{3}\right) = \frac{x}{3}$

Sol. :

$f(3x) - f(x) = x$

Replace $x \to \frac{x}{3} \Rightarrow f(x) - f\left(\frac{x}{3}\right) = \frac{x}{3}$

Again replace

$x \to \frac{x}{3} \Rightarrow f\left(\frac{x}{3}\right) - f\left(\frac{x}{3^2}\right) - f\left(\frac{x}{3^2}\right) = \frac{x}{3^2}$

$\Rightarrow \quad f(3x) - f(0) = \frac{3x}{2}$ putting $x = \frac{8}{3}$

$\Rightarrow \quad f(8) - f(0) = 4 \ \therefore f(0) = 3$

Also putting $x = \frac{14}{3}$ in $f(3x) - 3 = \frac{3x}{2}$

$\Rightarrow \quad F(14) - 3 = 7 \Rightarrow f(14) = 10$

13. Answer (3)

Hint :

$\frac{x}{1+x^2} \in \left[\frac{-1}{2}, \frac{1}{2}\right]$

Sol. :

$\frac{x}{1+x^2} \in \left[\frac{-1}{2}, \frac{1}{2}\right]$

$\frac{kx}{1+x^2} \in \left[\frac{-k}{2}, \frac{k}{2}\right]$

$\Rightarrow \frac{k}{2} \geq \pi$

$k \geq 2\pi$

$k = 7$

14. Answer (4)

Hint :

$|x - a|$ is not differentiable at $x = a$

Sol. :

$|x - a|$ is not differentiable at $x = a$. In the interval (0, 4), $f(x)$ is not derivable at $x = 0.5$, $x = 1$ and $x = 2$.

15. Answer (2)

Hint :

Given, $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

Sol. :

Given, $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

$\Rightarrow \quad \tan^{-1}\frac{(2x+3x)}{1-2x\cdot 3x} = \tan^{-1}(1)$

$\Rightarrow \quad \frac{5x}{1-6x^2} = 1$

$\Rightarrow \quad 6x^2 + 5x - 1 = 0$

$\Rightarrow \quad (6x-1)(x+1) = 0$

$\Rightarrow \quad x = \frac{1}{6}$ or -1

$\because$ $x = -1$ do not satisfies the given equation

$\Rightarrow$ $x = \frac{1}{6}$

16. Answer (3)

Hint :

Solve $p'(x) = 0$

Sol. :

$p'(x) = 6x^2 - 6ax - 12a^2 = 6(x^2 - ax - 2a^2)$

$= 6(x - 2a)(x + a)$

$p'(x) = 0$

$\Rightarrow$ $x = -a$ or $2a$

$\frac{s+t}{2} = \frac{-a+2a}{2} = \frac{a}{2}$

$p\left(\frac{s+t}{2}\right) = -54$

$\Rightarrow$ $p\left(\frac{a}{2}\right) = -54$

$\Rightarrow$ $2\left(\frac{a}{2}\right)^3 - 3a\left(\frac{a}{2}\right)^2 - 12a^2\left(\frac{a}{2}\right) - 2 = -54$

$\Rightarrow$ $\frac{a^3}{4} - \frac{3a^3}{4} - 6a^3 = -52$

$\Rightarrow$ $a^3\left(-\frac{13}{2}\right) = -52$

$\Rightarrow$ $a^3 = 8$

$\Rightarrow$ $a = 2$

$\therefore$ $p'(x) = 6(x^2 - 2x - 8)$, $p''(x) = 12(x - 1)$

$p''(-2) < 0$ and $p''(4) > 0$

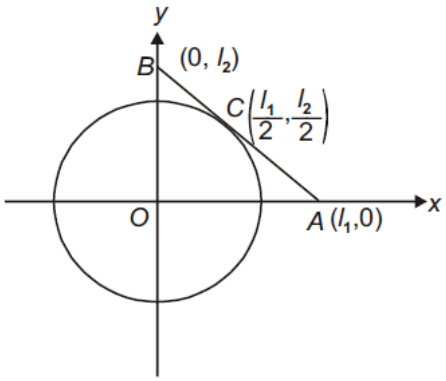
$\Rightarrow$ $s = -2$, $t = 4$

17. Answer (2)

Hint :

Use family of circles.

Sol. :



Equation of the circle is

$\left(x - \frac{l_1}{2}\right)^2 + \left(y - \frac{l_2}{2}\right)^2 + \lambda(l_2x + l_1y - l_1l_2) = 0$

It passes through $\left(\frac{l_1}{2}, 0\right)$

$\therefore$ $\lambda = \frac{l_2}{2l_1}$

So equation of circle will be

$x^2 + \frac{l_1^2}{4} + y^2 + \frac{l_2^2}{4} - l_1x - l_2y + \frac{l_2^2x}{2l_1} + \frac{l_2y}{2} - \frac{l_2^2}{2} = 0$

$\Rightarrow$ $x^2 + y^2 + x\left(-l_1 + \frac{l_2^2}{2l_1}\right) + y\left(\frac{l_2}{2} - l_2\right) + \frac{l_1^2}{4} - \frac{l_2^2}{4} = 0$

Radius = $\frac{l_2}{4l_1}\sqrt{l_1^2 + l_2^2}$

18. Answer (3)

Hint :

Let $x^2y^2 = t$

Sol. :

Let $x^2y^2 = t$

$\therefore$ $2xy^2 + 2x^2y\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore$ $\frac{dt}{dx} - 2xy^2 = \tan t - 2xy^2$

$\int \frac{dt}{\tan t} = \int dx$

$\therefore$ $\ln|\sin t| = x + C$ (C is constant)

$\ln|\sin(x^2y^2)| = x + C$

when $x = 1$, $y = \sqrt{\frac{\pi}{2}}$ then $C = -1$,

$\therefore \sin(x^2y^2) = e^{x-1}$

19. Answer (2)

Hint :

Given curves are $y = x^2 + 3$ and $y = 2x + 3$, points of intersection are (0, 3) and (2, 7)

Sol. :

Given curves are $y = x^2 + 3$ and $y = 2x + 3$, points of intersection are (0, 3) and (2, 7)

$\therefore$ Required area

$$= \left|\int_0^2 (x^2 - 2x)dx\right| = \left|\frac{x^3}{3} - \frac{2x^2}{2}\right|_0^2$$

$$= \left|\frac{8}{3} - 4\right| = \frac{4}{3}\text{sq. unit}$$

20. Answer (2)

Hint :

$(AP + PC)$ is minimised when P lies on AC and $BP + PD$ is minimised when P lies on BD

Sol. :

$(AP + PC)$ is minimised when P lies on AC and $BP + PD$ is minimised when P lies on BD

Thus $AP + PB + PC + PD = AC + BD = 2\sqrt{5}$

21. Answer (05.00)

Hint :

Direction ratio of median AD

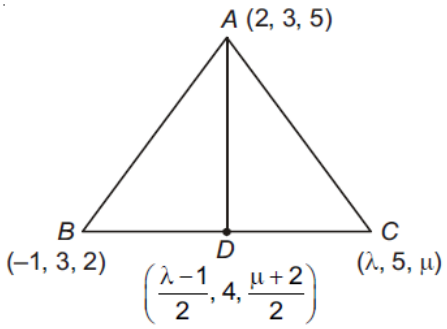
$$= < \frac{\lambda - 1}{2} - 2, 4 - 3, \frac{\mu + 2}{2} - 5 >$$

Sol. :

Direction ratio of median AD

$$= < \frac{\lambda - 1}{2} - 2, 4 - 3, \frac{\mu + 2}{2} - 5 >$$

$$= < \lambda - 5, 2, \mu - 8 >$$



But its direction ratio must be equivalent to <1, 1, 1>.

$\therefore \lambda = 7, \mu = 10$

22. Answer (05.00)

Hint :

Possibility of getting 7 are

(1, 6) (6, 1) (2, 5) (5, 2) (3, 4) (4, 3)

Sol. :

Possibility of getting 7 are

(1, 6) (6, 1) (2, 5) (5, 2) (3, 4) (4, 3)

$\therefore$ Required probability = probability of getting score of 7 exactly twice

$$= {}^3C_2\left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)$$

$$= \frac{3!}{2!(3-2)!}\frac{1}{36} \times \frac{5}{6}$$

$$= \frac{3}{36} \times \frac{5}{6}$$

$$= \frac{5}{72}$$

$\therefore \quad 72P = \frac{5}{72} \times 72 = 5$

23. Answer (05.00)

Hint & Sol. :

Number of points of discontinuity is 5

$\Rightarrow \sqrt{\frac{3}{2}}, \sqrt{2}, \sqrt{\frac{5}{2}}, \sqrt{3}, \sqrt{\frac{7}{2}}$

24. Answer (12.00)

Hint :

Use $\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$

Sol. :

$x+\frac{1}{x}=5 \quad \Rightarrow x^2+\frac{1}{x^2}=23$

$\Rightarrow x^4+\frac{1}{x^4}=527$

Now,

$x^5 + x^3 + x + x^{-5} + x^{-3} + x^{-1}$

$= x^4\left(x+\frac{1}{x}\right)+x^{-4}\left(x+\frac{1}{x}\right)+\left(x+\frac{1}{x}\right)$

$= \left(x+\frac{1}{x}\right)\left(x^4+x^{-4}+1\right)$

$= 5 \times 528$

$= 2640$

$\therefore$ Sum of digits = 2 + 6 + 4 + 0 = 12

25. Answer (03.00)

Hint :

$2^{2n+1} - 1 - 1 = 126$

Sol. :

$2^{2n+1} - 1 - 1 = 126$

$2^{2n+1} = 2^7$

$2n + 1 = 7$

$n = 3$

26. Answer (07.00)

Hint :

Let five terms in G.P. be $\frac{a}{r^2},\frac{a}{r},a,ar,ar^2$

Sol. :

Let five terms in G.P. be $\frac{a}{r^2},\frac{a}{r},a,ar,ar^2$

Then, $\frac{a(r^{-2}+r^{-1}+1+r+r^2)}{\left(\frac{1}{a}\right)(r^2+r+1+r^{-1}+r^{-2})}=49$

$\Rightarrow \quad a^2 = 49 \Rightarrow a = \pm 7$

Also, $\frac{a}{r^2}+a=35$

Therefore, $a = -7$ is not possible

Now, fifth term

$= ar^2 = a\left(\frac{a}{28}\right)-\frac{7}{4}=p \Rightarrow 4p=7$

27. Answer (23.00)

Hint :

Coefficient of x in

$(1+x)^p(1-x)^q = -{}^pC_0\,{}^qC_1+{}^pC_1\,{}^qC_0=-3$

Sol. :

Coefficient of x in

$(1+x)^p(1-x)^q = -{}^pC_0\,{}^qC_1+{}^pC_1\,{}^qC_0=-3$

$\Rightarrow \quad p - q = -3$

Coefficient of x^2 in

$(1+x)^p(1-x)^q = {}^pC_0\,{}^qC_2-{}^pC_1\,{}^qC_1-{}^pC_2\,{}^qC_0=-5$

$\Rightarrow \quad \frac{q(q-1)}{2}-pq+\frac{p(p-1)}{2}=-5$

$\Rightarrow \quad \frac{q(q-1)}{2}-(q-3)q+\frac{(q-3)(q-4)}{2}=-5$

$\Rightarrow \quad q = 11, p = 8$

Coefficient of x^3 in $(1 + x)^8(1 - x)^{11}$

$= -{}^{11}C_3 + {}^8C_1{}^{11}C_2 - {}^8C_2{}^{11}C_1 + {}^8C_3 = 23$

28. Answer (18.00)

Hint :

Five-digit combinations can be

(1,2,2,3,3), (1,4,3,3,1), (1,9,2,2,1), (1,4,9,1,1), (1,2,3,6,1), (1,6,6,1,1)

Sol. :

Factors of $36 = 2^2 \cdot 3^2 \cdot 1$

Five-digit combinations can be

(1,2,2,3,3), (1,4,3,3,1), (1,9,2,2,1), (1,4,9,1,1), (1,2,3,6,1), (1,6,6,1,1)

i.e, total numbers

$= \frac{5!}{2!2!}+\frac{5!}{2!2!}+\frac{5!}{2!2!}+\frac{5!}{3!}+\frac{5!}{2!}+\frac{5!}{3!2!}$

$= (30 \times 3) + 20 + 60 + 10 = 180.$

29. Answer (99.00)

Hint :

$f(x)=\frac{2e^{2x}}{e^{2x}+e}$ and $f(1-x)=\frac{2e^{2-2x}}{e^{2-2x}+e}$

Sol. :

$f(x)=\frac{2e^{2x}}{e^{2x}+e^{x}}$ and $f(1-x)=\frac{2e^{2-2x}}{e^{2-2x}+e}$

$\therefore \quad \frac{f(x)+f(1-x)}{2}=1$

i.e., $f(x) + f(1 - x) = 2$

$\therefore \quad f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+...+f\left(\frac{99}{100}\right)$

$= \sum_{x=1}^{49} f\left(\frac{x}{100}\right)+f\left(1-\frac{x}{100}\right)+f\left(\frac{1}{2}\right)$

$= 49 \times 2 + 1$

$= 99$

30. Answer (04.00)

Hint :

$2ab^2 - 4ab = 6b^2$

$b=\frac{2a}{a-3}$

Sol. :

$2ab^2 - 4ab = 6b^2$

$b=\frac{2a}{a-3}$

$\therefore \quad ab=\frac{2a^2}{a-3}=k$

$\therefore \quad a = 0, 6$

At $a = 6$, $\frac{dk^2}{da^2} = +\text{ve}$

$\therefore$ When $a = 6$ and $b = 4$, ab is minimum

$\therefore$ Minimum $\frac{ab}{6}=\frac{6\times 4}{6}=4$

PART - B (PHYSICS)

31. Answer (2)

Hint :

Apply Kirchhoff's rule

Sol. :

The equivalent circuit is a balanced wheat stone bridge

$R_{eq}=\frac{30\times 15}{30+15}=10\ \Omega$

Current, $I=\frac{10}{10}=1\ \text{A}$

32. Answer (4)

Hint :

L.C = 1 MSD – 1 VSD

Sol. :

Length of wire = 4.50 cm

Least count of measuring instrument = 0.01 cm

for vernier scale, 10 MSD = 1 cm = 10 mm

1 MSD = 1 mm

Also, 9 MSD = 10 VSD

L.C = 1 MSD – 1 VSD = (1 – 0.9) mm = 0.1 mm

33. Answer (3)

Hint :

$U=\frac{1}{2}\varepsilon_0 E^2$ (vol.)

Sol. :

$du = \frac{1}{2}\varepsilon_0(E_0x)^2(a^2dx)$

$u=\int du = \frac{1}{2}\varepsilon_0 E_0^2 a^2\left(\frac{8a^3}{3}\right) = \frac{4}{3}\varepsilon_0 E_0^2 a^5$

34. Answer (4)

Hint :

$\phi=\phi_0+(\text{K.E})_{max}$

Sol. :

$\phi = \frac{1240}{248}$ eV = 5 eV

$\therefore \quad \phi_0 = \phi - K.E_{max}$

= 5 – 2

= 3 eV

35. Answer (1)

Hint :

Use dimensional analysis

Sol. :

[Momentum] = $[MLT^{-1}]$

[Planck's constant] = $[ML^2T^{-1}]$

36. Answer (4)

Hint :

$\frac{dy}{dx}$ = slope = μ

Sol. :

Block is under limiting friction

$\mu = \tan\theta$ …(1)

$y = x^3$

Slope = $\frac{dy}{dx} = 3x^2$ …(2)

$\therefore$ From (1) and (2)

$0.5 = 3x^2$

$x^2 = \frac{1}{6}$

$x = \frac{1}{\sqrt{6}}$

$\therefore \quad y = \left(\frac{1}{\sqrt{6}}\right)^3 = \frac{1}{6^{3/2}}$

37. Answer (3)

Hint :

$V_{p-\text{side}} > V_{n-\text{side}}$

Sol. :

For forward biasing potential on *p*-side is greater than potential on *n*-side.

38. Answer (1)

Hint :

Speed is maximum at U_{min}

Sol. :

$\frac{dU}{dx} = \frac{4x^3}{4} - \frac{2x}{2}$

$= x^3 - x$

$= x(x^2 - 1)$

For $U \to \min, \frac{dU}{dx} = 0$

$\therefore \quad x = 0, \pm 1$

$\therefore$ at $x = \pm 1, U = -\frac{1}{4}$ J

$\therefore \quad U_{min} = -\frac{1}{4}$ J

$K.E_{max} = 4 - \left(-\frac{1}{4}\right) = \frac{17}{4}$ J

$\frac{17}{4} = \frac{1}{2}(2)v^2$

$v = \frac{\sqrt{17}}{2}$ m/s

39. Answer (3)

Hint :

Path is uniform circular motion.

Sol. :

Path is circular ($\because \vec{F} = q(\vec{v} \times \vec{B})$)

Speed is constant but direction changes.

40. Answer (2)

Hint :

$e = \frac{v_{\text{rel. sep.}}}{v_{\text{rel. app.}}}$

Sol. :

$\because$ $2 \times 1 + 1 \times 0 = 2v_1 + 1v_2$

$\Rightarrow$ $2v_1 + v_2 = 2$ …(i)

Also, $e = \frac{v_2 - v_1}{1}$

$1 = 2v_2 - 2v_1$ …(ii)

From (i) and (ii)

$v_2 = 1$ m/s

41. Answer (1)

Hint :

$X = \overline{\bar{A} \cdot \bar{B}}$

Sol. :

$X = \overline{\bar{A} \cdot \bar{B}}$

$= \bar{\bar{A}} + \bar{\bar{B}}$ (Using De Morgan Theorem)

$= A + B$

This is same as OR gate

42. Answer (3)

Hint & Sol. :

Theoretical

43. Answer (4)

Hint :

Two cells are in parallel

Sol. :

$\varepsilon_{eq} = \frac{\frac{5}{1} - \frac{2}{2}}{\frac{1}{1} + \frac{1}{2}} = \frac{4}{3/2} = \frac{8}{3}$ V

$r_{eq} = \frac{2}{3}\ \Omega$

$i_{5\,\Omega} = \frac{\frac{8}{3}}{\frac{2}{3} + 5} = \frac{8}{17}$ A

44. Answer (3)

Hint :

$v = \sqrt{2gh}$

Sol. :

Let v_1 and v_2 be the velocity of efflux from square and circular hole respectively. S_1 and S_2 be cross-section areas of square and circular holes.

$v_1 = \sqrt{8gy}$ and $v_2 = \sqrt{2gy}$

The volume of water coming out of square and circular hole per second is

$Q_1 = v_1 s_1 = \sqrt{8gy}\ L^2$

$Q_2 = v_2 s_2 = \sqrt{2gy}\ \pi R^2$

As $Q_1 = Q_2$

$\Rightarrow$ $R = \sqrt{\frac{2}{\pi}} . L$

45. Answer (1)

Hint :

$P = \frac{V^2}{R}$

Sol. :

$200 = (20)^2 \left(\frac{1}{R} + \frac{1}{4}\right)$

$\frac{1}{2} = \frac{1}{R} + \frac{1}{4}$

$\Rightarrow$ $R = 4\ \Omega$

46. Answer (1)

Hint :

$\Delta Q = nC\Delta T$

Sol. :

Heat extracted = $\Delta Q_{DA} + \Delta Q_{AB}$

$\Delta Q_{DA} = nC_v(T_A - T_D)$

$= n\left(\frac{3}{2}R\right)\left[\frac{3p_0v_0}{nR} - \frac{p_0v_0}{nR}\right]$

$= \frac{3}{2}(2p_0v_0) = 3p_0v_0$

$\Delta Q_{AB} = nC_p\Delta T$

$= nC_p(T_B - T_A)$

$= n\left(\frac{5}{2}R\right)\left[\frac{6p_0v_0}{nR} - \frac{3p_0v_0}{nR}\right]$

$= \frac{15}{2}p_0v_0$

Heat extracted = $\frac{21}{2}p_0v_0$

47. Answer (3)

Hint :

Theoretical

Sol. :

It is OR gate

48. Answer (2)

Hint :

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Sol. :

R = 20 cm

$\frac{1}{f_2} = (\mu - 1) \times \left(\frac{-1}{R}\right) = \left(\frac{4}{3} - 1\right) \times \left(\frac{-1}{20}\right) = -\frac{1}{60}$

$\therefore \quad \frac{1}{f_{eq}} = \frac{2}{20} + \frac{2 \times (-1)}{60} = \frac{1}{15}$

$\Rightarrow f_{eq}$ = 15 cm

Now, –2 = $\frac{15}{u - 15} \Rightarrow u$ = 7.5 cm

49. Answer (4)

Hint :

$m = \frac{f_o}{f_e}$

Sol. :

$m = \frac{f_o}{f_e} = \frac{120}{12} = 10$

$\therefore \quad \frac{\tan\alpha}{\tan\beta} = 12$

$\tan\alpha = \left(\frac{50}{1000}\right) \times 10 = \frac{1}{2}$

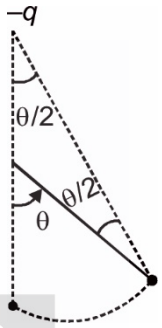
$\alpha = \tan^{-1}\left(\frac{1}{2}\right)$

50. Answer (2)

Hint :

$T = \frac{2\pi}{\omega}$

Sol. :



$\tau = mga\sin\theta - \frac{Kq^2}{4a^2\cos^2\frac{\theta}{2}}a\sin\frac{\theta}{2}$

For small θ

$\tau = \left[mga\theta - \frac{Kq^2\theta}{8a}\right]$ towards equilibrium

$\tau = ma^2\alpha$

$[\alpha \ \alpha \ \theta]$ towards equilibrium

T = 4 s

51. Answer (01.50)

Hint :

$L = I\omega$, K.E = $\frac{1}{2}I\omega^2$

Sol. :

$L = I\omega$, K.E = $\frac{1}{2}I\omega^2$

$$\therefore \quad \frac{L}{\text{K.E}} = \frac{2}{\omega}$$

$$\Rightarrow \quad L = \frac{2(\text{K.E})}{\omega}$$

$$\therefore \quad \frac{L'}{L} = \frac{2(2\text{K.E})}{3\omega} \times \frac{\omega}{2(\text{K.E})}$$

$$\boxed{L' = \frac{2}{3}L}$$

52. Answer (37.00)

Hint :

Apply parallel axis theorem

Sol. :

$$I_{XX'} = I_{\text{com}} + m(3R)^2$$

$$= \frac{mR^2}{4} + 9mR^2$$

$$= \frac{37}{4}mR^2$$

53. Answer (50.00)

Hint :

$\Delta U = nCv\Delta T$

Sol. :

$$\Delta U_{AB} = \frac{5R}{2}(500 - 700)$$

$$= -500R$$

54. Answer (00.50)

Hint :

$$\lambda = \frac{h}{\sqrt{2m(\text{K.E})}}$$

Sol. :

$$\boxed{\lambda' = \frac{\lambda}{2}}$$

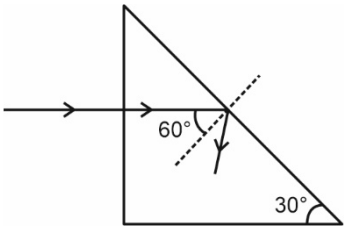
Factor is 0.5

55. Answer (03.00)

Hint :

$$\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$$

Sol. :



$$\mu > \frac{1}{\sin 60°}$$

$$\mu > \frac{2}{\sqrt{3}}$$

56. Answer (18.00)

Hint :

$$C = \frac{k\varepsilon_0 A}{d}$$

Sol. :

$$C_1 = \frac{6\varepsilon_0 A}{d/3} = \frac{18\varepsilon_0 A}{d} = 18C$$

$$C_2 = \frac{2\varepsilon_0 A}{2d/3} = \frac{3\varepsilon_0 A}{d} = 3C$$

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{18 \times 3}{18 + 3} = \frac{18}{7}C$$

$$= \frac{18}{7} \times 7 \text{ pF}$$

= 18 pF

57. Answer (02.00)

Hint :

$$\frac{1}{2}kx^2 = \frac{p^2}{2m}$$

Sol. :

$$\frac{1}{2}kx^2 = \frac{p^2}{2m}$$

$p = \sqrt{mk}x$

$= \sqrt{2\times 200}\times\frac{10}{100}$

$= 20\times\frac{1}{10}$

$= 2 \text{ kg ms}^{-1}$

58. Answer (03.00)

Hint :

$\vec{L} = \vec{r}\times\vec{p}_{com} + I_{com}\vec{\omega}$

Sol. :

$L = mv_0R + \frac{2}{5}mR^2\left(\frac{5v_0}{R}\right)$

$= 3mv_0R$

59. Answer (00.00)

Hint :

$a = -\omega^2 x$

Sol. :

$\phi = 5\pi$

$\Rightarrow a = 0$

$\because x = 0$

60. Answer (02.00)

Hint :

$\varepsilon_{ind} = B\ell v$

Sol. :

$R_{eq} = 2R$

$i = \frac{B\ell v}{2R}$

PART - C (CHEMISTRY)

61. Answer (3)

Hint :

1 mole of H_2 = 2 g = 22400 cm^3 at S.T.P.

Sol. :

22400 cm^3 of H_2 at STP have mass = 2 g

224 cm^3 of H_2 at S.T.P. have mass = $\frac{2\times 224}{22400}$

= 0.02 g

62. Answer (4)

Hint :

$\Delta H = [B.E._{(H-H)} + B.E._{(Cl-Cl)}] - [2 \times B.E.\ (H - Cl)]$

Sol. :

$H_2 + Cl_2 \rightarrow 2HCl$

$\Delta H = 2 \times (-z) = -2z \text{ kJ mol}^{-1}$

$-2z = [x + y] - 2\ B.E.\ (H - Cl)$

$B.E.\ (H - Cl) = \frac{x+y+2z}{2}$

63. Answer (1)

Hint :

Oxidising agent undergo self-reduction.

Sol. :

$Fe^{+2} \longrightarrow Fe^{+3}$ (Oxidation)

$MnO_4^{\ominus} \longrightarrow Mn^{2+}$ (Reduction)

$MnO_4^{\ominus}$ is oxidising agent in the reaction.

64. Answer (3)

Hint :

This is a rearrangement reaction.

Sol. :

$$CH_3-CH_2-CH_2-Cl \xrightarrow[573\ K]{\text{Anhy. } AlCl_3} CH_3-\underset{\displaystyle Cl}{\underset{|}{CH}}-CH_3$$

65. Answer (4)

Hint :

CN^- is strong field ligand and F^- is weak field ligand

Sol. :

$[Fe(CN)_6]^{4-}$, here

$\therefore$ CFSE = [–4 × 6 + (6 × 0)] Dq

= –24 Dq

For $[CoF_6]^{3-}$, F^-

$\therefore$ CFSE = [–4 × 4 + 6 × 2] Dq

= –16 + 12 = –4 Dq

66. Answer (3)

Hint :

Fe^{3+}, Ni^{2+}, Ba^{3+} does not give ppt with H_2S in dil HCl

Sol. :

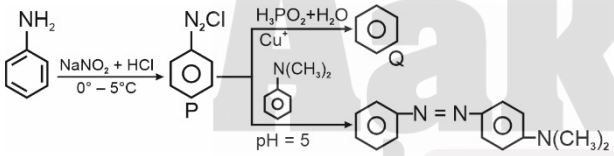
Hg^{2+}, Pb^{2+}, Cd^{2+}, Sn^{2+}, Sn^{4+} give ppt when treated with H_2S in presence of dil. HCl.

67. Answer (3)

Hint :

Product P is diazonium salt.

Sol. :



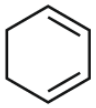
68. Answer (2)

Hint :

Some of the energy used against resonance energy.

Sol. :

Dehydrogenation of  = 29 kcal/mol

So, it should be 29 × 2 kcal/mol for 

but some energy is used against resonance energy.

R.E. = ((2 × 29) – 56) kcal

= 2 kcal

69. Answer (1)

Hint :

Ziegler-Natta catalyst is $TiCl_4 + Et(C_2H_5)_3$

Sol. :

$PdCl_2$ is used as catalyst in Wacker's process.

70. Answer (3)

Hint :

Starch is a polymer of α-(D)(+) Glucose consisting of amylose and amylopectin.

Sol. :

Amylopectin is a branched chain polysaccharide insoluble in water and it is formed by C1 – C4 glycosidic linkage and C1 – C6 glycosidic linkage. It constitutes about 80 – 85 % starch.

71. Answer (2)

Hint :

Fractional distillation is used to separate two liquids whose boiling points are very close to each other.

Sol. :

Mixture of methyl alcohol and acetone can be separated by fractional distillation.

72. Answer (2)

Hint :

$Na_4[Fe(CN)_5NOS]$ is sodium nitroprusside.

Sol. :

• Sodium nitroprusside is of purple colour

• $BaCO_3$ is white ppt.

73. Answer (3)

Hint :

Time period $(T) \propto \frac{n^3}{Z^2}$

Sol. :

$$\frac{T}{x} = \frac{2^3}{3^3} \times \frac{3^2}{2^2} = \frac{2}{3}$$

$$T = \frac{2}{3}x$$

74. Answer (2)

Hint :

Both chromate and dichromate ions are in equilibrium at pH = 4.

Sol. :

In alkaline solution, chromate ions are present while in acidic solution, dichromate ions are present

$$\left[2CrO_4^{2-} + 2H^+ \longrightarrow Cr_2O_7^{2-} + H_2O\right]$$

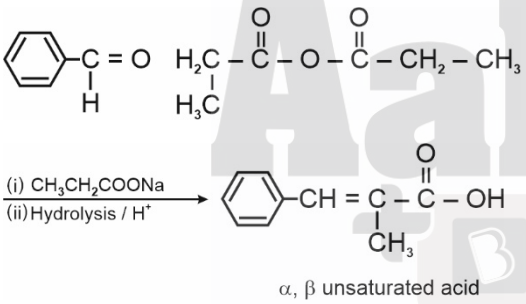
75. Answer (2)

Hint :

It is Perkin reaction.

Sol. :

Acid anhydride having α-hydrogen gives Perkin reaction with aromatic aldehydes in the presence of salt of the acid.



α, β unsaturated acid

76. Answer (4)

Hint :

Cr^{2+} acts as reducing agent as its configuration changes from d^4 to d^3 that having half – filled t_{2g} level.

Sol. :

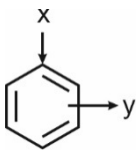
- In aqueous medium Cu^{2+} is more stable due to its high hydration energy than Cu^+ which compensate IE_2.
- Fe^{2+} have $3d^6$ configuration have 4 unpaired electron.

77. Answer (3)

Hint :

$$\mu = \sqrt{\mu_1^2 + \mu_2^2 - 2\mu_1\mu_2\cos\theta}$$

Sol. :



$\therefore$ dipole moment follows the order

$\therefore$ p-derivative > m-derivative > o-derivative

78. Answer (4)

Hint :

$$\text{Hydrated radius} \propto \frac{\text{charge}}{\text{size}}$$

Sol. :

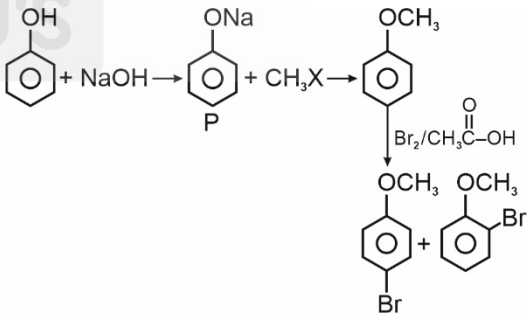
More the electronegativity difference more will be bond polarity.

79. Answer (3)

Hint :

$-OCH_3$ is o– and p–directing.

Sol. :



80. Answer (4)

Hint :

If reaction is added then their equilibrium constant is multiplied and if reaction is subtracted then their equilibrium constant value is divided.

Sol. :

$\frac{1}{2}N_2 + O_2 \longrightarrow NO_2;\ k_1$

$NO_2 \rightleftharpoons NO + \frac{1}{2}O_2;\ (k_2)^{\frac{1}{2}}$

$NO + \frac{1}{2}Br_2 \rightleftharpoons NOBr;\ \frac{1}{k_3}$

$\frac{1}{2}N_2 + \frac{1}{2}O_2 + \frac{1}{2}Br_2 \longrightarrow NOBr;\ k = \frac{k_1 \cdot (k_2)^{\frac{1}{2}}}{k_3}$

81. Answer (23.00)

Hint :

$\log K_C = \frac{nE^o_{cell}}{0.06}$

Sol. :

For n = 1

$\therefore\ E^o_{cell} = \log 10^{20} \times 0.06$

= 20 × 0.06 = 1.2 V

$\Delta G^o_r = -nFE^o_{cell} = -1 \times 96500\ C \times 1.2\ V$

= –115.8 kJ mol^{-1}

82. Answer (00.00)

Hint :

CFSE depends on number of t_{2g} & eg electrons

Sol. :

High spin complex $[FeCl_6]^{3-}$ has the d configuration as $t^3_{2g}e^2_g$.

CFSE: $3(-0.4)\Delta_0 + 2(0.6)\Delta_0 = 0$

83. Answer (10.00)

Hint :

$t_{1/2} = \frac{0.693}{k}$

Sol. :

Using = $\ln\frac{[SO_2Cl_2]_t}{[SO_2Cl_2]_0} = -kt$

$2.303 \times \log\frac{10}{5} = -k \times 10$

$\frac{2.303 \times 0.3010}{10} = k$

$0.693 \times 10^{-1}\ s^{-1} = k$

$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{0.693 \times 10^{-1}} = 10\,s$

84. Answer (04.00)

Hint :

SiF_4 have sp^3 hybridization and $SbCl_5^{2-}$ have sp^3d^2 hybridization.

Sol. :

$SF_4, I_3^-, PCl_5, XeF_2$ have sp^3d hybridisation

85. Answer (03.00)

Hint :

The correct statements are A, B, C

Sol. :

- At equilibrium, the rate of forward reaction becomes equal to the rate of backward reaction and hence the equilibrium is dynamic in nature.
- The equilibrium can be attained from either direction i.e. from the direction of reactants as well as from the direction of products.
- A catalyst does not change the state of equilibrium. This is because a catalyst increases the speed of forward reaction as well as of the backward reaction to same extent.

86. Answer (13.00)

Hint :

Pyrophosphoric acid is produced on heating phosphoric acid.

Sol. :

Pyrophosphoric acid = $H_4P_2O_7$

87. Answer (15.00)

Hint :

$\Delta T_b = ik_b \times m$

Sol. :

As $(101.536 - 100)°C = i \times 0.512\ K\ kg\ mol^{-1} \times 2$

$\frac{1.536}{0.512 \times 2} = i$

$Y = 1.5$

$10Y = 15.00$

88. Answer (03.00)

Hint :

Statements A, B, E are correct.

Sol. :

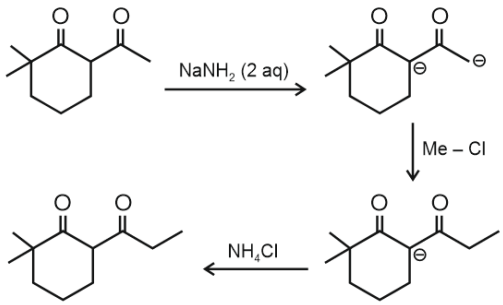
- Bohr's model could explain the line spectra of one-electron species only.
- It could not explain the shape of molecule.

89. Answer (01.00)

Hint :

$NaNH_2$ abstract acidic hydrogen.

Sol. :



Number of 4° carbon = 1

90. Answer (06.00)

Hint :

Use hybridisation concept

Sol. :

SO_2, NH_3, H_2O, SF_4, ClF_3, IF_5 contain lone pair(s) on their respective central atom.

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