All India Aakash Test Series for JEE (Main)-2024

TEST - 3

Test Date : 21/01/2024

ANSWERS

MATHEMATICS	PHYSICS	CHEMISTRY
1. (4)	31. (4)	61. (1)
2. (4)	32. (2)	62. (4)
3. (4)	33. (3)	63. (2)
4. (1)	34. (2)	64. (2)
5. (4)	35. (2)	65. (2)
6. (4)	36. (2)	66. (2)
7. (3)	37. (4)	67. (3)
8. (3)	38. (3)	68. (3)
9. (4)	39. (3)	69. (1)
10. (1)	40. (2)	70. (3)
11. (2)	41. (4)	71. (4)
12. (1)	42. (1)	72. (2)
13. (4)	43. (3)	73. (4)
14. (3)	44. (4)	74. (3)
15. (2)	45. (1)	75. (4)
16. (4)	46. (2)	76. (4)
17. (4)	47. (2)	77. (1)
18. (2)	48. (1)	78. (3)
19. (3)	49. (4)	79. (3)
20. (4)	50. (3)	80. (4)
21. (03.00)	51. (15.00)	81. (25.00)
22. (12.00)	52. (20.00)	82. (09.00)
23. (02.00)	53. (20.00)	83. (03.00)
24. (40.00)	54. (15.00)	84. (06.00)
25. (01.00)	55. (07.00)	85. (12.00)
26. (08.00)	56. (05.00)	86. (03.00)
27. (02.00)	57. (14.00)	87. (05.00)
28. (16.00)	58. (11.95)	88. (10.00)
29. (48.00)	59. (10.00)	89. (00.00)
30. (02.00)	60. (05.00)	90. (05.00)

PART - A (MATHEMATICS)

1. Answer (4)

Hint : Use substitution method.

Sol. : $\int \frac{2^x}{\sqrt{1-4^x}}dx$

Put $t = 2^x$

$dt = 2^x \ln 2dx$

$\Rightarrow 2^x dx = \frac{dt}{\ln 2}$

$\therefore \int \frac{2^x}{\sqrt{1-4^x}}dx = \frac{1}{\ln 2}\int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{1}{\ln 2}\sin^{-1}(2^x) + c$

$\therefore k = \frac{1}{\ln 2}$

2. Answer (4)

Hint : $f(-x) = -f(x)$, $f(x)$ is odd function.

Sol. : Let $f(x) = \frac{\sin^5 x\cos^3 x}{x^4}$

$f(-x) = \frac{\sin^5(-x)\cos^3(-x)}{(-x)^4} = \frac{-\sin^5 x\cos^3 x}{x^4}$

$f(-x) = -f(x)$

$\therefore$ $f(x)$ is odd function.

$\therefore \int_{-\pi/6}^{\pi/6} \frac{\sin^5 x\cos^3 x}{x^4} = 0$

3. Answer (4)

Hint : Co-efficient of x^r in $(1-x)^{-n} = {}^{r+n-1}C_{n-1}$

Sol. : $\therefore \vec{r}\cdot\vec{u} = 20$

$\Rightarrow x + y + z = 20$ ...(i)

$\therefore$ Total number of possible positions of point P

= Number of positive integral solutions of equation (i)

$= {}^{20-1}C_{3-1} = {}^{19}C_2 = \frac{19!}{2!\times 17!} = \frac{19\times 18\times 17!}{2\times 1\times 17!}$

$= 171$

4. Answer (1)

Hint : $(\vec{a}+\vec{b})\cdot\vec{c} = |\vec{a}+\vec{b}||\vec{c}|\cos\theta$

Sol. : Let $\vec{b} = p\hat{i} + q\hat{j} + r\hat{k}$

$\vec{a}\cdot\vec{b} = 27$

$\therefore 3p + 3q + 3r = 27$ …(i)

$\vec{a}\times\vec{b} = 9\vec{c}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 3 \\ p & q & r \end{vmatrix} = 9(\hat{j}-\hat{k})$$

$\hat{i}(3r-3q) - \hat{j}(3r-3p) + \hat{k}(3q-3p) = 9\hat{j} - 9\hat{k}$

On comparing,

$3r - 3q = 0 \Rightarrow r = q$ …(ii)

$3r - 3p = -9$

$r - p = -3$ …(iii)

$3q - 3p = -9$

$q - p = -3$ …(iv)

From (i), (ii), (iii) and (iv),

$p = 5, q = 2, r = 2$

$\therefore \vec{b} = 5\hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore \vec{a}+\vec{b} = 8\hat{i} + 5\hat{j} + 5\hat{k}$

Let θ be the angle between $(\vec{a}+\vec{b})$ and $\vec{c}$

$\therefore (\vec{a}+\vec{b})\cdot\vec{c} = |\vec{a}+\vec{b}||\vec{c}|\cos\theta$

$\Rightarrow 0+5-5 = \sqrt{8^2+5^2+5^2}\sqrt{1^2+(-1)^2}\cos\theta$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

5. Answer (4)

Hint : $\vec{r} = \vec{a} + \lambda(\vec{b}-\vec{a})$

Sol. : $\vec{r} = \vec{a} + \lambda(\vec{b}-\vec{a})$

$\vec{a} = -\hat{i} + 2\hat{k}$

$\vec{b} = 3\hat{i} + 4\hat{j} + 6\hat{k}$

$\therefore \vec{b}-\vec{a} = 4\hat{i} + 4\hat{j} + 4\hat{k}$

$\therefore \vec{r} = -\hat{i} + 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

6. Answer (4)

Hint : $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$

Sol. : $P\left(\dfrac{A \cap B}{\bar{A} \cup \bar{B}}\right) = P\left(\dfrac{A \cap B}{\overline{A \cap B}}\right) = 0$

7. Answer (3)

Hint : $\sigma^2 = \dfrac{\sum(x_i - \bar{x})^2}{n}$

Sol. : Variance, $\sigma^2 = \dfrac{\sum(x_i - \bar{x})^2}{n}$

If every number is multiplied by a number, new variance = (number)^2 × old variance.

$\bar{x} = 4, \ \sigma^2 = 2$

Number of observations, $n = 10$

New mean = 2(mean) = 2 × 4 = 8

New variance = $(2)^2 \times 2 = 8$

8. Answer (3)

Hint : Required probability

$= P(3H) + P(4H) + P(5H) + P(6H)$

Sol. : Probability of getting at least 3 heads = probability of getting 3 head + probability of getting 4 heads + probability of getting 5 heads + probability of getting 6 heads

$= \dfrac{({}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6)}{2^6}$

$= \dfrac{42}{64} = \dfrac{21}{32}$

9. Answer (4)

Hint : (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) are linearly dependent when $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$.

Sol. : (2, 3, –1), (–4, 2, –6), (5, –4, 9)

$$\begin{vmatrix} 2 & 3 & -1 \\ -4 & 2 & -6 \\ 5 & -4 & 9 \end{vmatrix}$$

$= 2(18 - 24) - 3(-36 + 30) - 1(16 - 10)$

$= 0$

$\therefore$ Vectors (2, 3, –1), (–4, 2, –6), (5, –4, 9) are linearly dependent.

10. Answer (1)

Hint : Area = $\int_0^4 (y_1 - y_2)\,dx$

Sol. : $y^2 = x$ and $2y = x$

$y^2 = 2y$

$y = 0, 2$

When $y = 0$, $x = 0$

$y = 2$, $x = 4$

Point of intersections are (0, 0) and (4, 2)

$$\text{Area} = \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{1}{2}\frac{x^2}{2}\right]_0^4$$

$$= \frac{2}{3}(4)^{3/2} - \frac{1}{2}\cdot\frac{4^2}{2} - 0$$

$$= \frac{16}{3} - 4 = \frac{4}{3} \text{ sq. units}$$

11. Answer (2)

Hint : Put $1 + x^3 = t$

Sol. : Put $1 + x^3 = t \Rightarrow x^2dx = \dfrac{1}{3}dt$

$$I = \frac{1}{3}\int \frac{dt}{\sqrt{t + t^{3/2}}} = \frac{1}{3}\int \frac{dt}{\sqrt{t}\cdot\sqrt{1+\sqrt{t}}}$$

$$= \frac{4}{3}\sqrt{1+\sqrt{t}} + C$$

$$= \frac{4}{3}\sqrt{1+\sqrt{1+x^3}} + C$$

12. Answer (1)

Hint : Let intersection point is $(\lambda, \lambda, \lambda)$.

Sol. : $\overrightarrow{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + (1+3)\hat{k}$

Let $(\lambda, \lambda, \lambda)$ be the point of intersection of given line.

$\therefore \quad \lambda(\sin A + \sin B + \sin C) = 2d^2 \qquad$ ...(i)

and $\lambda(\sin 2A + \sin 2B + \sin 2C) = d^2 \qquad$ ...(ii)

Divide (ii) by (i),

$$\frac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C}=\frac{1}{2}$$

$$\frac{2\sin\frac{2(A+B)}{2}\cos\frac{2(A-B)}{2}+2\sin C\cdot\cos C}{2\sin\frac{A+B}{2}\cdot\cos\frac{A-B}{2}+2\sin\frac{C}{2}\cdot\cos\frac{C}{2}}=\frac{1}{2}$$

$$\frac{2\sin(\pi-C)\cdot\cos(A-B)+2\sin C\cdot\cos C}{2\sin\left(90^\circ-\frac{C}{2}\right)\cdot\cos\frac{A-B}{2}+2\sin\left(90^\circ-\frac{A+B}{2}\right)\cdot\cos\frac{C}{2}}=\frac{1}{2}$$

$$\Rightarrow\quad \frac{2\sin C\left[\cos(A-B)+\cos(\pi-(A+B))\right]}{2\cos\frac{C}{2}\left[\cos\frac{A-B}{2}+\cos\frac{A+B}{2}\right]}=\frac{1}{2}$$

$$\Rightarrow\quad \frac{2\sin C\left[\cos(A-B)-\cos(A+B)\right]}{2\cos\frac{C}{2}\times 2\cos\frac{A}{2}\cdot\cos\frac{B}{2}}=\frac{1}{2}$$

$$\Rightarrow\quad \frac{2\sin C\times 2\sin A\sin B}{4\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\cos\frac{C}{2}}=\frac{1}{2}$$

$$\frac{2\sin\frac{A}{2}\cdot\cos\frac{A}{2}\cdot 2\sin\frac{B}{2}\cdot\cos\frac{B}{2}\cdot 2\sin\frac{C}{2}\cdot\cos\frac{C}{2}}{\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\cos\frac{C}{2}}=\frac{1}{2}$$

$$\Rightarrow\quad \sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}=\frac{1}{16}$$

$$\Rightarrow\quad \frac{4}{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}=\frac{4}{\frac{1}{16}}=64$$

13. Answer (4)

Hint : Put $\sin\theta + \cos\theta = t$

Sol. : Put $\sin\theta + \cos\theta = t$

$\Rightarrow\quad (\cos\theta - \sin\theta)d\theta = dt$

$$I=\int\frac{-dt}{t\sqrt{\frac{t^2-1}{2}+\left(\frac{t^2-1}{2}\right)^2}}$$

$$=-2\int\frac{dt}{t\sqrt{t^4-1}}$$

$$=-2\int\frac{t^3\,dt}{t^4(t^4-1)^{1/2}}$$

Put $t^4 - 1 = u^2 \Rightarrow 4t^3\,dt = 2u\,du$

$$I=-\int\frac{u\,du}{u\cdot(u^2+1)}=-\int\frac{du}{u^2+1}$$

$$=-\tan^{-1}u+c$$

$$=-\tan^{-1}\sqrt{(t^4-1)}+C$$

$$=-\tan^{-1}\sqrt{(\sin\theta+\cos\theta)^4-1}+C$$

$p = -1,\ q = 4,\ r = -1$

14. Answer (3)

Hint : Shortest distance = $\left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{n}_1\times\vec{n}_2)}{|\vec{n}_1\times\vec{n}_2|}\right|$

Sol. : Shortest distance

$$=\frac{\left|((\hat{i}-2\hat{j}-\hat{k})-(\hat{i}+2\hat{j}-\hat{k}))\cdot((4\hat{i}-3\hat{k})\times(2\hat{i}-4\hat{j}-5\hat{k}))\right|}{\left|(4\hat{i}-3\hat{k})\times(2\hat{i}-4\hat{j}-5\hat{k})\right|}$$

$$=\left|\frac{(-4)14}{\sqrt{12^2+14^2+16^2}}\right|$$

$$=\frac{56}{\sqrt{596}}=\frac{28}{\sqrt{149}}$$

15. Answer (2)

Hint : $x\,dx+y\,dy=\frac{1}{2}d(x^2+y^2)$

Sol. : Given diff. equation is

$(x^2 + y^2 + x)dx - (2x^2 + 2y^2 - y)dy = 0$

$\Rightarrow\quad (x^2 + y^2)(dx - 2dy) + xdx + ydy = 0$

$$\Rightarrow\quad \int 2(dx-2dy)+\int\frac{2(xdx+ydy)}{x^2+y^2}=0$$

$\Rightarrow\quad 2x - 4y + \log_e(x^2 + y^2) = c$

16. Answer (4)

Hint : $\vec{a}, \vec{b}$ and $\vec{c}$ are perpendicular to each other.

Sol. : $|(\vec{a}\times\vec{b})\cdot\vec{c}|=|\vec{a}||\vec{b}||\vec{c}|$ will hold only if $\vec{a}, \vec{b}, \vec{c}$ are perpendicular to each other.

i.e., $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

17. Answer (4)

Hint : $\frac{y\,dx-x\,dy}{y^2}=d\left(\frac{x}{y}\right)$

Sol. : From given equation,

$$\frac{xdx + ydy}{ydx - xdy} = \frac{x\sin^2(x^2+y^2)}{y^3}$$

$$\Rightarrow \int \frac{2(x\,dx + y\,dy)}{\sin^2(x^2+y^2)} = 2\frac{x}{y} \times \left(\frac{y\,dx - x\,dy}{y^2}\right)$$

$$= \int 2\left(\frac{x}{y}\right) d\left(\frac{x}{y}\right)$$

$$\Rightarrow \int \frac{dt}{\sin^2 t} = \frac{2\left(\frac{x}{y}\right)^2}{2}$$

Put $x^2 + y^2 = t$ in 1st integral

$\Rightarrow$ $2(xdx + ydy) = dt$

$$\Rightarrow -\cot t + c = \frac{x^2}{y^2}$$

$$\Rightarrow \frac{x^2}{y^2} + \cot(x^2+y^2) = c$$

18. Answer (2)

Hint : Mean deviation from mean = $\frac{1}{n}\Sigma|x - \bar{x}|$

Sol. : $n = 7$

$$\bar{x} = \frac{2+9+9+3+6+9+4}{7} = 6$$

Mean deviation from mean

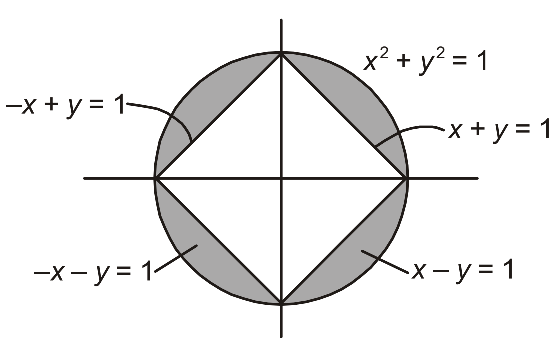
$$= \frac{1}{n}\Sigma|x - \bar{x}|$$

$$= \frac{4+3+3+3+0+3+2}{7} = \frac{18}{7} = 2.57$$

19. Answer (3)

Hint : Draw graph.

Sol. :



Required area = Area of circle – Area of square

$= \pi(1)^2 - (\sqrt{2})^2$

$= (\pi - 2)$ sq. units

20. Answer (4)

Hint : $\because$ Events are independent.

$\therefore$ $P(A \cap B) = P(A) \times P(B)$

Sol. : Odds in favour of A and B are 1 : 1 and 3 : 2.

$$P(A) = \frac{1}{2}, \quad P(B) = \frac{3}{5}$$

$P(A \cap B) = P(A) \times P(B)$

{$\because$ Events are independent}

$$= \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$$

Probability that only one of the two events occurs

$= [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)]$

$$= \left(\frac{1}{2} - \frac{3}{10}\right) + \left(\frac{3}{5} - \frac{3}{10}\right)$$

$$= \frac{1}{2}$$

21. Answer (03.00)

Hint : Area = $\int_0^{1/a} (y_2 - y_1)dx$

Sol. : $x = ay^2$ and $y = ax^2$

$a^3x^3 = 1$

$$\Rightarrow (x, y) = \left(\frac{1}{a}, \frac{1}{a}\right)$$

$$\text{Area} = \int_0^{1/a} \left|ax^2 - \sqrt{\frac{x}{a}}\right| dx = 1$$

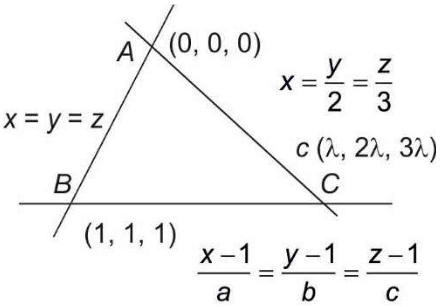
$$\Rightarrow \frac{1}{3a^2} = 1$$

$$\Rightarrow \frac{1}{a^2} = 3$$

22. Answer (12.00)

Hint : Find Δ_x, Δ_y, Δ_z then use $\Delta^2 = \Delta_x^2 + \Delta_y^2 + \Delta_z^2$

Sol. :



$$\Delta x = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 2\lambda & 3\lambda & 1 \end{vmatrix} = \frac{\lambda}{2}$$

$$\Delta y = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 3\lambda & \lambda & 1 \end{vmatrix} = \lambda$$

$$\Delta z = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ \lambda & 2\lambda & 1 \end{vmatrix} = \frac{\lambda}{2}$$

$$\Delta^2 = \Delta_x^2 + \Delta_y^2 + \Delta_z^2$$

$$6 = \frac{\lambda^2}{4} + \lambda^2 + \frac{\lambda^2}{4} \Rightarrow \lambda = \pm 2$$

$\therefore\ \alpha + \beta + \gamma = 2 + 4 + 6 = 12$

23. Answer (02.00)

Hint : Use substitution method and ILATE.

Sol. : Let $x^2 = t$

$dt = 2xdx$

$$\therefore\ \int \frac{1}{2} te^t dt = \frac{1}{2}\left[t\int e^t - \int \frac{d(t)}{dt}\int e^t\right)dt\right]$$

$$= \frac{1}{2}\left[te^t - \int e^t dt\right]$$

$$= \frac{1}{2}(te^t - e^t) + c$$

$$= \frac{1}{2}(x^2 e^{x^2} - e^{x^2}) + c$$

$$= \frac{1}{2}(x^2 - 1)e^{x^2} + c$$

$\therefore\ k = 2$

24. Answer (40.00)

Hint : COV = $\frac{\sigma}{\bar{x}} \times 100$

Sol. : $\sigma = 20$, COV = 50%

COV = $\frac{\sigma}{\bar{x}} \times 100$

$$50 = \frac{20}{\bar{x}} \times 100$$

$$\bar{x} = 40$$

25. Answer (01.00)

Hint : $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Sol. : $I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\cot x}} dx$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \ldots(i)$$

Now, $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$\therefore\ I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2 - x)} + \sqrt{\cos(\pi/2 - x)}} dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \ldots(ii)$$

From (i) + (ii),

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$= \int_0^{\pi/2} dx = [x]_0^{\pi/2} = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

$$\therefore\ \tan k = \tan\frac{\pi}{4} = 1$$

26. Answer (08.00)

Hint : Probability = $\frac{\text{Favourable cases}}{\text{Total cases}}$

Sol. : Total fruits = 6

Total possible ways = ${}^6C_2 = 15$

Favourable ways = ${}^3C_1 \times {}^3C_1 = 9$

Probability = $\frac{9}{15} = \frac{3}{5}$

$\therefore \quad a + b = 3 + 5 = 8$

27. Answer (02.00)

Hint : Use variable separable method.

Sol. : $x^2 \frac{dy}{dx} = 2$

$$\int dy = \int \frac{2}{x^2} dx$$

$$y = -\frac{2}{x} + c$$

$\therefore \quad k = 2$

28. Answer (16.00)

Hint : Area = $\int_2^3 y\,dx$

Sol. : Area $= \int_2^3 x^2 dx$

$$= \left[\frac{x^3}{3}\right]_2^3$$

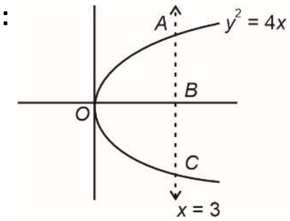
$$= \frac{(3)^3}{3} - \frac{(2)^3}{3} = \frac{19}{3}$$

$\therefore \quad \lambda - \mu = 19 - 3 = 16$

29. Answer (48.00)

Hint : Area = $\frac{1}{2}$ (area of OAB)

Sol. :



Required area = Area of $OACO$

= 2(area of OAB)

$$= 2\int_0^3 2\sqrt{x}\,dx$$

$$= \frac{8}{3}(3)^{3/2}$$

$$= 8\sqrt{3}$$

$\therefore \quad \lambda^2 = (8\sqrt{3})^2 = 192$

30. Answer (02.00)

Hint : Squaring both side.

Sol. : $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 4x$

$$1 + \left(\frac{dy}{dx}\right)^2 = 16x^2$$

Degree = 2

PART - B (PHYSICS)

31. Answer (4)

Hint : $\tau = -I\alpha$

Sol. : $\tau = -I\omega^2\theta \Rightarrow -MB\theta = -I\omega^2\theta$

$$\Rightarrow T^2 \propto \frac{1}{B} \Rightarrow \frac{T_f^2}{\left(\frac{60}{30}\right)^2} = \frac{1}{2} \Rightarrow T_f = \sqrt{2}$$

32. Answer (2)

Hint : Consider three lenses are in contact

Sol. : R = 20 cm

$$\therefore \quad \frac{1}{f_2} = (1.3 - 1)\left(\frac{-2}{20}\right) = \frac{-3}{100}$$

$$\therefore \quad \frac{1}{f_e} = \frac{1}{20} - \frac{3}{100} \Rightarrow f_e = 50 \text{ cm}$$

33. Answer (3)

Hint : $P_{avg} = \frac{V_{rms}^2}{R}$

Sol. : $V_{rms}^2 = \frac{2^2 + 1^2}{2} = \frac{5}{2}$

$$P_{avg} = \frac{5}{2 \times 20} = 0.125 \text{ W}$$

34. Answer (2)

Hint : Pressure due to radiation = $\frac{I}{C}$, where I is intensity.

Sol. : $F_{av} = \frac{2I\,A_0 \sin\theta}{c}$, $S = \frac{A_0}{\sin\theta}$

$\therefore \quad P = \frac{F_{av}}{S} = \frac{2I\sin^2\theta}{c}$

35. Answer (2)

Hint : $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Sol. : $\frac{1}{v} - \frac{1}{15} = -\frac{1}{10} \Rightarrow v = -30$ cm

$\because \quad \frac{v_p}{v^2} = \frac{u_p}{u^2} \Rightarrow v_p = -20\hat{i}$

36. Answer (2)

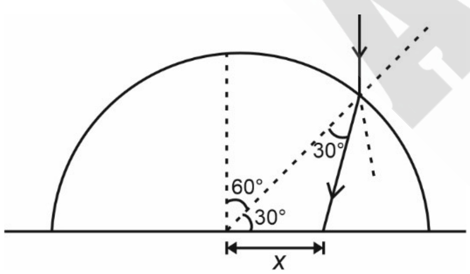
Hint : $\vec{E}\cdot\vec{B} = 0$

Sol. : $\vec{E}$ and $\vec{B}$ are always in same phase.

37. Answer (4)

Hint : Apply Snell's law and trace the path of refracted ray.

Sol. : $\frac{\sin 60^\circ}{\sin r} = \sqrt{3} \Rightarrow r = 30^\circ$



$\frac{\sin 30^\circ}{x} = \frac{\sin 120^\circ}{R}$

$x = \frac{R}{\sqrt{3}}$

38. Answer (3)

Hint : Tube length = $f_0 + f_e$

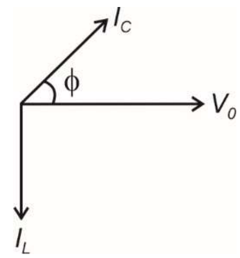
Sol. : $L = 20 + 5 = 25$

39. Answer (3)

Hint : Draw phase diagram of current in individual branch.

Sol. : $Z_L = X_L = \frac{2R}{\sqrt{3}}$

$Z_{R-C} = \sqrt{X_C^2 + R^2} = \frac{2R}{\sqrt{3}}$



$I_L = I_{R-C} = \frac{\sqrt{3}V_0}{2R}$

Also, $\tan\phi = \frac{X_C}{R} = \frac{1}{\sqrt{3}}$

$\Rightarrow \phi = 30°$

Power factor = cos30° = $\frac{\sqrt{3}}{2}$

40. Answer (2)

Hint : Ring rotates due to induced electric field.

Sol. : $E2\pi R = \frac{B\pi R^2}{\Delta t} \Rightarrow E = \frac{BR}{2\Delta t}$

$\because \quad \tau\Delta t = QER\Delta t = I\omega$

$\Rightarrow \quad \omega = \frac{QBR^2}{2(mR^2)} \Rightarrow \omega = \frac{QB}{2m}$

41. Answer (4)

Hint : At resonance, $X_C = X_L$

Sol. : $Z = R$ and $X_C = X_L$

$\Rightarrow$ Phase diff. = 0

42. Answer (1)

Hint : Apply KVL and KCL.

Sol. : $10 - 4 - 1.2\,I_2 = 0 \Rightarrow I_2 = 5$ mA

$4 - I_1 = 0 \Rightarrow I_1 = 4$ mA

$I_Z = I_2 - I_1 = 1$ mA

43. Answer (3)

Hint : $U = -\vec{M}\cdot\vec{B}, \vec{L} = \vec{M}\times\vec{B}$

Sol. : $W = \frac{MB}{2}$

$\tau = \frac{MB\sqrt{3}}{2} = \sqrt{3}W$

44. Answer (4)

Hint : Reading = MSR + $n(LC)$

Sol. : $LC = \frac{0.5}{100} = 0.005$ mm

Reading = 3 + 40 × 0.005 mm = 3.200 mm

45. Answer (1)

Hint : $h\nu = \phi + eV_0$

Sol. : $E = \frac{hc}{\lambda} = 6.2\text{ eV} > \phi$ (emission occurs)

$\Rightarrow V_0 = 1\text{ V} \Rightarrow \frac{ne}{4\pi\varepsilon_0 r} = 1$

$\Rightarrow n = \frac{2\times10^{-2}}{9\times10^9\times1.6\times10^{-19}} = 1.4\times10^7$

46. Answer (2)

Hint : $\frac{1}{2}mv^2 = \frac{K60e^2}{d}$

Sol. : $(mv)^2 = \frac{120Ke^2m}{d}$

$\Rightarrow \lambda = h\sqrt{\frac{d}{120mKe^2}} = \frac{h}{e}\sqrt{\frac{d}{120mK}}$

47. Answer (2)

Hint : Fringes formed will be symmetric on screen.

Sol. : Number of fringes are $\left[\frac{d}{\lambda}\right] = 5$

Shape of fringes will be circular.

48. Answer (1)

Hint : $E \propto \frac{1}{n^2}$

Sol. : $E_n - E_{n-1} = K\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$

$\Rightarrow \frac{hc}{\lambda} = \frac{2n-1}{n^2(n-1)^2} \Rightarrow \lambda \propto \frac{n^4}{n}$ (for $n >> 1$)

$\Rightarrow \lambda \propto n^3$

49. Answer (4)

Hint : $y = \overline{\overline{(A+B)}\cdot(A+B)}$

Sol. : $y = (A+B) + \overline{(A+B)} = 1$

50. Answer (3)

Hint : For polarizing angle reflected and refracted rays are mutually perpendicular.

Sol. : $\frac{\sin\theta_P}{\cos\theta_P} = \mu = \frac{1}{\sin\theta_C}$

$\Rightarrow \cot\theta_P = \sin\theta_C$

51. Answer (15.00)

Hint : Rays should fall normally on the mirror.

Sol. : $\frac{1.5}{\infty} - \frac{1}{-30} = \frac{(1.5-1)}{R} \Rightarrow R = 15$ cm

52. Answer (20.00)

Hint : Image formed by one will behave as object for other.

Sol. : $\frac{1}{v} + \frac{1}{60} = \frac{1}{20} \Rightarrow v = +30$ cm (focal point of mirror)

$\Rightarrow$ Rays will be parallel to principal axis after reflection.

53. Answer (20.00)

Hint : $V_L + V_R = 0$ after short circuiting

Sol. : $I_0 = \frac{\varepsilon}{R} = 1\text{ A}$

$\tau = \frac{L}{R} = 5\text{ ms}$

$\therefore I = I_0e^{-t/\tau} \Rightarrow I = \frac{1}{e}\text{ A}$

$\therefore |V_L| = V_R = IR = \frac{20}{e}$

54. Answer (15.00)

Hint : $E_0 = cB_0$

Sol. : $E_0 = 3 \times 10^8 \times 50 \times 10^{-9} = 15$ V/m

55. Answer (07.00)

Hint : $E_{bn} = \dfrac{\Delta mc^2}{N}$

Sol. : $(BE)_N = [2m_p + 2m_n - m({}_2\text{He}^4)]\dfrac{C^2}{4}$

56. Answer (05.00)

Hint : Revolving electron can be equivalent to a current carrying loop.

Sol. : $B_c \propto \dfrac{v}{r^2}$

$r \propto n^2$

$v \propto \dfrac{1}{n}$

57. Answer (14.00)

Hint : For intensity to be $\dfrac{1}{4}$th, $\phi = 2n\pi \pm \dfrac{2\pi}{3}$

Sol. : $5\lambda\cos\theta = \left(5\lambda - \dfrac{\lambda}{3}\right)$

$\Rightarrow \quad 5\lambda\cos\theta = \dfrac{14\lambda}{3}$

$\Rightarrow \quad \theta = \cos^{-1}\left(\dfrac{14}{15}\right)$

58. Answer (11.95)

Hint : $E = E_{\text{product}} - E_{\text{reactants}}$

Sol. : $E = E_1 - 2E_2$

59. Answer (10.00)

Hint : $n_e n_h = n_i^2$

Sol. : $n_h\, n_e = (4 \times 10^{16})^2$

$= \dfrac{16 \times 10^{32}}{4 \times 10^{22}}$

$n_e = 4 \times 10^{10}$

60. Answer (05.00)

Hint : $\dfrac{hc}{\lambda} = (13.6\text{ eV})\left[\dfrac{1}{n_1^2} - \dfrac{n}{n_2^2}\right]$

Sol. : For Lyman series, shortest wavelength,

$\dfrac{hc}{\lambda_0} = (13.6\text{ eV})\left[\dfrac{1}{1} - 0\right]$ …(i)

For largest wavelength in Balmer series,

$\dfrac{hc}{\lambda} = (13.6\text{ eV})\left[\dfrac{1}{2^2} - \dfrac{1}{3^2}\right]$ …(ii)

From (i) and (ii),

$\dfrac{\lambda}{\lambda_0} = \dfrac{36}{5}$

PART - C (CHEMISTRY)

61. Answer (1)

Hint : Chromate can be converted into dichromate in acidic condition.

Sol. : $\underset{\text{Yellow}}{CrO_4^{2-}} \underset{\text{Basic}}{\overset{\text{Acidic}}{\rightleftharpoons}} \underset{\text{Orange}}{Cr_2O_7^{2-}}$

62. Answer (4)

Hint : Catalytic activity is due to variable valency.

Sol. : The transition metals and their compounds are known for catalytic activity. This activity is due to their ability to adopt multiple oxidation state and to form complex.

63. Answer (2)

Hint : Stronger the ligand more will be Δ_0.

Sol. : Δ_0, *i.e.*, crystal field splitting energy is directly proportional to ligand strength, order of ligand strength.

$CN^- > NH_3 > H_2O > F^-$

So, crystal field splitting energy order Q > R > S > P.

64. Answer (2)

Hint : π-acceptor are those which can accept electron after donation.

Sol. : π-acceptor ligands are non-classical ligand, which can accept electron either in *d*-orbital or π^*.

65. Answer (2)

Hint : Oxidation state of sulphur in SO_2 is +4.

Sol. : CO_2 is already the highest oxidised form, so it cannot react with oxidising agent such as MnO_4^- and $Cr_2O_7^{2-}$. SO_2 decolourises the purple $KMnO_4$ (acidified) that is used to confirm the presence of SO_2.

66. Answer (2)

Hint : $Pb(OH)_2$ is soluble in excess NaOH solution.

Sol. :

$$Hg_2^{2+}(aq) + 2OH^-(aq) \longrightarrow \underset{(Black)}{Hg_2O} + H_2O(l)$$

$$2Ag^+ + 2OH^- \longrightarrow \underset{(Brown)}{Ag_2O} + H_2O$$

$$Pb^{2+} + 2NaOH \longrightarrow \underset{White}{Pb(OH)_2}\downarrow + 2Na^+$$

$$Pb(OH)_2 + 2NaOH \longrightarrow \underset{(Soluble)}{Na_2[Pb(OH)_2]}$$

67. Answer (3)

Hint : Outermost electronic configuration of Ta is $5d^36s^2$.

Sol. : Ta = $[Xe]5d^36s^2$.

68. Answer (3)

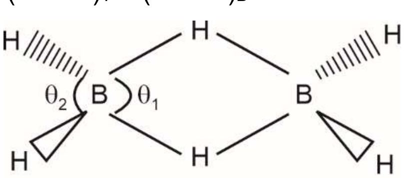
Hint : Given complex is square planar.

Sol. : In $[Cu(NH_3)_4]^{2+}$ complex ion, the four hybridised orbitals are arranged in square planar manner and experimentally it is found to be paramagnetic that can be explained by CFT.

69. Answer (1)

Hint : $(\angle HBH)_T > (\angle HBH)_B$

Sol. :



$\theta_1 = 90°$ $\theta_2 = 120°$

Maximum number of atoms in a plane in $Al_2(CH_3)_6$ is 10.

70. Answer (3)

Hint : Reaction occurs in acid base manner.

Sol. : $Na_2CO_3(aq) \xrightarrow{H_2SO_4} CO_2(g)$

$Na_2SO_3 \xrightarrow{H_2SO_4} SO_2(g)$

71. Answer (4)

Hint : AgCl is insoluble in aqua regia.

Sol. : A, B and C is Ag.

$$K_2S_2O_8 + 2AgNO_3 + 4KOH \longrightarrow \underset{(Black)}{2AgO} + 2KNO_3 + K_2SO_4 + 2H_2O$$

D is AgO, E is an insoluble substance.

72. Answer (2)

Hint : Kinoite is a single chain silicate.

Sol. :

○ Oxygen
◉ Silicon

Molecular formula of silicate anion.

$[Si_3O_{10}]^{8-}$

Charge of silicate anion = –8

73. Answer (4)

Hint : Tetrahedral compound cannot show geometrical isomerism.

Sol. :

(1) $Zn(Cl_2)(NH_3)_2 \Rightarrow sp^3$ (tetrahedral) cannot form cis/trans isomer.

(2) $[Zn(NH_3)_4]^{2+}$ is optically inactive due to presence of plane of symmetry.

(3) [PtClBr(Py)] $\Rightarrow$ optically inactive due to (square planar) presence of plane of symmetry.

(4) Brown ring complex is paramagnetic. It has 3 unpaired electrons and +1 oxidation state of Fe.

74. Answer (3)

Hint : Boron shows anomalous behaviour due to small size and high charge/size ratio.

Sol. : Al can also form Nitride at high temperature.

$2Al + N_2 \longrightarrow 2AlN$

75. Answer (4)

Hint : Both $PbCl_2$ and AgCl are soluble in excess of conc. HCl. So, cannot be separated.

Sol. : $PbCl_2(s) + \text{Conc. HCl} \longrightarrow PbCl_4^{2-}(aq)$

$AgCl(s) + \text{Conc. HCl} \longrightarrow [AgCl_2]^-(aq)$

76. Answer (4)

Hint : The decrease in atomic radii of lanthanide metals is not regular from left to right.

Sol. : $Ln \xrightarrow{\text{Acid}} H_2$

Ce^{4+} has zero 4*f* electrons.

77. Answer (1)

Hint : Actinides have higher tendency to form complexes.

Sol. : Actinides have more tendency to form complexes due to high charge density.

78. Answer (3)

Hint : π = iCRT, we have to check van't Hoff factor.

Sol. : According to Werner's theory, $CoCl_3{\cdot}6NH_3$ is $[Co(NH_3)_6]Cl_3$, when dissolved in water produces 4 ions, more the number of solute particles more will be osmotic pressure.

79. Answer (3)

Hint : For σ-donor use η^1, for π-donor use η^5 for cyclopentadienyl anion.

Sol. : $[Ti(\sigma{-}C_5H_5)_2(\pi{-}C_5H_5)_2]$

$\Rightarrow$ Bis(η^1–cyclopentadienyl) bis (η^5–cyclopenta-dienyl) Titanium(IV)

80. Answer (4)

Hint : Group II basic radicals.

Sol. : HCl (dil.) + H_2S is a group II reagent and form precipitate with Cu^{2+}, Cd^{2+}, Pb^{2+}, Hg^{2+}, Bi^{3+}, As^{3+}, Sb^{3+} and Sn^{2+}.

81. Answer (25.00)

Hint : Ions outside the ionisation sphere is free in solution.

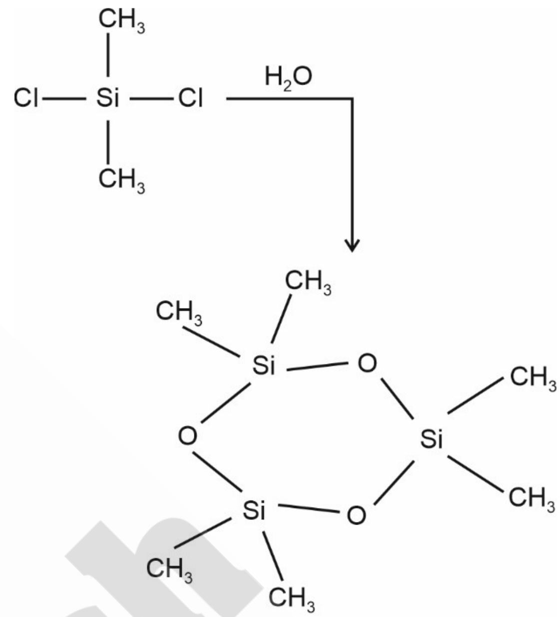
Sol. : 1 Cl^- can form precipitate with $AgNO_3$ solution out of 4.

So, % Cl^- precipitated = $\frac{1}{4} \times 100 = 25\%$

82. Answer (09.00)

Hint : Trimeric form is cyclic form.

Sol. :



lone pair = 6
Si – O – Si = 3

83. Answer (03.00)

Hint : Lead salt turn black due to formation of PbS by action of H_2S in atmosphere.

Sol. : (a), (b) and (e) are true statements.

$Ba(NO_3)_2$ is more soluble than $BaCl_2$, so solution formation is easy.

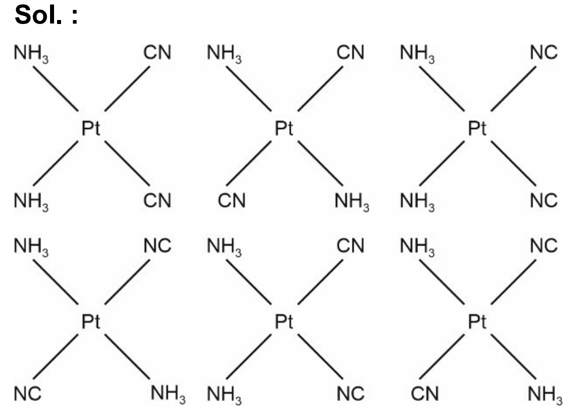
Cobalt chloride and ammonium thiocyanate forms cobalt thiocyanate

$Hg^{2+} + 2Co(SCN)_2 \rightarrow Co[Hg(SCN)_4]$ deep blue

84. Answer (06.00)

Hint : CN^- is an ambidentate ligand.

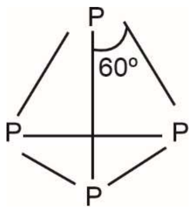
Sol. :



85. Answer (12.00)

Hint : All bond angles are of 60°.

Sol. :



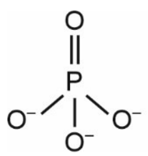
All form equilateral triangle

60° angle = 12.

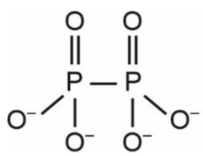
86. Answer (03.00)

Hint : Given anions are anions of oxyacids of phosphorus and sulphur.

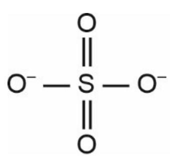
Sol. :



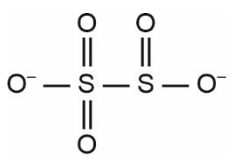
All the four P – O bonds are equivalent due to resonance.



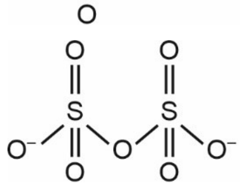
All P – O bonds are equivalent, hence having three equivalent X – O bonds per central atom.



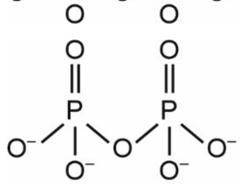
All the four S – O bonds are equivalent.



Not all S – O bonds are equivalent, 3 equivalent S – O bonds are present with only one central atom.



Not all S – O bonds are equivalent, but 3S – O equivalent bonds are per central atom.



(sigma bond)

3 equivalent P – O bonds per central atom.

87. Answer (05.00)

Hint : EAN = Atomic number of metal – O.S + (2 × C.N.)

Sol. : EAN = 26 – 0 + (2 × x) = 36

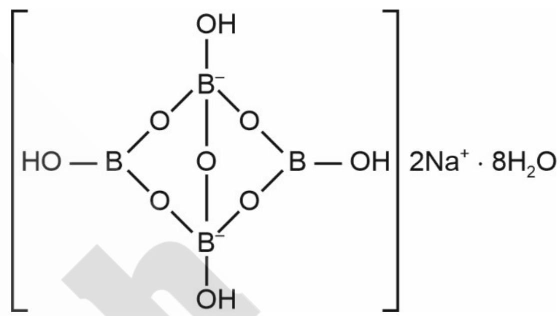
26 + 2x = 36

2x = 10

x = 5

88. Answer (10.00)

Hint : Structure of borax



Sol. : x = 2

y = 4

z = 5

$$\frac{y \times z}{x} = \frac{5 \times 4}{2} = \frac{20}{2} = 10$$

89. Answer (00.00)

Hint : $\mu = \sqrt{n(n+2)}$ B.M.

Sol. : In $[Co(H_2O)_6]^{3+}$, unpaired electron in $Co^{3+} = 0$

In $[Co(NH_3)_6]^{3+}$, unpaired electron in $Co^{3+} = 0$

Change in magnetic moment = 0

90. Answer (05.00)

Hint : Due to synergic bonding, stretching frequency of CO bond decreases.

Sol. : In all of the given carbonyl complexes, the synergic bonding is present.

❒ ❒ ❒