

All India Aakash Test Series for JEE (Advanced)-2024

TEST - 5A (Paper-1) - Code-C

Test Date : 02/04/2023

ANSWERS

PHYSICS	CHEMISTRY	MATHEMATICS
1. (37.00)	19. (08.00)	37. (07.00)
2. (21.00)	20. (04.00)	38. (05.00)
3. (00.85)	21. (04.00)	39. (11.00)
4. (16.00)	22. (17.00)	40. (06.00)
5. (01.72)	23. (50.00)	41. (04.00)
6. (04.00)	24. (21.00)	42. (02.00)
7. (50.00)	25. (02.00)	43. (04.00)
8. (15.00)	26. (05.00)	44. (00.00)
9. (A, C)	27. (C, D)	45. (A, C, D)
10. (D)	28. (A, D)	46. (A, C, D)
11. (B, C)	29. (B, C, D)	47. (C, D)
12. (B, C, D)	30. (B)	48. (B, C, D)
13. (B, D)	31. (A, C)	49. (A, B, C, D)
14. (A, B, C, D)	32. (A, D)	50. (D)
15. (D)	33. (B)	51. (B)
16. (A)	34. (C)	52. (C)
17. (C)	35. (A)	53. (D)
18. (D)	36. (D)	54. (A)

HINTS & SOLUTIONS

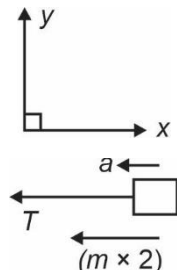
PART - I (PHYSICS)

1. Answer (37.00)

Hint : $a_p^2 = a_{px}^2 + a_{py}^2$

Sol. : $a_{px} = \frac{10}{5} = 2 \text{ m/s}^2$

For block 'm' with respect to M



$$\Rightarrow T + 2m = ma$$

$$\Rightarrow 10 + 2 = a$$

$$a = 12 \text{ m/s}^2$$

$$a = a_{py}$$

$$a_p = \sqrt{a_{px}^2 + a_{py}^2} = \sqrt{2^2 + 12^2} = 2\sqrt{37} \text{ m/s}^2$$

2. Answer (21.00)

Hint : $V_{\max} = \sqrt{Rg \tan(\theta + \phi)}$

Sol. : $V_{\max} = \sqrt{Rg \tan(\theta + \phi)}$

$$\frac{V_{\max}}{Rg} = \tan(\theta + \phi) = \sqrt{3}$$

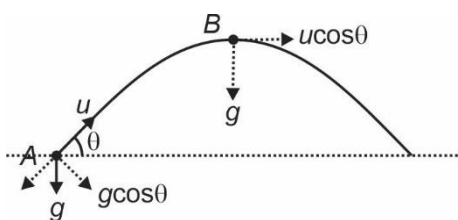
$$\theta + \phi = 60^\circ$$

$$\phi = 60^\circ - 39 = 21^\circ$$

3. Answer (00.85)

Hint : $\frac{v^2}{R} = \text{acceleration}$

Sol. :



$$\frac{u^2}{R_1} = g \cos \theta$$

$$R_1 = \frac{u^2}{g \cos \theta}$$

At point B,

$$\frac{u^2 \cos^2 \theta}{g} = R_2$$

$$\frac{R_2}{R_1} = \cos^3 \theta = \left(\frac{5}{8}\right)$$

$$\cos \theta = \frac{5^{\frac{1}{3}}}{2} = \frac{1.70}{2} = 0.85$$

4. Answer (16.00)

Hint : $F = -\frac{dU}{dr}$

Sol. : $F = -\frac{dU}{dr} = \left(\frac{\alpha}{r^4} - \frac{\beta}{r^3}\right)$

$$F = 0, \text{ at } \frac{\alpha}{r^4} = \frac{\beta}{r^3}$$

$$\frac{\alpha}{r} = \beta$$

$$r = \left(\frac{\alpha}{\beta}\right)$$

$$\text{For } r = \left(\frac{\alpha}{\beta} + x\right),$$

$$F = \frac{\alpha}{\left(\frac{\alpha}{\beta} + x\right)^4} - \frac{\beta}{\left(\frac{\alpha}{\beta} + x\right)^3}$$

$$= \alpha \left[\frac{\beta^4}{\alpha^4} \left(1 - \frac{4\beta x}{\alpha}\right) \right] - \beta \left[\frac{\beta^3}{\alpha^3} \left(1 - \frac{3\beta x}{\alpha}\right) \right]$$

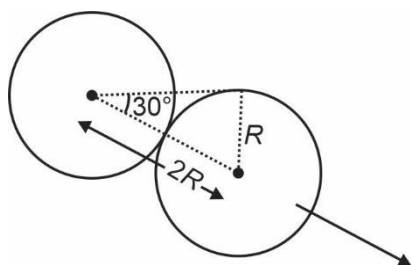
$$= \frac{\beta^4}{\alpha^3} \left[1 - \frac{4\beta x}{\alpha} - 1 + \frac{3\beta x}{\alpha} \right]$$

$$= \frac{\beta^4}{\alpha^3} \left[-\frac{\beta}{\alpha} x \right] = -\left(\frac{\beta^5}{\alpha^4}\right) x$$

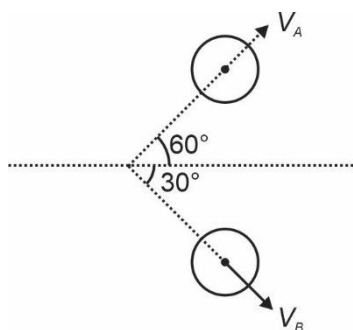
$$\Rightarrow K = \frac{\beta^5}{\alpha^4 \cdot m} = 16$$

5. Answer (01.72)

Hint : Apply momentum conservation.

Sol. :


After collision



Conserving momentum in horizontal and vertical direction

$$\frac{V_A}{2} + \frac{\sqrt{3}V_B}{2} = u$$

$$\Rightarrow V_A + \sqrt{3} V_B = u \quad \dots(1)$$

$$\frac{\sqrt{3}V_A}{2} = \frac{V_B}{2}$$

$$\Rightarrow V_B = \sqrt{3} V_A \quad \dots(2)$$

Solving,

$$4V_A = u$$

$$V_A = \left(\frac{u}{4}\right)$$

$$V_B = \left(\frac{\sqrt{3}u}{2}\right)$$

$$\Rightarrow \frac{V_B}{V_A} = \frac{\frac{\sqrt{3}}{2}u}{\frac{u}{4}} = \frac{\frac{\sqrt{3}}{2} \times 4}{1} = 2\sqrt{3}$$

$$= \frac{2\sqrt{3}}{1} = \sqrt{3} = 1.72$$

6. Answer (04.00)

$$\text{Hint : } \dot{Q} = -KA \frac{dT}{dx}$$

$$\text{Sol. : } \dot{Q} = -KA \frac{dT}{dx}$$

$$\Rightarrow \int_0^L \dot{Q} \frac{dx}{\left(1 + \frac{x^2}{L^2}\right)} = \int_{T_1}^{T_2} -K_0 A dT$$

$$\Rightarrow \dot{Q} L \left[\tan^{-1} \left(\frac{x}{L} \right) \right]_0^L = K_0 A (T_1 - T_2)$$

$$\Rightarrow \dot{Q} L \left(\frac{\pi}{4} \right) = K_0 A (T_1 - T_2)$$

$$\dot{Q} = \left(\frac{4}{\pi} \right) \frac{K_0 A}{L} (T_1 - T_2)$$

7. Answer (50.00)

$$\text{Hint : } C = C_v - \frac{R}{n-1}$$

$$\text{Sol. : } C = C_v - \frac{R}{n-1}$$

$$P V^n = \text{constant} \Rightarrow T V^{n-1} = \text{constant}$$

$$T V^{-2} = \text{constant}$$

$$n-1 = -2$$

$$n = -1$$

$$C = C_v - \frac{R}{-1-1} = C_v + \frac{R}{2}$$

$$\frac{3R}{2} + \frac{R}{2} = 2R$$

8. Answer (15.00)

$$\text{Hint : } I_{\text{hollow}} = \int \frac{2}{3} dm r^2$$

$$\text{Sol. : } dm = \rho (4\pi r^2 dr) \\ = \frac{4\pi K}{R^5} (r^7 dr)$$

$$I_{\text{solid}} = \int \frac{2}{3} (dm) r^2 \\ = \frac{2}{3} \times \frac{4\pi K}{R^5} \int_0^R r^9 dr \\ = \frac{2}{3} \times \frac{4\pi K}{R^5} \times \frac{R^{10}}{10} \\ = \frac{4\pi}{15} (KR^5)$$

9. Answer (A, C)

Hint : Range is maximum, when height is half of effective height.

$$\text{Sol. : } x = \frac{h}{2}$$

$$T = \sqrt{\frac{h}{g}}$$

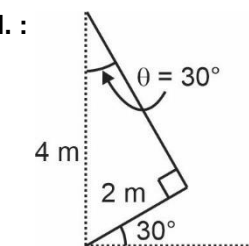
$$R = \sqrt{2g \frac{h}{2} \times \frac{h}{g}}$$

$$R = h$$

10. Answer (D)

Hint : Balance torque about point P.

Sol. :



$$fr = \frac{T}{2}$$

Balancing torque about 'P'

$$T \times 2 = mg \cos 30$$

$$2T = 4g \times \frac{\sqrt{3}}{2}$$

$$T = \sqrt{3}g$$

$$fr = \frac{\sqrt{3}}{2}g$$

$$N + T \sin 60 = mg$$

$$N = 4g - \sqrt{3}g \times \frac{\sqrt{3}}{2}$$

$$= \frac{8g - 3g}{2} = \left(\frac{5g}{2}\right)$$

11. Answer (B, C)

$$\text{Hint : } \vec{E} = -\frac{dv}{dr} \hat{r}$$

$$\text{Sol. : } V = \frac{r}{r^2 + n^2}$$

$$\vec{E} = -\frac{dv}{dr} \hat{r}$$

$$\vec{E} = \frac{(r^2 + n^2) - 2r^2}{(r^2 + n^2)^2} = \frac{r^2 - n^2}{(r^2 + n^2)^2} \hat{r}$$

At $r = n$, $E = 0$

$$r = 2n \quad E = \frac{3n^2}{25n^4} = \left(\frac{3}{25n^2}\right)$$

$$r = 3n \quad E = \frac{8n^2}{100n^4} = \left(\frac{2}{25n^2}\right)$$

12. Answer (B, C, D)

$$\text{Hint : } \text{Slope} \propto \frac{1}{\text{volume}}$$

$$\text{Sol. : } PdT + TdP = 0$$

$$\Rightarrow \frac{dP}{dT} = -\frac{P}{T}$$

$$\text{Slope} \propto \frac{1}{\text{Volume}} \Rightarrow V_2 > V_1$$

13. Answer (B, D)

Hint : Equate dimensional formula on both side.

$$\text{Sol. : } y = ML^{-1}T^{-2} = (MLT^{-2})^\gamma (LT^{-1})^\alpha (LT^{-2})^\beta$$

$$ML^{-1}T^{-2} = M^\gamma L^{\alpha+\beta+\gamma} T^{(-\alpha-2\beta-2\gamma)}$$

$$\gamma = 1$$

$$\alpha + \beta + \gamma = -1$$

$$\alpha + \beta = -2$$

$$-\alpha - 2\beta - 2\gamma = -2$$

$$-\alpha - 2\beta - 2 = -2$$

$$\alpha + 2\beta = 0$$

$$\alpha + \beta + \beta = 0$$

$$\Rightarrow \beta = 2$$

$$\alpha = -4$$

14. Answer (A, B, C, D)

$$\text{Hint : } v = \frac{\partial y}{\partial t} = A\omega \cos\left(\omega t - \pi x - \frac{\pi}{6}\right)$$

$$\text{Sol. : } y = A \sin(\omega t - kx + \phi)$$

$$\lambda = \frac{2\pi}{K} = \frac{2\pi}{\pi} = 2 \text{ m}$$

$$v = \frac{\partial y}{\partial t} = A\omega \cos\left(\omega t - \pi x - \frac{\pi}{6}\right)$$

$$\text{At } t = 0, x = \frac{\pi}{4}$$

$$v = A\omega \cos\left(-\pi \times \left(\frac{2}{4}\right) - \frac{\pi}{6}\right)$$

$$= A\omega \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$$

$$|v| = \left(\frac{A\omega}{2}\right) = \frac{A \times 2\pi}{2} = (A\pi) \text{ m/s}$$

$$\text{At } t = 0, x = \frac{\pi}{2}$$

$$v = A\omega \cos\left(-\pi - \frac{\pi}{6}\right)$$

$$|v| = A\omega \cos \frac{\pi}{6}$$

$$= \frac{\sqrt{3}}{2} A\omega$$

$$\text{At } t = \frac{1}{12} \text{ s, } x = \frac{\lambda}{4}$$

$$v = A\omega \cos\left(\frac{2\pi}{12} - \frac{\pi}{2} - \frac{\pi}{6}\right)$$

$$v = A\omega \cos\left(-\frac{\pi}{2}\right)$$

$$v = 0 \text{ m/s}$$

$$\text{At } t = \frac{1}{12} \text{ s, } x = \frac{\lambda}{2}$$

$$v = A\omega \cos(-\pi)$$

$$|v| = A\omega$$

15. Answer (D)

$$\text{Hint : } \vec{v}_0 = \vec{v}_A \text{ implies } \omega = 0$$

$$\vec{v}_0 \neq \vec{v}_A \Rightarrow \omega \neq 0$$

Sol. : Angular momentum and linear momentum of system remains conserved.

16. Answer (A)

Hint & Sol. : To escape, total energy ≥ 0

17. Answer (C)

$$\text{Hint : } F = -\frac{dU}{dX}$$

$$\text{Sol. : } U = 2\pi^2 x^2$$

$$F = -\frac{dU}{dX} = -4\pi^2 x$$

$$F = -4\pi^2 x$$

$$K = 4\pi^2$$

$$\omega^2 = 4\pi^2$$

$$\omega = 2\pi$$

$$\left[T = \left(\frac{2\pi}{\omega}\right) = 1\right]$$

$$E\left(\frac{1}{2}\right) = 2\pi^2 \left(\frac{1}{2}\right)^2$$

$$= 2\pi^2 \times \frac{1}{4}$$

$$= \left(\frac{\pi^2}{2}\right)$$

$$E\left(\frac{1}{\sqrt{2}}\right) = (\pi^2)$$

18. Answer (D)

$$\text{Hint : } I = \int r^2 dm$$

Sol. : Moment of Inertia of solid sphere $= \frac{2}{5} MR^2$ and also passes through centre of mass.

$$\text{Moment of Inertia of circular arc} = \frac{1}{2} MR^2$$

PART - II (CHEMISTRY)

19. Answer (08.00)

Hint : Element is polonium**Sol. :** Electronic configuration as [Xe] $4f^{14}5d^{10}6s^26p^4$ for last electron

$$n = 6, l = 1, m = +1$$

$$\text{Hence, } n + l + m \text{ is } 6 + 1 + 1 = 8$$

20. Answer (04.00)

Hint : Species with zero bond order are He_2 , Be_2 , Ne_2 i.e. 3**Sol. :** H_2 , H_2^+ , O_2^{2-} , Li_2 , F_2 have bond order ≤ 1 i.e. 5 O_2 have a bond order of 2.And C_2^{2-} , N_2 , O_2^+ , O_2^{2+} have a bond order 3, CO^+ have a bond order of 3.5.

$$a = 8, b = 1, c = 5$$

21. Answer (04.00)

Hint : Effective nuclear charge (z^*) is given by

$$z^* = Z - \sigma$$

peripheral electron corresponds to last e^- .**Sol. :** Calculation for σ (screening constant) as $\therefore$ last electron is in 4s-subshell so,

$$\sigma = 1 \times 0.35 + 15 \times 0.85 + 10 \times 1$$

$$\sigma = 23.1$$

$$z^* = 3.9 \approx 4$$

22. Answer (17.00)

$$\text{Hint : } \frac{C_p}{C_v} = \gamma$$

$$\text{Sol. : } \gamma = \frac{0.206}{0.148} \approx 1.4$$

 $\therefore$ gas is diatomic = B = 2

$$\text{Also, } C_p - C_v = R$$

$$R = 0.206 - 0.148 = 0.058 \text{ cal/g}$$

$$\text{For 1 mol, } R = 2 \text{ cal}$$

$$\text{Number of moles} = \frac{1}{2} \times 0.058 = 0.029 \text{ moles}$$

$$\therefore \text{molecular mass} = \frac{1}{\text{number of moles}} = 34.48 = A$$

$$\text{i.e. } \frac{A}{B} = 17.24 \approx 17$$

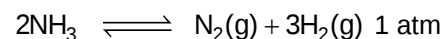
23. Answer (50.00)

Hint : 1 atm = 760 mm of Hg**Sol. :** Initially, $P_{\text{NH}_3} = 1 \text{ atm}$

After sparking

$$P_{\text{NH}_3} + P_{\text{H}_2} + P_{\text{N}_2} = 1 + \frac{38}{76} = \frac{3}{2} \text{ atm}$$

From dissociation reaction,



$$(1 - P) \quad \frac{P}{2} \quad \frac{3P}{2}$$

$$\text{Final pressure as } 1 - P + \frac{P}{2} + \frac{3P}{2} = \frac{3}{2}$$

$$1 + P = \frac{3}{2}$$

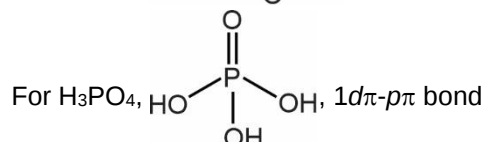
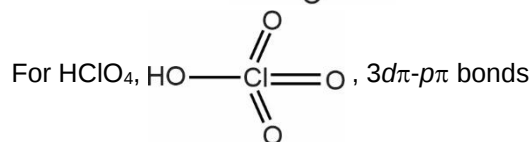
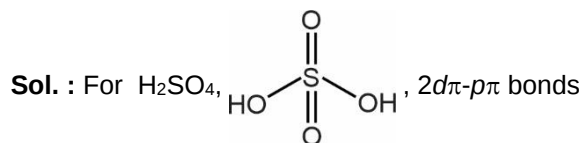
$$P = \frac{1}{2} \text{ atm}$$

$$\% \text{ dissociation is } \frac{1/2}{1} \times 100 = 50\%$$

24. Answer (21.00)

Hint : Fluorapatite is $[\text{Ca}_5(\text{PO}_4)_3\text{F}]$ **Sol. :** Molecular formula can also be written as $\text{Ca}_5\text{FO}_{12}\text{P}_3$

25. Answer (02.00)

Hint : Lone pair on each oxygen atom contributes to H-bonding

26. Answer (05.00)

Hint : Compound with molecular formula $C_6H_{12}O$ can show metamerism, keto-enol

Sol. : Also, it can show positional, chain and functional group isomerism.

27. Answer (C, D)

Hint : Stannane or SnH_4 is inert towards hydrolysis as $Sn - H$ bond has insufficient bond polarity.

Sol. : $Te(OH)_6$ exists as, there is no steric crowding to accommodate six OH^- groups around it.

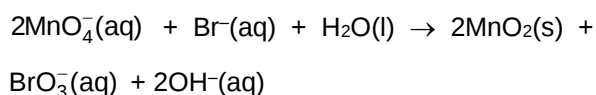
28. Answer (A, D)

Hint : Be and Mg does not impart colour in flame test.

Sol. : Na gives yellow colour in flame test

29. Answer (B, C, D)

Hint : The balanced redox reaction is



Sol. : Hence option B, C and D are correct

30. Answer (B)

Hint : Coordinated water molecule present in $[Cr(H_2O)_6]Cl_3$

Sol. : $BaCl_2 \cdot 2H_2O$ water molecule present only in interstitials.

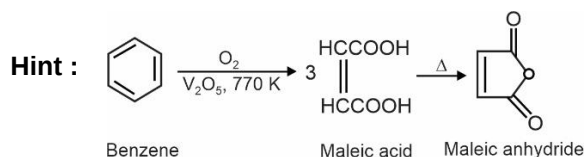
In $CuSO_4 \cdot 5H_2O$, $4H_2O$ molecules are coordinated

31. Answer (A, C)

Hint : Temporary hardness can be removed by Clark's method

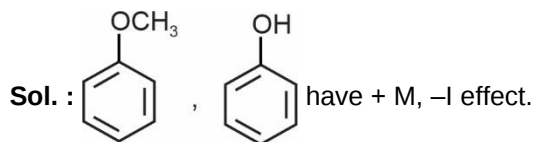
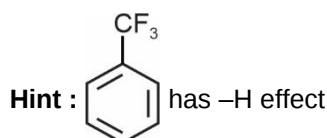
Soln. : Permanent hardness can be removed by Calgon's method

32. Answer (A, D)



Sol. : A and B are not isomers

33. Answer (B)



34. Answer (C)

Hint : The atomic number greater than 100 given by IUPAC nomenclature, but some common names.

Sol. : Atomic No.	IUPAC official name
110	Darmstadtium (Ds)
108	Hassium (Hs)
104	Rutherfordium (Rf)
105	Dubnium (Db)

35. Answer (A)

Hint : Coulombic force of attraction $\propto \frac{1}{r^2}$

Sol. : Dipole – Dipole forces

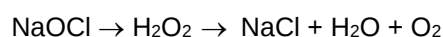
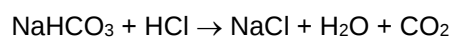
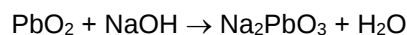
(i) In stationary state $\propto \frac{1}{r^3}$

(ii) In rotation state $\propto \frac{1}{r^6}$

36. Answer (D)

Hint : PbO_2 is amphoteric in nature

Sol. : $Sn + NaOH \rightarrow Na_2SnO_2 + H_2$



PART - III (MATHEMATICS)

37. Answer (07.00)

Hint : Make cases and count.**Sol. :** Let $x + y + z = 30$ and $x \neq y \neq z$ Let $x < y < z$ If $x = 1$,

y	z
2	27
$\vdots$	
14	15

13 triplets

If $x = 2$,

y	z
3	25
$\vdots$	
13	15

11 triplets

If $x = 3$,

y	z
4	23
5	22
$\vdots$	
13	14

10 triplets

If $x = 4$,

y	z
5	21
$\vdots$	
12	14

8 triplets

If $x = 5$,

y	z
6	19
$\vdots$	
12	13

7 triplets

If $x = 6$,

y	z
7	17
$\vdots$	
11	13

5 triplets

If $x = 7$,

y	z
8	15
$\vdots$	
11	12

4 triplets

If $x = 8$,

y	z
9	13
10	12

2 triplets

If $x = 9, y = 10, z = 11 \rightarrow 1$ triplet

Total triplets :

$$2 + 1 + 4 + 5 + 7 + 8 + 10 + 11 + 13 = 61$$

$$\therefore 6 + 1 = 7$$

38. Answer (05.00)

Hint : Find $z_1 z_2$ and equate for values of x and y .**Sol. :** $z_1 = (8 \sin \theta + 7 \cos \theta) + i(\sin \theta + 4 \cos \theta)$

$$z_2 = (\sin \theta + 4 \cos \theta) + i(8 \sin \theta + 7 \cos \theta)$$

$$z_1 = a + ib$$

$$z_2 = b + ia$$

$$z_1 z_2 = (a + ib)(b + ia)$$

$$= ab - ba + i(a^2 + b^2)$$

$$= (a^2 + b^2)i = x + iy$$

$$\Rightarrow x = 0; y = a^2 + b^2$$

$$\text{Now, } x + y = a^2 + b^2$$

$$M = (x + y)_{\max}$$

$$x + y = (8 \sin \theta + 7 \cos \theta)^2 + (\sin \theta + 4 \cos \theta)^2$$

$$= 65 + 60 \sin 2\theta$$

$$(x + y)_{\max} = 125$$

$$M^{1/3} = (125)^{1/3} = 5$$

39. Answer (11.00)

Hint : Use Newton's formula.**Sol. :** By Newton's formula,

$$P_n = aP_{n-1} - bP_{n-2}$$

 a = sum of roots b = product of roots

$$P_n = P_{n-1} + P_{n-2}$$

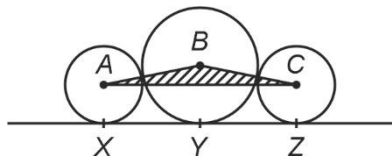
$$P_n = 7 + 4$$

$$P_n = 11$$

40. Answer (06.00)

Hint : Let radius of circles with centres A, B, C are a, b and c respectively, then

$$XZ = \sqrt{AC^2 - (a - c)^2} \quad \dots(i)$$



Sol. : Area of quadrilateral ACZX

$$= \frac{1}{2}(a + c)40 = 300$$

$$\Rightarrow a + c = 15 \quad \dots(ii)$$

Hence, $a = 12$ and $c = 3$

Now, for finding the radius of circle touching both circles with centre A and C.

$$XZ = XY + YZ$$

$$40 = \sqrt{(b + 12)^2 - (b - 12)^2} + \sqrt{(b + 3)^2 - (b - 3)^2}$$

$$40 = \sqrt{48b} + \sqrt{12b}$$

$$\Rightarrow b = \frac{400}{27}$$

Now, $\text{ar}(\triangle ABC) = \text{Area of quad. ABYX}$

+ Area of quad. BCZY - Area of quad. ACZX

$$\begin{aligned} \Delta &= \frac{1}{2} \left(12 + \frac{400}{27} \right) \left(\frac{80}{3} \right) \\ &\quad + \frac{1}{2} \left(3 + \frac{400}{27} \right) \left(\frac{40}{3} \right) - 300 \\ &= 476 + \frac{8}{27} - 300 \end{aligned}$$

$$[\Delta] = 176 + \frac{8}{27}$$

$$[\Delta] - 170 = 176 - 170 = 6$$

41. Answer (04.00)

Hint : Substitute the value of $2y$ in given expression, then apply $AM \geq GM$.

Sol. : $\because x, y, z$ are in AP,

$x + z = 2y$ and hence

$$\frac{x+y}{2y-x} + \frac{y+z}{2y-z} = \frac{x+y}{z} + \frac{y+z}{x}$$

$$= \frac{y}{x} + \frac{y}{z} + \frac{x}{z} + \frac{z}{x}$$

$$= \frac{x}{z} + \frac{x+z}{2z} + \frac{x+z}{2x} + \frac{z}{x}$$

$$= 1 + \frac{3}{2} \left(\frac{x}{z} + \frac{z}{x} \right)$$

$$\geq 1 + 3 \quad [AM \geq GM]$$

$\Rightarrow$ Minimum value of expression is 4.

42. Answer (02.00)

Hint : $|z - a| = r$ represents equation of circle and $|z - z_1| + |z - z_2| = k$ is equation of ellipse.

Sol. : $|z - 3 - i| = 2$ represents a circle with centre $3 + i$ and radius 2.

$|z - 2 + i| + |z - 4 - 3i| = 6$ represents ellipse with centre $\left| \frac{2-i+4+3i}{2} \right| = 3+i$ and length of major axis = 6.

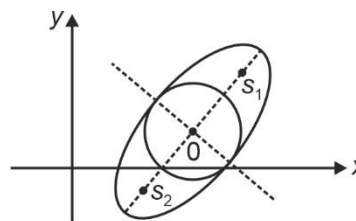
Also, distance between foci = $2\sqrt{5}$

$$\Rightarrow \text{Eccentricity} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3} < 1$$

$$\therefore \text{Minor axis} = 2 \times 3 \sqrt{1 - \frac{5}{9}} = 4$$

$\therefore$ Diameter of circle = minor axis

So, the circle touches the ellipse internally.



43. Answer (04.00)

Hint : Represent b in terms of a .

$$\text{Sol. : } a = \sum_{r=1}^{\infty} \frac{1}{r^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \infty$$

$$b = \sum \frac{1}{(2r-1)^2} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \infty$$

$$= \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \infty \right)$$

$$- \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \dots \infty \right)$$

$$= \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \infty \right)$$

$$- \frac{1}{2^2} \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \infty \right)$$

$$b = a - \frac{1}{4}a$$

$$b = \frac{3}{4}a$$

$$\therefore \frac{3a}{b} = 4$$

44. Answer (00.00)

Hint : Draw graph to find number of solutions.

$$\text{Sol. : } [y + [y]] = 2 \cos x$$

$$\Rightarrow [y] = \cos x$$

$$y = \frac{1}{3} [\sin x + [\sin x + [\sin x]]] = [\sin x]$$

$$\Rightarrow [\sin x] = \cos x$$

Number of solution in $[0, 2\pi]$ is 0.

45. Answer (A, C, D)

Hint : Factorise the given equation.

Sol. : Given equation is

$$\sin^2 x + (\cos x - 1) \sin x - \cos x - k \sin x + k = 0$$

Given equation can be rewritten as

$$\sin^2 x + \sin x \cdot \cos x - k \sin x - \sin x - \cos x + k = 0$$

$$\Rightarrow (\sin x - 1)(\sin x + \cos x - k) = 0$$

$$\Rightarrow \sin x = 1 \text{ or } \sin x + \cos x = k$$

$\sin x = 1$ is one real root in $(0, 2\pi)$

$\sin x + \cos x = k$ has two real roots $(0, 2\pi)$

$$\Rightarrow -\sqrt{2} < k < \sqrt{2}$$

Integral value of k is $-1, 0, 1$.

But $k \neq 1$

$$\therefore k = 1 \Rightarrow x = \frac{\pi}{2}$$

$$\Rightarrow k = 0, -1$$

Hence, two possible integral values of k .

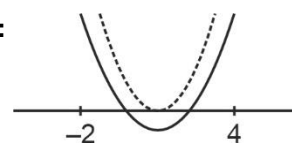
46. Answer (A, C, D)

Hint : Use location of roots

Sol. : Let $A = [-2, 4]$

$B \leq A \Rightarrow$ Both roots of $x^2 - ax - 4 = 0$ should lie in $[-2, 4]$, by using location of roots two cases are possible.

Case I :



Both roots lie in $(-2, 4)$.

$$D \geq 0 \Rightarrow a^2 + 16 > 0 \rightarrow a \in \mathbb{R} \quad \dots(i)$$

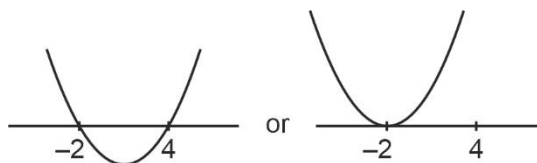
$$-2 \in \frac{b}{2a} < 4 \Rightarrow -2 < \frac{a}{2} < 4 \rightarrow a \in [-4, 8] \quad \dots(ii)$$

$$f(-2) > 0 \Rightarrow 4 + 2a - 4 > 0 \rightarrow a \in (0, \infty) \quad \dots(iii)$$

$$f(4) > 0 \Rightarrow 12 - 4a > 0 \rightarrow a \in (-\infty, 3) \quad \dots(iv)$$

Equations (i) $\cap$ (ii) $\cap$ (iii) $\cap$ (iv) gives $a \in (0, 3)$.

Case II : At least one root is -2 .



By putting $x = -2$ into $x^2 - ax - 4 = 0$, we get

$$a = 0$$

Hence, $x^2 - 4 = 0$ have 2, roots 2 and -2 both lying in $[-2, 4]$.

Hence, $a = 0$ is also included

$$\therefore S = [0, 3)$$

47. Answer (C, D)

Hint : Assume

$$b = a + p$$

$$c = a + 2p \quad p = \text{common difference}$$

$$d = a + 3p$$

Sol. : Let $b = a + p$

$c = a + 2p$ p = common difference

$d = a + 3p$

$$\frac{\frac{1}{a} + \frac{1}{d}}{\frac{1}{b} + \frac{1}{c}} = \frac{\frac{1}{a} + \frac{1}{a+3p}}{\frac{1}{a+p} + \frac{1}{a+2p}}$$

$$= \frac{(a+p)(a+2p)}{a(a+3p)}$$

$$= \frac{a^2 + 3ap + 2p^2}{a^2 + 3ap} > 1$$

$$\therefore \frac{1}{a} + \frac{1}{d} > \frac{1}{b} + \frac{1}{c}$$

$$\begin{aligned} \left(\frac{1}{b} + \frac{1}{c}\right)(a+d) &= \left(\frac{1}{a+p} + \frac{1}{a+2p}\right)(a+a+3p) \\ &= \frac{(2a+3p)^2}{a^2 + 3pa + 2p^2} \\ &= 4 + \frac{p^2}{a^2 + 3ap + 2p^2} > 4 \end{aligned}$$

48. Answer (B, C, D)

Hint : Count all the cases for equality and inequality.

Sol. : Triplets with $x = y < z$, $x < y < z$, $y < x < z$ can be chosen in

$${}^{n+1}C_2, {}^{n+1}C_3, {}^{n+1}C_3 \text{ ways}$$

$$\begin{aligned} \text{There are } {}^{n+1}C_2 + 2({}^{n+1}C_3) &= {}^{n+2}C_2 + {}^{n+1}C_3 \\ &= 2({}^{n+2}C_3) - {}^{n+1}C_2 \end{aligned}$$

49. Answer (A, B, C, D)

$$\text{Hint : } \cos \alpha = \frac{1}{2} \left(x + \frac{1}{x} \right)$$

$$\cos \beta = \frac{1}{2} \left(y + \frac{1}{y} \right)$$

$$\therefore xy > 0 \Rightarrow x + \frac{1}{x} \geq 2 \text{ or } \leq -2$$

$$y + \frac{1}{y} \geq 2 \text{ or } \leq -2$$

$$\Rightarrow \cos \alpha = 1, \cos \beta = 1 \quad \dots (i)$$

$$\text{or } \cos \alpha = -1, \cos \beta = -1 \quad \dots (ii)$$

Sol. : $\therefore \cos \alpha \cos \beta = 1$

$$(\cos \alpha + \cos \beta)^2 = 4$$

$\Rightarrow \alpha + \beta$ is an even multiple of π .

$$\begin{aligned} \Rightarrow \sin(\alpha + \beta + \gamma) &= \sin(2n\pi + \gamma) \\ &= \sin \gamma \end{aligned}$$

Also, $\sin \alpha = \sin \beta = 0$

50. Answer (D)

Hint : Expression can be written as sum of squares.

Sol. : $x^2 + 9y^2 + 25z^2 = 15yz + 5zx + 3xy$

$$= \frac{1}{2} [(x-3y)^2 + (3y-5z)^2 + (5z-x)^2] = 0$$

$$\Rightarrow x = 3y = \frac{1}{x} = \frac{1}{3y}$$

$$3y = 5z = \frac{1}{z} = \frac{5}{3y}$$

$$5z = x = \frac{1}{5z} = \frac{1}{x}$$

$$\frac{1}{x} + \frac{1}{z} = \frac{1}{3y} + \frac{5}{3y} = \frac{2}{y}$$

$$\therefore \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \text{ are in AP}$$

$$\therefore x, y, z \rightarrow \text{HP}$$

51. Answer (B)

Hint : Use cosine rule.

Sol. :

(I) Let the sides a and b are the roots of $x^2 - 5x + 3 = 0$, then $a + b = 5$ and $ab = 3$

$$\text{Also, } \angle c = \frac{\pi}{3}$$

$$\therefore \frac{1}{2} = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow ab = (a+b)^2 - 2ab - c^2$$

$$\Rightarrow c = 4$$

$$\text{Now, } rR = \frac{abc}{2(a+b+c)} = \frac{3 \times 4}{2(5+4)} = \frac{2}{3}$$

(II) We have,

$$a = \frac{r_1(r_2 + r_3)}{\sqrt{r_1r_2 + r_2r_3 + r_3r_1}} = \frac{8 \times 36}{24} = 12$$

(III) Given $1 \leq a \leq 10$

Also, for roots of opposite sign

$$2a^2 - 7a + 3 < 0$$

$$\text{or } \frac{1}{2} < a < 3 \Rightarrow a = 1 \text{ or } 2$$

$$\therefore \text{Desired probability} = \frac{2}{10} = \frac{1}{5}$$

 (IV) (α, β) lie on director circle of an ellipse, i.e., on $x^2 + y^2 = 9$

 So, we assume $\alpha = 3 \cos \theta$, $\beta = 3 \sin \theta$

 Thus, $F = 12 \cos \theta + 9 \sin \theta$

$$= 3 [4 \cos \theta + 3 \sin \theta]$$

$$-15 \leq F \leq 15$$

52. Answer (C)

Hint : Product of roots is negative.

Sol. :

 (I) Product < 0

$$\frac{b^2 - 3b + 2}{3} < 0$$

$$b \in (1, 2)$$

(II) Probability problem is solved.

$$1 - P(\bar{A}) P(\bar{B}) P(\bar{C})$$

$$1 - \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) (1 - \alpha) = \frac{2}{3} + \frac{\alpha}{3} \in \left(\frac{2}{3}, 1\right)$$

 (III) $x = 5 - (y + z)$

$$yz + x(y + z) = 8$$

$$yz + (y + z)(5 - (y + z)) - 8 = 0$$

$$y^2 + y(z - 5) + (z^2 - 5z + 8) = 0$$

For real solution,

$$(z - 5)^2 - 4(z^2 - 5z + 8) \geq 0$$

$$(z - 1) \left(z - \frac{7}{3} \right) \leq 0$$

$$z \in \left[1, \frac{7}{3} \right]$$

 (IV) $|\sec x| + |\operatorname{cosec} x|$

$$\text{Range } [2\sqrt{2}, \infty)$$

53. Answer (D)

Hint : Use cosine rule.

Sol. :

$$(I) \frac{b^2 + c^2 - a^2}{2bc} = \frac{b}{2c} \Rightarrow c = a$$

$$(II) 1 + a \geq 2\sqrt{a} \quad (\text{AM} \geq \text{GM})$$

$$1 + b \geq 2\sqrt{b}$$

$$1 + c \geq 2\sqrt{c}$$

$$1 + d \geq 2\sqrt{d}$$

$$\therefore (1 + a)(1 + b)(1 + c)(1 + d) \geq 16\sqrt{abcd}$$

$$\geq 16$$

$$\therefore \text{Minimum} = 16$$

 (III) $5^{x+2} > 5^{-2/x}$

$$\Rightarrow x + 2 > \frac{-2}{x}$$

$$\frac{x^2 + 2x + 2}{x} > 0$$

$$\frac{(x+1)^2 + 1}{x} > 0$$

$$\Rightarrow x \in (0, \infty)$$

 (IV) $x^2 - (3k - 1)x + 2k^2 - 3k - 2 \leq 0$

$$D = (k + 3)^2$$

$$\therefore \forall x \in R, f(x) \leq 0$$

$$\Rightarrow D \leq 0$$

$$\Rightarrow D = 0$$

$$\Rightarrow k = -3$$

$$S = \{-3\}$$

54. Answer (A)

$$\text{Hint : } t_2 = -t_1 - \frac{2}{t_1}$$

Sol. :

(I) Equation of normal is $y = -tx + 2at + at^2$ at $P(t)$.

It intersect the curve again at point $Q(t_1)$ on the parabola such that

$$t_2 = -t - \frac{2}{t}$$

Again, slope of OP is $\frac{2}{t} = M_{OP}$

Also, slope of OQ is $\frac{2}{t_1} = M_{OQ}$

$$\therefore M_{OP} \cdot M_{OQ} = -1 = \frac{4}{t t_1}$$

$$\Rightarrow t t_1 = -4$$

$$t \left(-t - \frac{2}{t} \right) = -4$$

$$\Rightarrow t^2 = 2$$

(II) $P(1, 2), Q(4, 4), R(16, 8)$

Now ar $(\Delta PQR) = 6$ sq. unit

(III) Equation of normal from any point $P(am^2, -2m)$ is $y = mx - am^3 - 2am$

It passes through $\left(\frac{11}{4}, \frac{1}{4}\right)$

$$\Rightarrow 4m^3 + 8m - 11m + 1 = 0$$

$$\Rightarrow m = \frac{1}{2}, -1 \text{ two distinct normals}$$

(IV) $\therefore$ Normal at $P(t_1)$ if meets the curve again at

(t_2) , then $t_2 = -t_1 - \frac{2}{t_1}$

Such that here normal at $P(1)$ meets the curve again at $Q(t)$.

$$\Rightarrow t = -1 - \frac{2}{1} = -3$$

