All India Aakash Test Series for JEE (Advanced)-2024

TEST - 4A (Paper-1) - Code-A

Test Date : 02/04/2023

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(37.00)	19.	(08.00)	37.	(07.00)
2.	(21.00)	20.	(04.00)	38.	(05.00)
3.	(00.85)	21.	(04.00)	39.	(11.00)
4.	(16.00)	22.	(17.00)	40.	(06.00)
5.	(01.72)	23.	(50.00)	41.	(04.00)
6.	(04.00)	24.	(21.00)	42.	(02.00)
7.	(50.00)	25.	(02.00)	43.	(04.00)
8.	(15.00)	26.	(05.00)	44.	(00.00)
9.	(A, C)	27.	(C, D)	45.	(A, C, D)
10.	(D)	28.	(A, D)	46.	(A, C, D)
11.	(B, C)	29.	(B, C, D)	47.	(C, D)
12.	(B, C, D)	30.	(B)	48.	(B, C, D)
13.	(B, D)	31.	(A, C)	49.	(A, B, C, D)
14.	(A, B, C, D)	32.	(A, D)	50.	(D)
15.	(D)	33.	(B)	51.	(B)
16.	(A)	34.	(C)	52.	(C)
17.	(C)	35.	(A)	53.	(D)
18.	(D)	36.	(D)	54.	(A)

HINTS & SOLUTIONS

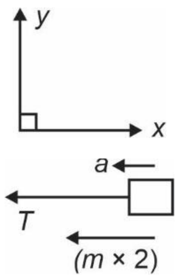
PART - I (PHYSICS)

1. Answer (37.00)

Hint : $a_p^2 = a_{px}^2 + a_{py}^2$

Sol. : $a_{px} = \frac{10}{5} = 2$ m/s^2

For block 'm' with respect to M



$\Rightarrow T + 2m = ma$

$\Rightarrow 10 + 2 = a$

$a = 12$ m/s^2

$a = a_{py}$

$$a_p = \sqrt{a_{px}^2 + a_{py}^2} = \sqrt{2^2 + 12^2} = 2\sqrt{37} \text{ m/s}^2$$

2. Answer (21.00)

Hint : $V_{\max} = \sqrt{Rg\tan(\theta+\phi)}$

Sol. : $V_{\max} = \sqrt{Rg\tan(\theta+\phi)}$

$$\frac{V_{\max}}{Rg} = \tan(\theta+\phi) = \sqrt{3}$$

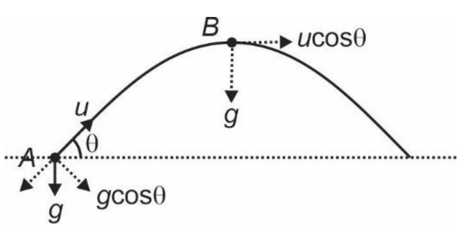
$\theta + \phi = 60°$

$\phi = 60° - 39 = 21°$

3. Answer (00.85)

Hint : $\frac{v^2}{R} = \text{acceleration}$

Sol. :



$$\frac{u^2}{R_1} = g\cos\theta$$

$$R_1 = \frac{u^2}{g\cos\theta}$$

At point B,

$$\frac{u^2\cos^2\theta}{g} = R_2$$

$$\frac{R_2}{R_1} = \cos^3\theta = \left(\frac{5}{8}\right)$$

$$\cos\theta = \frac{5^{\frac{1}{3}}}{2} = \frac{1.70}{2} = 0.85$$

4. Answer (16.00)

Hint : $F = -\frac{dU}{dr}$

Sol. : $F = -\frac{dU}{dr} = \left(\frac{\alpha}{r^4} - \frac{\beta}{r^3}\right)$

$F = 0$, at $\frac{\alpha}{r^4} = \frac{\beta}{r^3}$

$$\frac{\alpha}{r} = \beta$$

$$r = \left(\frac{\alpha}{\beta}\right)$$

For $r = \left(\frac{\alpha}{\beta} + x\right)$,

$$F = \frac{\alpha}{\left(\frac{\alpha}{\beta}+x\right)^4} - \frac{\beta}{\left(\frac{\alpha}{\beta}+x\right)^3}$$

$$= \alpha\left[\frac{\beta^4}{\alpha^4}\left(1 - \frac{4\beta x}{\alpha}\right)\right] - \beta\left[\frac{\beta^3}{\alpha^3}\left(1 - \frac{3\beta x}{\alpha}\right)\right]$$

$$= \frac{\beta^4}{\alpha^3}\left[1 - \frac{4\beta x}{\alpha} - 1 + \frac{3\beta x}{\alpha}\right]$$

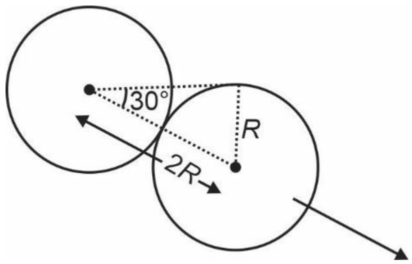
$$= \frac{\beta^4}{\alpha^3}\left[-\frac{\beta}{\alpha}x\right] = -\left(\frac{\beta^5}{\alpha^4}\right)x$$

$$\Rightarrow K = \frac{\beta^5}{\alpha^4 \cdot m} = 16$$

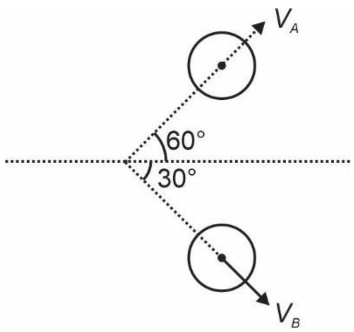
5. Answer (01.72)

Hint : Apply momentum conservation.

Sol. :



After collision



Conserving momentum in horizontal and vertical direction

$$\frac{V_A}{2}+\frac{\sqrt{3}V_B}{2}=u$$

$\Rightarrow V_A + \sqrt{3}\, V_B = u$ …(1)

$$\frac{\sqrt{3}V_A}{2}=\frac{V_B}{2}$$

$\Rightarrow V_B = \sqrt{3}\, V_A$ …(2)

Solving,

$4V_A = u$

$$V_A=\left(\frac{u}{2}\right)$$

$$V_B=\left(\frac{\sqrt{3}u}{2}\right)$$

$$\Rightarrow \frac{V_B}{V_A}=\frac{\frac{\sqrt{3}}{2}u}{\left(\frac{u^2}{4}\right)}=\frac{\frac{\sqrt{3}}{2}\times 4}{u}$$

$$=\frac{2\sqrt{3}}{u}=\sqrt{3}=1.72$$

6. Answer (04.00)

Hint : $\dot{Q}=-KA\dfrac{dT}{dx}$

Sol. : $\dot{Q}=-KA\dfrac{dT}{dx}$

$$\Rightarrow \int_0^L \dot{Q}\frac{dx}{\left(1+\frac{x^2}{L^2}\right)}=\int_{T_1}^{T_2}-K_0AdT$$

$$\Rightarrow \dot{Q}L\left[\tan^{-1}\left(\frac{x}{L}\right)\right]_0^L=K_0A\left(T_1-T_2\right)$$

$$\Rightarrow \dot{Q}L\left(\frac{\pi}{4}\right)=K_0A\left(T_1-T_2\right)$$

$$\dot{Q}=\left(\frac{4}{\pi}\right)\frac{K_0A}{L}\left(T_1-T_2\right)$$

7. Answer (50.00)

Hint : $C=C_v-\dfrac{R}{n-1}$

Sol. : $C=C_v-\dfrac{R}{n-1}$

PV^n = constant $\Rightarrow TV^{n-1}$ = constant

TV^{-2} = constant

$n-1=-2$

$n=-1$

$$C=C_v-\frac{R}{-1-1}=C_v+\frac{R}{2}$$

$$\frac{3R}{2}+\frac{R}{2}=2R$$

8. Answer (15.00)

Hint : $I_{\text{hollow}}=\int\dfrac{2}{3}dmr^2$

Sol. : $dm=\rho\left(4\pi r^2dr\right)$

$$=\frac{4\pi K}{R^5}\left(r^7dr\right)$$

$$I_{\text{solid}}=\int\frac{2}{3}(dm)r^2$$

$$=\frac{2}{3}\times\frac{4\pi K}{R^5}\int_0^R r^9dr$$

$$=\frac{2}{3}\times\frac{4\pi K}{R^5}\times\frac{R^{10}}{10}$$

$$=\frac{4\pi}{15}(KR^5)$$

9. Answer (A, C)

Hint : Range is maximum, when height is half of effective height.

Sol. : $x = \frac{h}{2}$

$T = \sqrt{\frac{h}{g}}$

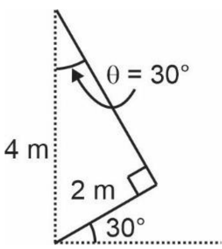
$R = \sqrt{2g\frac{h}{2} \times \frac{h}{g}}$

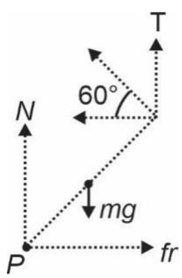
$R = h$

10. Answer (D)

Hint : Balance torque about point *P*.

Sol. :





$fr = \frac{T}{2}$

Balancing torque about '*P*'

$T \times 2 = mg\cos 30$

$2T = 4g \times \frac{\sqrt{3}}{2}$

$T = \sqrt{3}g$

$fr = \frac{\sqrt{3}}{2}g$

$N + T\sin 60 = mg$

$N = 4g - \sqrt{3}g \times \frac{\sqrt{3}}{2}$

$= \frac{8g - 3g}{2} = \left(\frac{5g}{2}\right)$

11. Answer (B, C)

Hint : $\vec{E} = -\frac{dv}{dr}\hat{r}$

Sol. : $V = \frac{r}{r^2 + n^2}$

$\vec{E} = -\frac{dv}{dr}\hat{r}$

$\vec{E} = \frac{\left(r^2 + n^2\right) - 2r^2}{\left(r^2 + n^2\right)^2} = \frac{r^2 - n^2}{\left(r^2 + n^2\right)^2}\hat{r}$

At $r = n$, $E = 0$

$r = 2n \quad E = \frac{3n^2}{25n^4} = \left(\frac{3}{25n^2}\right)$

$r = 3n \quad E = \frac{8n^2}{100n^4} = \left(\frac{2}{25n^2}\right)$

12. Answer (B, C, D)

Hint : Slope $\propto \frac{1}{\text{volume}}$

Sol. : $PdT + TdP = 0$

$\Rightarrow \frac{dP}{dT} = -\frac{P}{T}$

Slope $\propto \frac{1}{\text{Volume}} \Rightarrow V_2 > V_1$

13. Answer (B, D)

Hint : Equate dimensional formula on both side.

Sol. : $y = ML^{-1}T^{-2} = (MLT^{-2})^{\gamma}\,(LT^{-1})^{\alpha}\,(LT^{-2})^{\beta}$

$ML^{-1}T^{-2} = M^{\gamma}\,L^{\alpha+\beta+\gamma}\,T^{(-\alpha-2\beta-2\gamma)}$

$\gamma = 1$

$\alpha + \beta + \gamma = -1$

$\alpha + \beta = -2$

$-\alpha - 2\beta - 2\gamma = -2$

$-\alpha - 2\beta - 2 = -2$

$\alpha + 2\beta = 0$

$\alpha + \beta + \beta = 0$

$\Rightarrow \beta = 2 \qquad \alpha = -4$

14. Answer (A, B, C, D)

Hint : $v = \frac{\partial y}{\partial t} = A\omega\cos\left(\omega t - \pi x - \frac{\pi}{6}\right)$

Sol. : $y = A\sin(\omega t - kx + \phi)$

$\lambda = \frac{2\pi}{K} = \frac{2\pi}{\pi} = 2\text{ m}$

$v = \frac{\partial y}{\partial t} = A\omega\cos\left(\omega t - \pi x - \frac{\pi}{6}\right)$

At $t = 0$, $x = \frac{\pi}{4}$

$v = A\omega\cos\left(-\pi \times \left(\frac{2}{4}\right) - \frac{\pi}{6}\right)$

$= A\omega\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$

$|v| = \left(\frac{A\omega}{2}\right) = \frac{A \times 2\pi}{2} = (A\pi)\text{ m/s}$

At $t = 0$, $x = \frac{\pi}{2}$

$v = A\omega\cos\left(-\pi - \frac{\pi}{6}\right)$

$|v| = A\omega\cos\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}A\omega$

At $t = \frac{1}{12}\text{ s}$, $x = \frac{\lambda}{4}$

$v = A\omega\cos\left(\frac{2\pi}{12} - \frac{\pi}{2} - \frac{\pi}{6}\right)$

$v = A\omega\cos\left(-\frac{\pi}{2}\right)$

$v = 0$ m/s

At $t = \frac{1}{12}\text{ s}$, $x = \frac{\lambda}{2}$

$v = A\omega\cos(-\pi)$

$|v| = A\omega$

15. Answer (D)

Hint : $\vec{v}_0 = \vec{v}_A$ implies $\omega = 0$

$\vec{v}_0 \neq \vec{v}_A \Rightarrow \omega \neq 0$

Sol. : Angular momentum and linear momentum of system remains conserved.

16. Answer (A)

Hint & Sol. : To escape, total energy ≥ 0

17. Answer (C)

Hint : $F = -\frac{dU}{dX}$

Sol. : $U = 2\pi^2 x^2$

$F = -\frac{dU}{dX} = -4\pi^2 x$

$F = -4\pi^2 x$ $\qquad K = 4\pi^2$

$\omega^2 = 4\pi^2$

$\omega = 2\pi$

$\left[T = \left(\frac{2\pi}{\omega}\right) = 1\right]$

$E\left(\frac{1}{2}\right) = 2\pi^2\left(\frac{1}{2}\right)^2$

$= 2\pi^2 \times \frac{1}{4}$

$= \left(\frac{\pi^2}{2}\right)$

$E\left(\frac{1}{\sqrt{2}}\right) = \left(\pi^2\right)$

18. Answer (D)

Hint : $I = \int r^2 dm$

Sol. : Moment of Inertia of solid sphere $= \frac{2}{5}MR^2$ and also passes through centre of mass.

Moment of Inertia of circular arc = $\frac{1}{2}MR^2$

PART - II (CHEMISTRY)

19. Answer (08.00)

Hint : Element is polonium

Sol. : Electronic configuration as [Xe] $4f^{14}5d^{10}\ 6s^2\ 6p^4$ for last electron

n = 6, l = 1, m = +1

Hence, n + l + m is 6 + 1 + 1 = 8

20. Answer (04.00)

Hint : Species with zero bond order are He_2, Be_2, Ne_2 *i.e.* 3

Sol. : H_2, H_2^+, O_2^{2-}, Li_2, F_2 have bond order ≤ 1 *i.e.* 5

O_2 have a bond order of 2.

And $C_2^{2-}, N_2, O_2^+, O_2^{2+}$ have a bond order 3, CO^+ have a bond order of 3.5.

a = 8, b = 1, c = 5

21. Answer (04.00)

Hint : Effective nuclear charge (z^*) is given by

$z^* = z - \sigma$

peripheral electron corresponds to last e^-.

Sol. : Calculation for σ (screening constant) as

$\because$ last electron is in 4*s*-subshell so,

$\sigma = 1 \times 0.35 + 15 \times 0.85 + 10 \times 1$

$\sigma = 23.1$

$z^* = 3.9 \simeq 4$

22. Answer (17.00)

Hint : $\dfrac{C_p}{C_v} = \gamma$

Sol. : $\gamma = \dfrac{0.206}{0.148} \simeq 1.4$

$\because$ gas is diatomic = B = 2

Also, $C_p - C_v = R$

R = 0.206 – 0.148 = 0.058 cal/g

For 1 mol, R = 2 cal

Number of moles = $\dfrac{1}{2} \times 0.058$ = 0.029 moles

$\therefore$ molecular mass = $\dfrac{1}{\text{number of moles}}$

= 34.48 = A

i.e. $\dfrac{A}{B} = 17.24 \simeq 17$

23. Answer (50.00)

Hint : 1 atm = 760 mm of Hg

Sol. : Initially, $P_{NH_3} = 1\,\text{atm}$

After sparking

$$P_{NH_3} + P_{H_2} + P_{N_2} = 1 + \frac{38}{76} = \frac{3}{2}\ \text{atm}$$

From dissociation reaction,

$$2NH_3 \rightleftharpoons N_2(g) + 3H_2(g)\ \ 1\ \text{atm}$$

$$(1 - P) \qquad \frac{P}{2} \qquad \frac{3P}{2}$$

Final pressure as $1 - P + \dfrac{P}{2} + \dfrac{3P}{2} = \dfrac{3}{2}$

$$1 + P = \frac{3}{2}$$

$$P = \frac{1}{2}\ \text{atm}$$

% dissociation is $\dfrac{1/2}{1} \times 100 = 50\%$

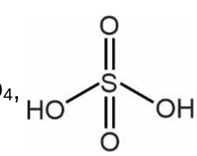
24. Answer (21.00)

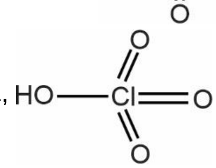
Hint : Fluorapatite is $[Ca_5(PO_4)_3F]$

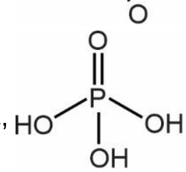
Sol. : Molecular formula can also be written as $Ca_5FO_{12}P_3$

25. Answer (02.00)

Hint : Lone pair on each oxygen atom contributes to H-bonding

Sol. : For H_2SO_4, , 2*d*π-*p*π bonds

For $HClO_4$, , 3*d*π-*p*π bonds

For H_3PO_4, , 1*d*π-*p*π bond

26. Answer (05.00)

Hint : Compound with molecular formula $C_6H_{12}O$ can show metamerism, keto-enol

Sol. : Also, it can show positional, chain and functional group isomerism.

27. Answer (C, D)

Hint : Stannane or SnH_4 is inert towards hydrolysis as Sn – H bond has insufficient bond polarity.

Sol. : $Te(OH)_6$ exists as, there is no steric crowding to accommodate six OH^- groups around it.

28. Answer (A, D)

Hint : Be and Mg does not impart colour in flame test.

Sol. : Na gives yellow colour in flame test

29. Answer (B, C, D)

Hint : The balanced redox reaction is

$$2MnO_4^-(aq) + Br^-(aq) + H_2O(l) \rightarrow 2MnO_2(s) + BrO_3^-(aq) + 2OH^-(aq)$$

Sol. : Hence option B, C and D are correct

30. Answer (B)

Hint : Coordinated water molecule present in $[Cr(H_2O)_6]Cl_3$

Sol. : $BaCl_2 \cdot 2H_2O$ water molecule present only in interstitials.

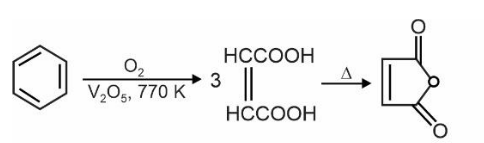
In $CuSO_4 \cdot 5H_2O$, $4H_2O$ molecules are coordinated

31. Answer (A, C)

Hint : Temporary hardness can be removed by Clark's method

Soln. : Permanent hardness can be removed by Calgon's method

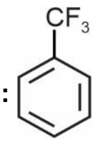
32. Answer (A, D)

Hint : 

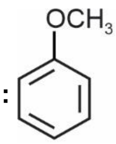
Benzene Maleic acid Maleic anhydride

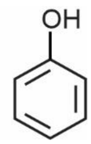
Sol. : A and B are not isomers

33. Answer (B)

Hint : 

has –H effect

Sol. : 

, 

have + M, –I effect.

34. Answer (C)

Hint : The atomic number greater than 100 given by IUPAC nomenclature, but some common names.

Sol. :

Atomic No.	IUPAC official name
110	Darmstadtium (Ds)
108	Hassium (Hs)
104	Rutherfordium (Rf)
105	Dubnium (Db)

35. Answer (A)

Hint : Coulombic force of attraction $\propto \frac{1}{r^2}$

Sol. : Dipole – Dipole forces

(i) In stationary state $\propto \frac{1}{r^3}$

(ii) In rotation state $\propto \frac{1}{r^6}$

36. Answer (D)

Hint : PbO_2 is amphoteric in nature

Sol. : $Sn + NaOH \rightarrow Na_2SnO_2 + H_2$

$PbO_2 + NaOH \rightarrow Na_2PbO_3 + H_2O$

$NaHCO_3 + HCl \rightarrow NaCl + H_2O + CO_2$

$NaOCl \rightarrow H_2O_2 \rightarrow NaCl + H_2O + O_2$

PART - III (MATHEMATICS)

37. Answer (07.00)

Hint : Make cases and count.

Sol. : Let $x + y + z = 30$ and $x \neq y \neq z$

Let $x < y < z$

If $x = 1$,

$$\left.\begin{array}{|c|c|} \hline y & z \\ \hline 2 & 27 \\ \vdots & \\ 14 & 15 \\ \hline \end{array}\right\} \text{13 triplets}$$

If $x = 2$,

$$\left.\begin{array}{|c|c|} \hline y & z \\ \hline 3 & 25 \\ \vdots & \\ 13 & 15 \\ \hline \end{array}\right\} \text{11 triplets}$$

If $x = 3$,

$$\left.\begin{array}{|c|c|} \hline y & z \\ \hline 4 & 23 \\ 5 & 22 \\ \vdots & \\ 13 & 14 \\ \hline \end{array}\right\} \text{10 triplets}$$

If $x = 4$,

$$\left.\begin{array}{|c|c|} \hline y & z \\ \hline 5 & 21 \\ \vdots & \\ 12 & 14 \\ \hline \end{array}\right\} \text{8 triplets}$$

If $x = 5$,

$$\left.\begin{array}{|c|c|} \hline y & z \\ \hline 6 & 19 \\ \vdots & \\ 12 & 13 \\ \hline \end{array}\right\} \text{7 triplets}$$

If $x = 6$,

$$\left.\begin{array}{|c|c|} \hline y & z \\ \hline 7 & 17 \\ \vdots & \\ 11 & 13 \\ \hline \end{array}\right\} \text{5 triplets}$$

If $x = 7$,

$$\left.\begin{array}{|c|c|} \hline y & z \\ \hline 8 & 15 \\ \vdots & \\ 11 & 12 \\ \hline \end{array}\right\} \text{4 triplets}$$

If $x = 8$,

$$\left.\begin{array}{|c|c|} \hline y & z \\ \hline 9 & 13 \\ 10 & 12 \\ \hline \end{array}\right\} \text{2 triplets}$$

If $x = 9$, $y = 10$, $z = 11 \rightarrow$ 1 triplet

Total triplets :

$2 + 1 + 4 + 5 + 7 + 8 + 10 + 11 + 13 = 61$

$\therefore \quad 6 + 1 = 7$

38. Answer (05.00)

Hint : Find z_1z_2 and equate for values of x and y.

Sol. : $z_1 = (8 \sin\theta + 7 \cos\theta) + i(\sin\theta + 4 \cos\theta)$

$z_2 = (\sin\theta + 4 \cos\theta) + i(8 \sin\theta + 7 \cos\theta)$

$z_1 = a + ib$

$z_2 = b + ia$

$z_1z_2 = (a + ib)(b + ia)$

$= ab - ba + i(a^2 + b^2)$

$= (a^2 + b^2)i = x + iy$

$\Rightarrow \quad x = 0;\ \ y = a^2 + b^2$

Now, $x + y = a^2 + b^2$

$M = (x + y)_{max}$

$x + y = (8 \sin\theta + 7 \cos\theta)^2 + (\sin\theta + 4 \cos\theta)^2$

$= 65 + 60 \sin 2\theta$

$(x + y)_{max} = 125$

$M^{1/3} = (125)^{1/3} = 5$

39. Answer (11.00)

Hint : Use Newton's formula.

Sol. : By Newton's formula,

$P_n = aP_{n-1} - bP_{n-2}$

a = sum of roots

b = product of roots

$P_n = P_{n-1} + P_{n-2}$

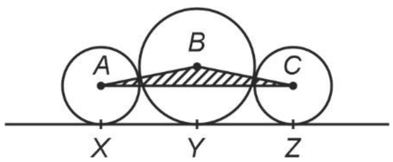
$P_n = 7 + 4$

$P_n = 11$

40. Answer (06.00)

Hint : Let radius of circles with centres A, B, C are a, b and c respectively, then

$$XZ = \sqrt{AC^2 - (a-c)^2} \quad \ldots\text{(i)}$$



Sol. : Area of quadrilateral $ACZX$

$$= \frac{1}{2}(a+c)40 = 300$$

$\Rightarrow$ $a + c = 15$ …(ii)

Hence, $a = 12$ and $c = 3$

Now, for finding the radius of circle touching both circles with centre A and C.

$XZ = XY + YZ$

$$40 = \sqrt{(b+12)^2 - (b-12)^2} + \sqrt{(b+3)^2 - (b-3)^2}$$

$$40 = \sqrt{48b} + \sqrt{12b}$$

$$\Rightarrow \quad b = \frac{400}{27}$$

Now, ar(ΔABC) = Area of quad. $ABYX$ + Area of quad. $BCZY$ – Area of quad. $ACZX$

$$\Delta = \frac{1}{2}\left(12 + \frac{400}{27}\right)\left(\frac{80}{3}\right) + \frac{1}{2}\left(3 + \frac{400}{27}\right)\left(\frac{40}{3}\right) - 300$$

$$= 476 + \frac{8}{27} - 300$$

$$[\Delta] = 176 + \frac{8}{27}$$

$[\Delta] - 170 = 176 - 170 = 6$

41. Answer (04.00)

Hint : Substitute the value of $2y$ in given expression, then apply AM $\geq$ GM.

Sol. : $\because$ x, y, z are in AP,

$x + z = 2y$ and hence

$$\frac{x+y}{2y-x} + \frac{y+z}{2y-z} = \frac{x+y}{z} + \frac{y+z}{x}$$

$$= \frac{y}{x} + \frac{y}{z} + \frac{x}{z} + \frac{z}{x}$$

$$= \frac{x}{z} + \frac{x+z}{2z} + \frac{x+z}{2x} + \frac{z}{x}$$

$$= 1 + \frac{3}{2}\left(\frac{x}{z} + \frac{z}{x}\right)$$

$$\geq 1 + 3 \qquad [\text{AM} \geq \text{GM}]$$

$\Rightarrow$ Minimum value of expression is 4.

42. Answer (02.00)

Hint : $|z - a| = r$ represents equation of circle and $|z - z_1| + |z - z_2| = k$ is equation of ellipse.

Sol. : $|z - 3 - i| = 2$ represents a circle with centre $3 + i$ and radius 2.

$|z - 2 + i| + |z - 4 - 3i| = 6$ represents ellipse with centre $\left|\frac{2-i+4+3i}{2}\right| = 3+i$ and length of major axis = 6.

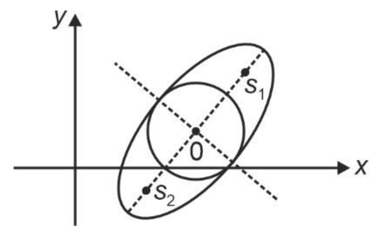
Also, distance between foci = $2\sqrt{5}$

$\Rightarrow$ Eccentricity = $\frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3} < 1$

$\therefore$ Minor axis = $2 \times 3\sqrt{1 - \frac{5}{9}} = 4$

$\because$ Diameter of circle = minor axis

So, the circle touches the ellipse internally.



43. Answer (04.00)

Hint : Represent b in terms of a.

Sol. : $a = \sum_{r=1}^{\infty} \frac{1}{r^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots \infty$

$$b = \sum \frac{1}{(2r-1)^2} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \infty$$

$$= \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots \infty\right) - \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \ldots \infty\right)$$

$$= \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots \infty\right) - \frac{1}{2^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots \infty\right)$$

$$b = a - \frac{1}{4}a$$

$$b = \frac{3}{4}a$$

$$\therefore \quad \frac{3a}{b} = 4$$

44. Answer (00.00)

Hint : Draw graph to find number of solutions.

Sol. : $[y + [y]] = 2\cos x$

$\Rightarrow \quad [y] = \cos x$

$$y = \frac{1}{3}[\sin x + [\sin x + [\sin x]]] = [\sin x]$$

$\Rightarrow \quad [\sin x] = \cos x$

Number of solution in $[0, 2\pi]$ is 0.

45. Answer (A, C, D)

Hint : Factorise the given equation.

Sol. : Given equation is

$\sin^2 x + (\cos x - 1)\sin x - \cos x - k\sin x + k = 0$

Given equation can be rewritten as

$\sin^2 x + \sin x \cdot \cos x - k\sin x - \sin x - \cos x + k = 0$

$\Rightarrow \quad (\sin x - 1)(\sin x + \cos x - k) = 0$

$\Rightarrow \quad \sin x = 1$ or $\sin x + \cos x = k$

$\sin x = 1$ is one real root in $(0, 2\pi)$

$\sin x + \cos x = k$ has two real roots $(0, 2\pi)$

$\Rightarrow \quad -\sqrt{2} < k < \sqrt{2}$

Integral value of k is –1, 0, 1.

But $k \pm 1$

$\because \quad k = 1 \Rightarrow x = \frac{\pi}{2}$

$\Rightarrow \quad k = 0, -1$

Hence, two possible integral values of k.

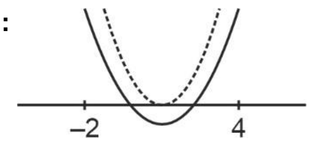
46. Answer (A, C, D)

Hint : Use location of roots

Sol. : Let $A = [-2, 4)$

$B \le A \Rightarrow$ Both roots of $x^2 - ax - 4 = 0$ should lie in $[-2, 4]$, by using location of roots two cases are possible.

Case I :



Both roots lie in $(-2, 4)$.

$D \ge 0 \Rightarrow a^2 + 16 > 0 \rightarrow a \in R$ …(i)

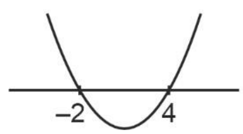
$-2 \in \frac{b}{2a} < 4 \Rightarrow -2 < \frac{a}{2} < 4 \rightarrow a \in [-4, 8]$ …(ii)

$f(-2) > 0 \Rightarrow 4 + 2a - 4 > 0 \rightarrow a \in (0, \infty)$ …(iii)

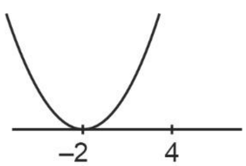
$f(4) > 0 \Rightarrow 12 - 4a > 0 \rightarrow a \in (-\infty, 3)$ …(iv)

Equations (i) $\cap$ (ii) $\cap$ (iii) $\cap$ (iv) gives $a \in (0, 3)$.

Case II : At least one root is –2.



or



By putting $x = -2$ into $x^2 - ax - 4 = 0$, we get

$a = 0$

Hence, $x^2 - 4 = 0$ have 2, roots 2 and –2 both lying in $[-2, 4)$.

Hence, $a = 0$ is also included

$\therefore \quad S = [0, 3)$

47. Answer (C, D)

Hint : Assume

$b = a + p$

$c = a + 2p \quad p =$ common difference

$d = a + 3p$

Sol. : Let $b = a + p$

$c = a + 2p$ p = common difference

$d = a + 3p$

$$\frac{\frac{1}{a}+\frac{1}{d}}{\frac{1}{b}+\frac{1}{c}} = \frac{\frac{1}{a}+\frac{1}{a+3p}}{\frac{1}{a+p}+\frac{1}{a+2p}}$$

$$= \frac{(a+p)(a+2p)}{a(a+3p)}$$

$$= \frac{a^2+3ap+2p^2}{a^2+3ap} > 1$$

$$\therefore \quad \frac{1}{a}+\frac{1}{d} > \frac{1}{b}+\frac{1}{c}$$

$$\left(\frac{1}{b}+\frac{1}{c}\right)(a+d) = \left(\frac{1}{a+p}+\frac{1}{a+2p}\right)(a+a+3p)$$

$$= \frac{(2a+3p)^2}{a^2+3pa+2p^2}$$

$$= 4 + \frac{p^2}{a^2+3ap+2p^2} > 4$$

48. Answer (B, C, D)

Hint : Count all the cases for equality and inequality.

Sol. : Triplets with $x = y < z$, $x < y < z$, $y < x < z$ can be chosen in

${}^{n+1}C_2$, ${}^{n+1}C_3$, ${}^{n+1}C_3$ ways

There are ${}^{n+1}C_2 + 2({}^{n+1}C_3) = {}^{n+2}C_2 + {}^{n+1}C_3$

$= 2({}^{n+2}C_3) - {}^{n+1}C_2$

49. Answer (A, B, C, D)

Hint : $\cos\alpha = \frac{1}{2}\left(x+\frac{1}{x}\right)$

$$\cos\beta = \frac{1}{2}\left(y+\frac{1}{y}\right)$$

$$\because \quad xy > 0 \ \Rightarrow \ x+\frac{1}{x} \ge 2 \text{ or } \le -2$$

$$y+\frac{1}{y} \ge 2 \text{ or } \le -2$$

$\Rightarrow$ $\cos\alpha = 1$, $\cos\beta = 1$ …(i)

or $\cos\alpha = -1$, $\cos\beta = -1$ …(ii)

Sol. : $\therefore$ $\cos\alpha \cos\beta = 1$

$(\cos\alpha + \cos\beta)^2 = 4$

$\Rightarrow$ $\alpha + \beta$ is an even multiple of π.

$\Rightarrow$ $\sin(\alpha + \beta + \gamma) = \sin(2n\pi + \gamma)$

$= \sin\gamma$

Also, $\sin\alpha = \sin\beta = 0$

50. Answer (D)

Hint : Expression can be written as sum of squares.

Sol. : $x^2 + 9y^2 + 25z^2 = 15yz + 5zx + 3xy$

$$= \frac{1}{2}\left[(x-3y)^2+(3y-5z)+(5z-x)^2\right] = 0$$

$$\Rightarrow \quad x = 3y = \frac{1}{x} = \frac{1}{3y}$$

$$3y = 5z = \frac{1}{z} = \frac{5}{3y}$$

$$5z = x = \frac{1}{5z} = \frac{1}{x}$$

$$\frac{1}{x}+\frac{1}{z} = \frac{1}{3y}+\frac{5}{3y} = \frac{2}{y}$$

$\therefore$ $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in AP

$\therefore$ $x, y, z \to$ HP

51. Answer (B)

Hint : Use cosine rule.

Sol. :

(I) Let the sides a and b are the roots of $x^2 - 5x + 3 = 0$, then $a + b = 5$ and $ab = 3$

Also, $\angle c = \frac{\pi}{3}$

$$\therefore \quad \frac{1}{2} = \frac{a^2+b^2-c^2}{2ab}$$

$\Rightarrow$ $ab = (a + b)^2 - 2ab - c^2$

$\Rightarrow$ $c = 4$

Now, $rR = \frac{abc}{2(a+b+c)} = \frac{3\times 4}{2(5+4)} = \frac{2}{3}$

(II) We have,

$$a = \frac{r_1(r_2+r_3)}{\sqrt{r_1r_2+r_2r_3+r_3r_1}} = \frac{8\times 36}{24} = 12$$

(III) Given $1 \le a \le 10$

Also, for roots of opposite sign

$2a^2 - 7a + 3 < 0$

or $\frac{1}{2} < a < 3 \Rightarrow a = 1$ or 2

$\therefore$ Desired probability = $\frac{2}{10} = \frac{1}{5}$

(IV) (α, β) lie on director circle of an ellipse, *i.e.*, on $x^2 + y^2 = 9$

So, we assume $\alpha = 3\cos\theta$, $\beta = 3\sin\theta$

Thus, $F = 12\cos\theta + 9\sin\theta$

$= 3[4\cos\theta + 3\sin\theta]$

$-15 \le F \le 15$

52. Answer (C)

Hint : Product of roots is negative.

Sol. :

(I) Product < 0

$$\frac{b^2 - 3b + 2}{3} < 0$$

$b \in (1, 2)$

(II) Probability problem is solved.

$1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})$

$$1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)(1 - \alpha) = \frac{2}{3} + \frac{\alpha}{3} \in \left(\frac{2}{3}, 1\right)$$

(III) $x = 5 - (y + z)$

$yz + x(y + z) = 8$

$yz + (y + z)(5 - (y + z) - 8 = 0$

$y^2 + y(z - 5) + (z^2 - 5z + 8) = 0$

For real solution,

$(z - 5)^2 - 4(z^2 - 5z + 8) \ge 0$

$$(z - 1)\left(z - \frac{7}{3}\right) \le 0$$

$$z \in \left[1, \frac{7}{3}\right]$$

(IV) $|\sec x| + |\mathrm{cosec}\, x|$

Range $[2\sqrt{2}, \infty)$

53. Answer (D)

Hint : Use cosine rule.

Sol. :

(I) $\frac{b^2 + c^2 - a^2}{2bc} = \frac{b}{2c} \Rightarrow c = a$

(II) $1 + a \ge 2\sqrt{a}$ (AM $\ge$ GM)

$1 + b \ge 2\sqrt{b}$

$1 + c \ge 2\sqrt{c}$

$1 + d \ge 2\sqrt{d}$

$\therefore$ $(1 + a)(1 + b)(1 + c)(1 + d) \ge 16\sqrt{abcd}$

≥ 16

$\therefore$ Minimum = 16

(III) $5^{x + 2} > 5^{-2/x}$

$$\Rightarrow x + 2 > \frac{-2}{x}$$

$$\frac{x^2 + 2x + 2}{x} > 0$$

$$\frac{(x + 1)^2 + 1}{x} > 0$$

$\Rightarrow x \in (0, \infty)$

(IV) $x^2 - (3k - 1)x + 2k^2 - 3k - 2 \le 0$

$D = (k + 3)^2$

$\because$ $\forall\, x \in R$, $f(x) \le 0$

$\Rightarrow D \le 0$

$\Rightarrow D = 0$

$\Rightarrow k = -3$

$S = \{-3\}$

54. Answer (A)

Hint : $t_2 = -t_1 - \frac{2}{t_1}$

Sol. :

(I) Equation of normal is $y = -tx + 2at + at^2$ at $P(t)$.

It intersect the curve again at point $Q(t_1)$ on the parabola such that

$$t_2 = -t - \frac{2}{t}$$

Again, slope of OP is $\frac{2}{t} = M_{OP}$

Also, slope of OQ is $\frac{2}{t_1} = M_{OQ}$

$\because \quad M_{OP} \cdot M_{OQ} = -1 = \frac{4}{t\,t_1}$

$\Rightarrow \quad tt_1 = -4$

$$t\left(-t - \frac{2}{t}\right) = -4$$

$\Rightarrow \quad t^2 = 2$

(II) $P(1, 2)$, $Q(4, 4)$, $R(16, 8)$

Now ar (ΔPQR) = 6 sq. unit

(III) Equation of normal from any point $P(am^2, -2m)$ is $y = mx - am^3 - 2am$

It passes through $\left(\frac{11}{4}, \frac{1}{4}\right)$

$\Rightarrow \quad 4m^3 + 8m - 11m + 1 = 0$

$\Rightarrow \quad m = \frac{1}{2}, -1$ two distinct normals

(IV) $\because$ Normal at $P(t_1)$ if meets the curve again at (t_2), then $t_2 = -t_1 - \frac{2}{t_1}$

Such that here normal at $P(1)$ meets the curve again at $Q(t)$.

$\Rightarrow \quad t = -1 - \frac{2}{1} = -3$

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