



Aakash

Medical | IIT-JEE | Foundations

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Answers & Hints

for

WBJEE-2019

MATHEMATICS

CATEGORY - I (Q1 to Q50)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, $\frac{1}{4}$ mark will be deducted.

1. $\lim_{x \rightarrow 0^+} (x^n \ln x), n > 0$
- (A) does not exist
(B) exists and is zero
(C) exists and is 1
(D) exists and is e^{-1}

Ans : (B)

Hint : $\lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{x^n}} \left(\frac{\infty}{\infty} \right)$. Applying LH rule

$$\Rightarrow \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{-n}{x^{n+1}}} = 0$$

2. If $\int \cos x \log \left(\tan \frac{x}{2} \right) dx = \sin x \log \left(\tan \frac{x}{2} \right) + f(x)$ then $f(x)$ is equal to, (assuming c is an arbitrary real constant)
- (A) c (B) $c - x$ (C) $c + x$ (D) $2x + c$

Ans : (B)

Hint : IBP $\Rightarrow I = \sin x \log \left(\tan \frac{x}{2} \right) - x + c$

$$\therefore f(x) = c - x$$

3. $y = \int \cos \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\} dx$ is an equation of a family of

(A) straight lines (B) circles (C) ellipses (D) parabolas

Ans : (D)

Hint : Putting $x = \cos 2\theta \therefore \cos \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\} = \cos 2\theta = x$

$$\therefore y = \int x dx = \frac{x^2}{2} + c$$

4. The value of the integration $\int_{-\pi/4}^{\pi/4} \left(\lambda |\sin x| + \frac{\mu \sin x}{1 + \cos x} + \gamma \right) dx$

(A) is independent of λ only (B) is independent of μ only
(C) is independent of γ only (D) depends on λ, μ and γ

Ans : (B)

$$\text{Hint : } I = 2\lambda \int_0^{\pi/4} \sin x dx + \mu \int_{-\pi/4}^{\pi/4} \tan \frac{x}{2} dx + \gamma \int_{-\pi/4}^{\pi/4} dx$$

$$= 2\lambda \left(1 - \frac{1}{\sqrt{2}} \right) - 0 + \gamma \left(\frac{\pi}{2} \right) \quad \left(\because \tan \frac{x}{2} \text{ is an odd function} \right)$$

5. The value of $\lim_{x \rightarrow 0} \frac{1}{x} \left[\int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t} dt \right]$ is equal to

(A) $e^{\sin^2 y}$ (B) $e^{2 \sin y}$ (C) $e^{|\sin y|}$ (D) $e^{\operatorname{cosec}^2 y}$

Ans : (A)

$$\text{Hint : } = \lim_{x \rightarrow 0} \frac{1}{x} \left(\int_y^a e^{\sin^2 t} dt + \int_a^{x+y} e^{\sin^2 t} dt \right)$$

$$= \lim_{x \rightarrow 0} \frac{1}{x} \int_y^{x+y} e^{\sin^2 t} dt \quad \left(\text{form } \frac{0}{0} \right)$$

Applying LH rule

$$= \lim_{x \rightarrow 0} \frac{e^{\sin^2(x+y)} \cdot 1 - 0}{1} = e^{\sin^2 y}$$

6. If $\int 2^{2^x} \cdot 2^x dx = A \cdot 2^{2^x} + c$, then $A =$

(A) $\frac{1}{\log 2}$ (B) $\log 2$ (C) $(\log 2)^2$ (D) $\frac{1}{(\log 2)^2}$

Ans : (D)

$$\text{Hint : } 2^x = z \Rightarrow 2^x dx = \frac{dz}{\ln 2}$$

$$\Rightarrow \frac{1}{\ln 2} \int 2^z dz = \frac{1}{(\ln 2)^2} 2^{2^x} + c$$

7. The value of the integral $\int_{-1}^1 \left\{ \frac{x^{2015}}{e^{|x|}(x^2 + \cos x)} + \frac{1}{e^{|x|}} \right\} dx$ is equal to

(A) 0 (B) $1 - e^{-1}$ (C) $2e^{-1}$ (D) $2(1 - e^{-1})$

Ans : (D)

Hint : $\Rightarrow \int_{-1}^1 \left(\text{odd} + \frac{1}{e^{|x|}} \right) dx$
 $= 0 + 2 \int_0^1 \frac{dx}{e^x} = 2(1 - e^{-1})$

8. $\lim_{n \rightarrow \infty} \frac{3}{n} \left\{ 1 + \sqrt{\frac{n}{n+3}} + \sqrt{\frac{n}{n+6}} + \sqrt{\frac{n}{n+9}} + \dots + \sqrt{\frac{n}{n+3(n-1)}} \right\}$

(A) does not exit (B) is 1 (C) is 2 (D) is 3

Ans : (C)

Hint : $\lim_{n \rightarrow \infty} \frac{3}{n} \sum_{r=0}^{n-1} \sqrt{\frac{1}{1+3\left(\frac{r}{n}\right)}}$
 $= 3 \int_0^1 \frac{dx}{\sqrt{1+3x}} = 2 \left[\sqrt{1+3x} \right]_0^1 = 2$

9. The general solution of the differential equation $\left(1 + e^{\frac{x}{y}}\right) dx + \left(1 - \frac{x}{y}\right) e^{\frac{x}{y}} dy = 0$ is (c is an arbitrary constant)

(A) $x - ye^{\frac{x}{y}} = c$ (B) $y - xe^{\frac{x}{y}} = c$ (C) $x + ye^{\frac{x}{y}} = c$ (D) $y + xe^{\frac{x}{y}} = c$

Ans : (C)

Hint : Putting $\frac{x}{y} = v \Rightarrow x = vy$

$$\Rightarrow \frac{dx}{dt} = v + y \frac{dv}{dy} \Rightarrow y \frac{dv}{dy} = - \left(\frac{v + e^v}{1 + e^v} \right) \Rightarrow \left(\frac{1 + e^v}{v + e^v} \right) dv + \frac{dy}{y} = 0$$

Integrating

$$\Rightarrow \ln(v + e^v) + \ln y = c_1$$

$$\Rightarrow y \left(\frac{x}{y} + e^{\frac{x}{y}} \right) = c \Rightarrow x + ye^{\frac{x}{y}} = c$$

10. General solution of $(x + y)^2 \frac{dy}{dx} = a^2$, $a \neq 0$ is (c is an arbitrary constant)

(A) $\frac{x}{a} = \tan \frac{y}{a} + c$ (B) $\tan xy = c$ (C) $\tan(x + y) = c$ (D) $\tan \frac{y+c}{a} = \frac{x+y}{a}$

Ans : (D)

Hint : $x + y = z, \Rightarrow \frac{dy}{dx} = \frac{dz}{dx} - 1$

$$\Rightarrow \frac{z^2 dz}{a^2 + z^2} = dx$$

Integrating

$$\Rightarrow x + y - a \tan^{-1} \frac{x+y}{a} = x + c_1 \Rightarrow \tan \left(\frac{y - c_1}{a} \right) = \frac{x+y}{a}$$

$$\Rightarrow \tan \left(\frac{y+c}{a} \right) = \frac{x+y}{a} \quad (c_1 = -c)$$

11. Let P (4, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at P intersects the X-axis at (16, 0), then the eccentricity of the hyperbola is

- (A) $\frac{\sqrt{5}}{2}$ (B) 2 (C) $\sqrt{2}$ (D) $\sqrt{3}$

Ans : (B)

Hint : Normal at P(4, 3)

$$\frac{a^2 x}{4} + \frac{b^2 y}{3} = a^2 + b^2 \text{ through } (16, 0)$$

$$\Rightarrow 4a^2 = a^2 + b^2$$

$$\Rightarrow \frac{b^2}{a^2} = 3 \therefore e = 2$$

12. If the radius of a spherical balloon increases by 0.1%, then its volume increases approximately by
(A) 0.2% (B) 0.3% (C) 0.4% (D) 0.05%

Ans : (B)

$$\text{Hint : } v = \frac{4}{3} \pi r^3, \therefore \Delta r = \left(\frac{0.1}{100} \right) r \therefore \Delta v = \left(\frac{dv}{dr} \right) \Delta r = 4\pi r^2 \cdot \left(\frac{0.1}{100} \right) r = \left(\frac{0.3}{100} \right) v$$

13. The three sides of a right-angled triangle are in G.P (geometric progression). If the two acute angles be α and β , then $\tan \alpha$ and $\tan \beta$ are

- (A) $\frac{\sqrt{5}+1}{2}$ and $\frac{\sqrt{5}-1}{2}$ (B) $\sqrt{\frac{\sqrt{5}+1}{2}}$ and $\sqrt{\frac{\sqrt{5}-1}{2}}$ (C) $\sqrt{5}$ and $\frac{1}{\sqrt{5}}$ (D) $\frac{\sqrt{5}}{2}$ and $\frac{2}{\sqrt{5}}$

Ans : (B)

$$\text{Hint : } \therefore \left(\frac{a}{r} \right)^2 + a^2 = a^2 r^2 \quad (r > 1), a \neq 0 \Rightarrow r^4 - r^2 - 1 = 0 \Rightarrow r^2 = \frac{1 \pm \sqrt{5}}{2} \Rightarrow r = \pm \sqrt{\frac{\sqrt{5}+1}{2}}$$

$$\Rightarrow r = \sqrt{\frac{\sqrt{5}+1}{2}} (r > 1), \frac{1}{r} = \sqrt{\frac{\sqrt{5}-1}{2}} \left(\because \alpha + \beta = 90^\circ \Rightarrow \tan \alpha = \cot \beta = \frac{1}{\tan \beta} \right)$$

14. If $\log_2 6 + \frac{1}{2x} = \log_2 \left(2^{\frac{1}{x}} + 8 \right)$, then the values of x are

- (A) $\frac{1}{4}, \frac{1}{3}$ (B) $\frac{1}{4}, \frac{1}{2}$ (C) $-\frac{1}{4}, \frac{1}{2}$ (D) $\frac{1}{3}, -\frac{1}{2}$

Ans : (B)

$$\text{Hint : } \log_2 6 + \log_2 2^{\frac{1}{2x}} = \log_2 \left(2^{\frac{1}{x}} + 8 \right) \Rightarrow 6 \cdot 2^{\frac{1}{2x}} = 2^{\frac{1}{x}} + 8, \text{ let } 2^{\frac{1}{2x}} = a$$

$$\Rightarrow a^2 - 6a + 8 = 0 \Rightarrow a = 4, a = 2 \Rightarrow x = \frac{1}{4}, \frac{1}{2}$$

15. Let z be a complex number such that the principal value of argument, $\arg z > 0$. Then $\arg z - \arg(-z)$ is

(A) $\frac{\pi}{2}$ (B) $\pm\pi$ (C) π (D) $-\pi$

Ans : (C)

Hint : $\arg(z) - \arg(-z) = \arg\left(\frac{z}{-z}\right) = \arg(-1) = \pi$

16. The general value of the real angle θ , which satisfies the equation, $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)\dots(\cos n\theta + i\sin n\theta) = 1$ is given by, (assuming k is an integer)

(A) $\frac{2k\pi}{n+2}$ (B) $\frac{4k\pi}{n(n+1)}$ (C) $\frac{4k\pi}{n+1}$ (D) $\frac{6k\pi}{n(n+1)}$

Ans : (B)

Hint : $e^{i\theta} \cdot e^{i(2\theta)} \cdot e^{i(3\theta)} \dots e^{i(n\theta)} = 1 \Rightarrow e^{i\left(\frac{n(n+1)}{2}\theta\right)} = e^{i2k\pi} \Rightarrow \theta = \frac{4k\pi}{n(n+1)}$

17. Let a, b, c be real numbers such that $a + b + c < 0$ and the quadratic equation $ax^2 + bx + c = 0$ has imaginary roots. Then

(A) $a > 0, c > 0$ (B) $a > 0, c < 0$ (C) $a < 0, c > 0$ (D) $a < 0, c < 0$

Ans : (D)

Hint : $f(x) = ax^2 + bx + c$, $f(1) < 0$ so $f(x) < 0 \forall x \in \mathbb{R} \Rightarrow f(0) < 0 \Rightarrow c < 0 \Rightarrow a < 0$ and $c < 0$

18. A candidate is required to answer 6 out of 12 questions which are divided into two parts A and B, each containing 6 questions and he/she is not permitted to attempt more than 4 questions from any part. In how many different ways can he/she make up his/her choice of 6 questions ?

(A) 850 (B) 800 (C) 750 (D) 700

Ans : (A)

Hint : A B

$$4 \quad 2 \rightarrow {}^6C_4 \times {}^6C_2$$

$$2 \quad 4 \rightarrow {}^6C_2 \times {}^6C_4$$

$$3 \quad 3 \rightarrow {}^6C_3 \times {}^6C_3$$

$$\text{Total} = 2 \times {}^6C_2 \times {}^6C_4 \times {}^6C_3 \times {}^6C_3 = 450 + 400 = 850$$

19. There are 7 greetings cards, each of a different colour and 7 envelopes of same 7 colours as that of the cards. The number of ways in which the cards can be put in envelopes, so that exactly 4 of the cards go into envelopes of respective colour is,

(A) ${}^7C_3$ (B) $2 \cdot {}^7C_3$ (C) $3! \cdot {}^4C_4$ (D) $3! \cdot {}^7C_3 \cdot {}^4C_3$

Ans : (B)

Hint : ${}^7C_4 \times 3! \left(\frac{1}{2!} - \frac{1}{3!} \right) = {}^7C_3 \times 2$

20. $7^{2n} + 16n - 1 (n \in \mathbb{N})$ is divisible by

(A) 65 (B) 63 (C) 61 (D) 64

Ans : (D)

Hint : $(1+48)^n + 16n - 1 = 64n + {}^nC_2 48^2 + \dots = 64k$

21. The number of irrational terms in the expansion of $\left(3^{\frac{1}{8}} + 5^{\frac{1}{4}}\right)^{84}$ is

(A) 73 (B) 74 (C) 75 (D) 76

Ans : (B)

Hint : $T_{r+1} = {}^{84}C_r \left(3^{\frac{1}{8}}\right)^{84-r} \left(5^{\frac{1}{4}}\right)^r = {}^{84}C_r \cdot 3^{\left(\frac{84-r}{8}\right)} \cdot 5^{\frac{r}{4}}$

T_{r+1} is rational for $r = 4, 12, 20, 28, 36, 44, 52, 60, 68, 76, 84$, Number of Irrational terms = $85 - 11 = 74$

22. Let A be a square matrix of order 3 whose all entries are 1 and let I_3 be the identity matrix of order 3. Then the matrix $A - 3I_3$ is

(A) invertible (B) orthogonal
(C) non-invertible (D) real Skew Symmetric matrix

Ans : (A)

Hint : $A - 3I_3 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $|A - 3I_3| \neq 0$

23. If M is any square matrix of order 3 over $\mathbb{R}$ and If M' be the transpose of M , then $\text{adj}(M') - (\text{adj } M)'$ is equal to

(A) M (B) M' (C) null matrix (D) identity matrix

Ans : (C)

Hint : $\text{adj}(M') = (\text{adj}(M))'$

24. If $A = \begin{pmatrix} 5 & 5x & x \\ 0 & x & 5x \\ 0 & 0 & 5 \end{pmatrix}$ and $|A^2| = 25$, then $|x|$ is equal to

(A) $\frac{1}{5}$ (B) 5 (C) 5^2 (D) 1

Ans : (A)

Hint : $|A^2| = |A|^2 = 25$

$\Rightarrow (25x)^2 = 25$

$\Rightarrow x = \frac{1}{5}$

25. Let A and B be two square matrices of order 3 and $AB = O_3$, where O_3 denotes the null matrix of order 3. Then,

(A) must be $A = O_3$, $B = O_3$ (B) if $A \neq O_3$, must be $B \neq O_3$
(C) if $A = O_3$, must be $B \neq O_3$ (D) may be $A \neq O_3$, $B \neq O_3$

Ans : (D)

Hint : $AB = O_3$

A, B may not be null matrix

26. Let P and T be the subsets of $X - Y$ plane defined by

$P = \{(x, y) : x > 0, y > 0 \text{ and } x^2 + y^2 = 1\}$

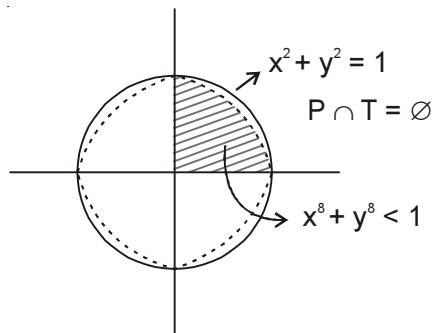
$T = \{(x, y) : x > 0, y > 0 \text{ and } x^8 + y^8 < 1\}$

Then $P \cap T$ is

(A) the void set Φ (B) P (C) T (D) $P - T^c$

Ans : (A)

Hint :



27. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 - \frac{x^2}{1+x^2}$ for all $x \in \mathbb{R}$. Then

- (A) f is one – one but not onto mapping
 (B) f is onto but not one – one mapping
 (C) f is both one – one and onto
 (D) f is neither one – one nor onto

Ans : (D)

Hint : $f(-x) = f(x)$, so many-one and into as Codomain is $\mathbb{R}$

28. Let the relation ρ be defined on $\mathbb{R}$ as $a\rho b$ iff $1 + ab > 0$. Then

- (A) ρ is reflexive only
 (B) ρ is equivalence relation
 (C) ρ is reflexive and transitive but not symmetric
 (D) ρ is reflexive and symmetric but not transitive

Ans : (D)

Hint : $1 + a^2 > 0$, so reflexive $1 + ab = 1 + ba > 0$ so symmetric
 $1 + ab > 0$ and $1 + bc > 0$ does not
 imply $1 + ac > 0$ so not transitive

29. A problem in mathematics is given to 4 students whose chances of solving individually are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$. The probability that the problem will be solved at least by one student is

- (A) $\frac{2}{3}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{3}{4}$

Ans : (C)

Hint : $P(A \cup B \cup C \cup D) = 1 - P(A^c \cap B^c \cap C^c \cap D^c)$

$$= 1 - P(A^c) \cdot P(B^c) \cdot P(C^c) \cdot P(D^c)$$

$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$$

$$= 1 - \frac{1}{5} = \frac{4}{5}$$

30. If X is a random variable such that $\sigma(X) = 2.6$, then $\sigma(1 - 4X)$ is equal to,

- (A) 7.8 (B) -10.4 (C) 13 (D) 10.4

Ans : (D)

Hint : $\sigma(a + bx) = |b| \sigma(x)$

31. If $e^{\sin x} - e^{-\sin x} - 4 = 0$, then the number of real values of x is

- (A) 0 (B) 1 (C) 2 (D) 3

Ans : (A)

Hint : $e^{\sin x} - \frac{1}{e^{\sin x}} - 4 = 0$

$$\Rightarrow e^{\sin x} = 2 \pm \sqrt{5}$$

$$\therefore -1 \leq \sin x \leq 1$$

$$\Rightarrow \frac{1}{e} \leq e^{\sin x} \leq e$$

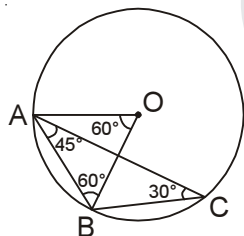
$\Rightarrow$ No solutions exist

32. The angles of a triangle are in the ratio 2 : 3 : 7 and the radius of the circumscribed circle is 10 cm. The length of the smallest side is

- (A) 2 cm (B) 5 cm (C) 7 cm (D) 10 cm

Ans : (D)

Hint :



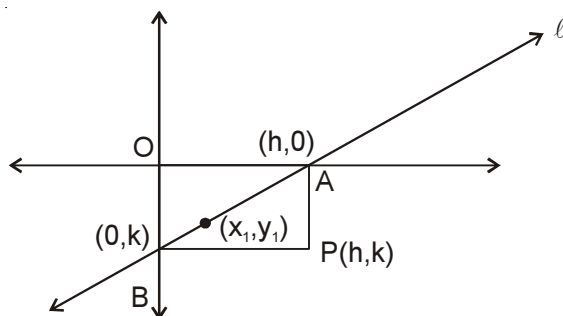
In the figure $\triangle ABC$ is the respective triangle, O is the circumcentre. $\angle AOB = 2\angle ACB = 2 \times 30^\circ = 60^\circ \Rightarrow \angle OAB = \angle OBA = 60^\circ \Rightarrow \triangle OAB$ is equilateral $\Rightarrow OA = AB = OB = 10$ cm.

33. A variable line passes through a fixed point (x_1, y_1) & meets the axes at A and B . If the rectangle $OAPB$ be completed, the locus of P is, (O being the origin of the system of axes)

- (A) $(y - y_1)^2 = 4(x - x_1)$ (B) $\frac{x_1}{x} + \frac{y_1}{y} = 1$ (C) $x^2 + y^2 = x_1^2 + y_1^2$ (D) $\frac{x^2}{2x_1^2} + \frac{y^2}{y_1^2} = 1$

Ans : (B)

Hint :



From the figure, ℓ passes through $(h, 0)$, $(0, k)$

Thus, $\frac{x}{h} + \frac{y}{k} = 1$, passes through (x_1, y_1)

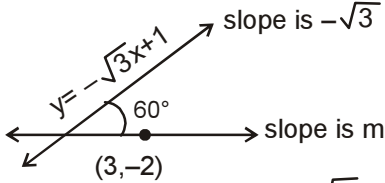
then, $\frac{x_1}{h} + \frac{y_1}{k} = 1 \Rightarrow$ locus is $\frac{x_1}{x} + \frac{y_1}{y} = 1$

34. A straight line through the point $(3, -2)$ is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If it intersects the X-axis, then its equation will be

(A) $y + x\sqrt{3} + 2 + 3\sqrt{3} = 0$ (B) $y - x\sqrt{3} + 2 + 3\sqrt{3} = 0$ (C) $y - x\sqrt{3} - 2 - 2\sqrt{3} = 0$ (D) $x - x\sqrt{3} + 2 - 3\sqrt{3} = 0$

Ans : (B)

Hint :



$$\sqrt{3} = \left| \frac{(-\sqrt{3}) - m}{1 + (-\sqrt{3})m} \right|$$

$$\Rightarrow m = \sqrt{3} \quad (\because \text{it is non parallel to x axis})$$

$$\Rightarrow \frac{y + 2}{x - 3} = \sqrt{3}$$

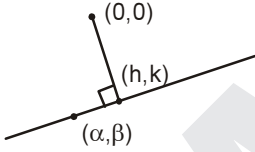
$$\Rightarrow y - x\sqrt{3} + 2 + 3\sqrt{3} = 0$$

35. A variable line passes through the fixed point (α, β) . The locus of the foot of the perpendicular from the origin on the line is,

(A) $x^2 + y^2 - \alpha x - \beta y = 0$ (B) $x^2 - y^2 + 2\alpha x + 2\beta y = 0$ (C) $\alpha x + \beta y \pm \sqrt{(\alpha^2 + \beta^2)} = 0$ (D) $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$

Ans : (A)

Hint :



$$\frac{k - \beta}{h - \alpha} \times \frac{k - 0}{h - 0} = -1$$

$$\Rightarrow \text{locus is } y(y - \beta) = -x(x - \alpha)$$

$$\Rightarrow x^2 + y^2 - \alpha x - \beta y = 0$$

36. If the point of intersection of the lines $2ax + 4ay + c = 0$ and $7bx + 3by - d = 0$ lies in the 4th quadrant and is equidistant from the two axes, where a, b, c and d are non-zero numbers, then $ad : bc$ equals to

(A) 2:3 (B) 2:1 (C) 1:1 (D) 3:2

Ans : (B)

Hint :

$$2ax + 4ay + c = 0$$

$$7bx + 3by - d = 0$$

$$x = \frac{4ad + 3bc}{+22ab}, y = \frac{2ad + 7bc}{22ab}$$

$$x = y \Rightarrow 4ad + 3bc = 2ad + 7bc$$

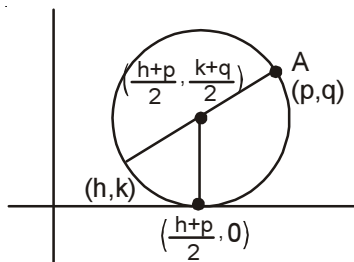
$$\Rightarrow 2ad = 4bc \Rightarrow \frac{ad}{bc} = \frac{2}{1}$$

37. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis. The locus of the other end of the diameter through A is

(A) $(x - p)^2 = 4qy$ (B) $(x - q)^2 = 4py$ (C) $(y - p)^2 = 4qx$ (D) $(y - q)^2 = 4px$

Ans : (A)

Hint :



$$\Rightarrow (h-p)^2 + (k-q)^2 = 4 \left(\frac{k+q}{2} \right)^2$$

$$= k^2 + 2kq + q^2 \Rightarrow (x-p)^2 = 4qy$$

38. If $P(0,0)$, $Q(1,0)$ and $R\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ are three given points, then the centre of the circle for which the lines PQ, QR and RP are the tangents is

- (A) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (B) $\left(\frac{1}{2}, \frac{\sqrt{3}}{4}\right)$ (C) $\left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$ (D) $\left(\frac{1}{2}, \frac{-1}{\sqrt{3}}\right)$

Ans : (C)

Hint : ΔPQR is equilateral, thus, the centroid is the incentre = $\left(\frac{0+1+\frac{1}{2}}{3}, \frac{0+0+\frac{\sqrt{3}}{2}}{3}\right) = \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$

39. For the hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$, which of the following remains fixed when α varies ?

- (A) directrix (B) vertices (C) foci (D) eccentricity

Ans : (C)

Hint : foci $(\pm ae, 0) = (\pm \cos \alpha \cdot \sec \alpha, 0) = (\pm 1, 0)$ which is fixed

40. S and T are the foci of an ellipse and B is the end point of the minor axis. If STB is equilateral triangle, the eccentricity of the ellipse is

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

Ans : (C)

Hint : $2ae = \sqrt{(ae)^2 + b^2} \Rightarrow \frac{b^2}{a^2} = 3e^2 \Rightarrow e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2}$

41. The equation of the directrices of the hyperbola $3x^2 - 3y^2 - 18x + 12y + 2 = 0$ is

- (A) $x = 3 \pm \sqrt{\frac{13}{6}}$ (B) $x = 3 \pm \sqrt{\frac{6}{13}}$ (C) $x = 6 \pm \sqrt{\frac{13}{3}}$ (D) $x = 6 \pm \sqrt{\frac{3}{13}}$

Ans : (A)

Hint : $3x^2 - 3y^2 - 18x + 12y + 2 = 0 \Rightarrow \frac{(x-3)^2}{\left(\sqrt{\frac{13}{3}}\right)^2} - \frac{(y-2)^2}{\left(\sqrt{\frac{13}{3}}\right)^2} = 1$

Here $a = b = \sqrt{\frac{13}{3}}$ and $e = \sqrt{2} \therefore$ the equations of the directrices are $x = 3 \pm \sqrt{\frac{13}{6}}$

42. P is the extremity of the latusrectum of ellipse $3x^2 + 4y^2 = 48$ in the first quadrant. The eccentric angle of P is

- (A) $\frac{\pi}{8}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Ans : (C)

Hint : $3x^2 + 4y^2 = 48 \Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1$. Here $a = 4$, $b = 2\sqrt{3}$ and $e = \frac{1}{2}$. $\therefore$ (co-ordinates of P are (2,3)

$$\therefore (4\cos\theta, 2\sqrt{3}\sin\theta) \equiv (2, 3) \Rightarrow \theta = \frac{\pi}{3}$$

43. The direction ratios of the normal to the plane passing through the points (1, 2, -3), (-1, -2, 1) and parallel to

$$\frac{x-2}{2} = \frac{y+1}{3} = \frac{z}{4} \text{ is}$$

- (A) (2,3,4) (B) (14,-8,-1) (C) (-2,0,-3) (D) (1,-2,-3)

Ans : (B)

Hint : Let the direction ratios of the normal to the plane be (a,b,c), then

$$\left. \begin{array}{l} 2a + 3b + 4c = 0 \\ \text{and } -2a - 4b + 4c = 0 \end{array} \right\} \Rightarrow \frac{a}{28} = \frac{b}{-16} = \frac{c}{-2} \therefore (a, b, c) \equiv (14, -8, -1)$$

44. The equation of the plane, which bisects the line joining the points (1,2,3) and (3,4,5) at right angles is,

- (A) $x+y+z = 0$ (B) $x+y-z = 9$ (C) $x+y+z = 9$ (D) $x+y-z+9 = 0$

Ans : (C)

Hint : Midpoint of the line segment joining (1,2,3) and (3,4,5) is (2,3,4) and the d.r.'s joining them are (2,2,2) $\equiv$ (1,1,1)
 $\therefore$ the required equation of the plane is $(x-2)+(y-3)+(z-4) = 0 \Rightarrow x+y+z = 9$

45. The limit of the interior angle of a regular polygon of n sides as $n \rightarrow \infty$ is

- (A) π (B) $\frac{\pi}{3}$ (C) $\frac{3\pi}{2}$ (D) $\frac{2\pi}{3}$

Ans : (A)

Hint : The limit of the interior angle of a regular polygon of n sides as $n \rightarrow \infty$ is π

46. Let $f(x) > 0$ for all x and $f'(x)$ exists for all x. If f is the inverse function of h and $h'(x) = \frac{1}{1+\log x}$. Then $f'(x)$ will be

- (A) $1+\log(f(x))$ (B) $1+f(x)$ (C) $1-\log(f(x))$ (D) $\log f(x)$

Ans : (A)

$$\text{Hint : } h(f(x)) = x \Rightarrow h'(f(x)) \cdot f'(x) = 1 \Rightarrow f'(x) = \frac{1}{h'(f(x))} \Rightarrow f'(x) = 1+\log(f(x))$$

47. Consider the function $f(x) = \cos x^2$. Then

- (A) f is of period 2π (B) f is of period $\sqrt{2\pi}$ (C) f is not periodic (D) f is of period π

Ans : (C)

Hint : $f(x) = \cos x^2$, $f(x+T) = \cos(x+T)^2 \neq f(x)$ for any $x \in \mathbb{R}$ and $T > 0$. So f(x) is not periodic

48. $\lim_{x \rightarrow 0^+} (e^x + x)^{\frac{1}{x}}$

- (A) Does not exist finitely (B) is 1 (C) is e^2 (D) is 2

Ans : (C)

$$\text{Hint : } \lim_{x \rightarrow 0^+} (e^x + x)^{\frac{1}{x}} = e^{\lim_{x \rightarrow 0^+} \frac{e^x + x - 1}{x}} = e^{\lim_{x \rightarrow 0^+} \left(\frac{e^x - 1}{x} + 1 \right)} = e^2$$

49. Let $f(x)$ be a derivable function, $f'(x) > f(x)$ and $f(0)=0$. Then

- (A) $f(x) > 0$ for all $x > 0$ (B) $f(x) < 0$ for all $x > 0$
(C) no sign of $f(x)$ can be ascertained (D) $f(x)$ is a constant function

Ans : (A)

Hint : Let $g(x) = e^{-x} f(x) \Rightarrow g'(x) = e^{-x} (f'(x) - f(x)) > 0$

$\Rightarrow g(x)$ is increasing function $\Rightarrow g(x) > g(0)$ for $x > 0 \Rightarrow e^{-x} f(x) > 0$, for $x > 0 \Rightarrow f(x) > 0$, for $x > 0$

50. Let $f : [1, 3] \rightarrow \mathbb{R}$ be a continuous function that is differentiable in $(1, 3)$ and $f'(x) = |f(x)|^2 + 4$ for all $x \in (1, 3)$. Then,

- (A) $f(3) - f(1) = 5$ is true (B) $f(3) - f(1) = 5$ is false
(C) $f(3) - f(1) = 7$ is false (D) $f(3) - f(1) < 0$ only at one point of $(1, 3)$

Ans : (B,C)*

Hint : By applying LMVT, there exist at least one $c \in (1, 3)$ such that $\frac{f(3) - f(1)}{3 - 1} = f'(c)$

$$\Rightarrow f(3) - f(1) = 2 \cdot |f(c)|^2 + 8 \Rightarrow f(3) - f(1) \geq 8$$

CATEGORY - II (Q51 to Q65)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, $\frac{1}{2}$ mark will be deducted.

51. Let $a = \min\{x^2 + 2x + 3 : x \in \mathbb{R}\}$ and $b = \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta^2}$. Then $\sum_{r=0}^n a^r b^{n-r}$ is

- (A) $\frac{2^{n+1} - 1}{3 \cdot 2^n}$ (B) $\frac{2^{n+1} + 1}{3 \cdot 2^n}$ (C) $\frac{4^{n+1} - 1}{3 \cdot 2^n}$ (D) $\frac{1}{2}(2^n - 1)$

Ans : (C)

Hint : $a = \min\{(x+1)^2 + 2\} = 2$

$$b = \lim_{\theta \rightarrow 0} \frac{2 \sin^2 \frac{\theta}{2}}{4 \left(\frac{\theta}{2}\right)^2} = \frac{1}{2}, \quad a^r b^{n-r} = \frac{2^r}{2^{n-r}} = 2^{2r-n} = \frac{4^r}{2^n}, \quad \sum_{r=0}^n a^r b^{n-r} = \frac{1}{2^n} \sum_{r=0}^n 4^r = \frac{1}{2^n} \left(\frac{1 - 4^{n+1}}{1 - 4} \right) = \frac{4^{n+1} - 1}{3 \times 2^n}$$

52. Let $a > b > 0$ and $I(n) = a^{\frac{1}{n}} - b^{\frac{1}{n}}$, $J(n) = (a - b)^{\frac{1}{n}}$ for all $n \geq 2$. Then

- (A) $I(n) < J(n)$ (B) $I(n) > J(n)$ (C) $I(n) = J(n)$ (D) $I(n) + J(n) = 0$

Ans : (A)

Hint : $\frac{I(n)}{J(n)} = \frac{a^{\frac{1}{n}} - b^{\frac{1}{n}}}{(a - b)^{\frac{1}{n}}} = \frac{x^{\frac{1}{n}} - 1}{(x - 1)^{\frac{1}{n}}} = \lim_{x \rightarrow 1^+} \frac{I(n)}{J(n)} = \lim_{x \rightarrow 1^+} \left(1 - \frac{1}{x} \right)^{\left(\frac{1 - \frac{1}{n}}{1 - 1} \right)} \therefore x > 0 \text{ \& } n \geq 2$

53. Let $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ be three unit vectors such that $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = \frac{1}{2}(\hat{\beta} + \hat{\gamma})$ where $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = (\hat{\alpha} \cdot \hat{\gamma})\hat{\beta} - (\hat{\alpha} \cdot \hat{\beta})\hat{\gamma}$. If $\hat{\beta}$ is not parallel to $\hat{\gamma}$, then the angle between $\hat{\alpha}$ and $\hat{\beta}$ is

- (A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Ans : (D)

Hint : $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = \frac{1}{2}(\hat{\beta} + \hat{\gamma})$, $(\hat{\alpha} \cdot \hat{\gamma})\hat{\beta} - (\hat{\alpha} \cdot \hat{\beta})\hat{\gamma} = \frac{1}{2}(\hat{\beta} + \hat{\gamma})$, $-(\hat{\alpha} \cdot \hat{\beta}) = \frac{1}{2}$, $-\cos \theta = \frac{1}{2}$, $\cos \theta = -\frac{1}{2}$, $\theta = 120^\circ = \frac{2\pi}{3}$

54. The position vectors of the points A, B, C and D are $3\hat{i} - 2\hat{j} - \hat{k}$, $2\hat{i} - 3\hat{j} + 2\hat{k}$, $5\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + \lambda\hat{k}$ respectively.

If the points A, B, C and D lie on a plane, the value of λ is

- (A) 0 (B) 1 (C) 2 (D) -4

Ans : (D)

Hint : $\vec{AB} = -\vec{i} - \vec{j} + 3\vec{k}$

$$\begin{vmatrix} -1 & -1 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & \lambda+1 \end{vmatrix} = 0$$

$$\vec{AC} = 2\vec{i} + \vec{j} + 3\vec{k}$$

$$\vec{AD} = \vec{i} + \vec{j} + (\lambda+1)\vec{k}$$

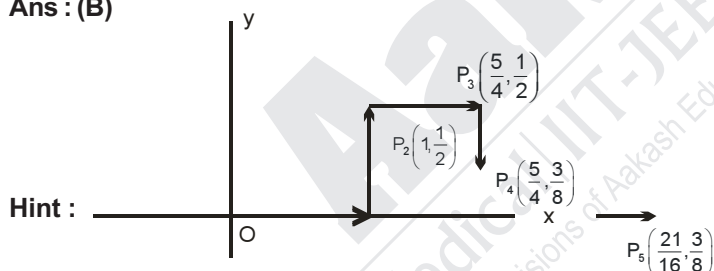
$$\begin{vmatrix} 0 & -1 & 3 \\ 1 & 1 & 3 \\ 0 & 1 & \lambda+1 \end{vmatrix} = 0$$

$$-(-\lambda - 1 - 3) = 0, (\lambda+4) = 0, \lambda = -4$$

55. A particle starts at the origin and moves 1 unit horizontally to the right and reaches P_1 , then it moves $\frac{1}{2}$ unit vertically up and reaches P_2 , then it moves $\frac{1}{4}$ unit horizontally to right and reaches P_3 , then it moves $\frac{1}{8}$ unit vertically down and reaches P_4 , then it moves $\frac{1}{16}$ unit horizontally to right and reaches P_5 and so on. Let $P_n = (x_n, y_n)$ and $\lim_{n \rightarrow \infty} x_n = \alpha$ and $\lim_{n \rightarrow \infty} y_n = \beta$. Then (α, β) is

- (A) (2, 3) (B) $\left(\frac{4}{3}, \frac{2}{5}\right)$ (C) $\left(\frac{2}{5}, 1\right)$ (D) $\left(\frac{4}{3}, 3\right)$

Ans : (B)



$$x_1 = 1, x_2 = 1, x_3 = \frac{5}{4}, x_4 = \frac{5}{4}, x_5 = \frac{21}{16}$$

$$x_n = 1 + \frac{1}{4} + \frac{1}{4^2} + \dots \infty = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

$$y_n = 0 + \frac{1}{2} - \frac{1}{8} + \dots \infty = \frac{\frac{1}{2}}{1 - \left(-\frac{1}{4}\right)} = \frac{2}{5} \quad (\alpha, \beta) = \left(\frac{4}{3}, \frac{2}{5}\right)$$

56. For any non-zero complex number z , the minimum value of $|z| + |z - 1|$ is

- (A) 1 (B) $\frac{1}{2}$ (C) 0 (D) $\frac{3}{2}$

Ans : (A)

Hint : $(|z| + |z-1|)_{\min} = 1$

57. The system of equations

$$\lambda x + y + 3z = 0$$

$$2x + \mu y - z = 0$$

$$5x + 7y + z = 0$$

has infinitely many solutions in $\mathbb{R}$. Then,

(A) $\lambda = 2, \mu = 3$

(B) $\lambda = 1, \mu = 2$

(C) $\lambda = 1, \mu = 3$

(D) $\lambda = 3, \mu = 1$

Ans : (C)

Hint :
$$\begin{vmatrix} \lambda & 1 & 3 \\ 2 & \mu & -1 \\ 5 & 7 & 1 \end{vmatrix} = 0$$

$$\lambda(\mu + 7) - 1(2 + 5) + 3(14 - 5\mu) = 0 \text{ or, } \lambda\mu + 7\lambda - 7 + 42 - 15\mu = 0 \text{ or, } \lambda\mu + 7\lambda - 15\mu = -35$$

if $\lambda = 1, \mu = 3$, satisfies.

58. Let $f : X \rightarrow Y$ and A, B are non-void subsets of Y , then (where the symbols have their usual interpretation)

(A) $f^{-1}(A) - f^{-1}(B) \supset f^{-1}(A - B)$ but the opposite does not hold

(B) $f^{-1}(A) - f^{-1}(B) \subset f^{-1}(A - B)$ but the opposite does not hold

(C) $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$

(D) $f^{-1}(A - B) = f^{-1}(A) \cup f^{-1}(B)$

Ans : (C)

Hint : According to definition $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$.

59. Let S, T, U be three non-void sets and $f : S \rightarrow T, g : T \rightarrow U$ be so that $g \circ f : S \rightarrow U$ is surjective. Then

(A) g and f are both surjective

(B) g is surjective, f may not be so

(C) f is surjective, g may not be so

(D) f and g both may not be surjective

Ans : (B)

Hint : $g \circ f : S \rightarrow U$ is onto

Let z be an arbitrary element of $U \therefore g \circ f : S \rightarrow U$ onto

there exists $x \in S$

$$g \circ f(x) = z \Rightarrow g(f(x)) = z; g(y) = z, \text{ where } y = f(x) \in T \text{ for all } z \in U, \text{ there exists } y = f(x) \in T \text{ such that } g(y) = z$$

$g : T \rightarrow U$ onto.

60. The polar coordinate of a point P is $\left(2, -\frac{\pi}{4}\right)$. The polar coordinate of the point Q , which is such that the line joining PQ is bisected perpendicularly by the initial line, is

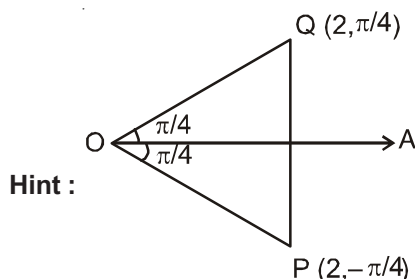
(A) $\left(2, \frac{\pi}{4}\right)$

(B) $\left(2, \frac{\pi}{6}\right)$

(C) $\left(-2, \frac{\pi}{4}\right)$

(D) $\left(-2, \frac{\pi}{6}\right)$

Ans : (A)



Point $Q = (2, \pi/4)$

61. The length of conjugate axis of a hyperbola is greater than the length of transverse axis. Then the eccentricity e is,

- (A) $=\sqrt{2}$ (B) $>\sqrt{2}$ (C) $<\sqrt{2}$ (D) $<\frac{1}{\sqrt{2}}$

Ans : (B)

Hint : $2b > 2a$ or, $b > a$ or, $\frac{b}{a} > 1$ or, $\frac{b^2}{a^2} > 1$ or, $e^2 - 1 > 1$ [$\because b^2 = a^2(e^2 - 1)$ or, $\frac{b^2}{a^2} = e^2 - 1$] or, $e^2 > 2$ or, $e > \sqrt{2}$

62. The value of $\lim_{x \rightarrow 0^+} \frac{x}{p} \left[\frac{q}{x} \right]$ is

- (A) $\frac{[q]}{p}$ (B) 0 (C) 1 (D) ∞

Ans : (A)

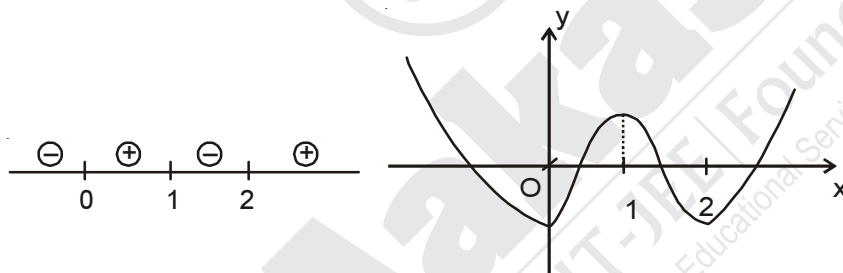
Hint : $= \lim_{x \rightarrow 0^+} \frac{x}{p} \left(\frac{q}{x} - \left\{ \frac{q}{x} \right\} \right) = \lim_{x \rightarrow 0^+} \frac{q}{p} - \lim_{x \rightarrow 0^+} \frac{x}{p} \left\{ \frac{q}{x} \right\} = \frac{q}{p} - 0 \times (\text{finite}) \left(0 \leq \left\{ \frac{q}{x} \right\} < 1 \right) = \frac{q}{p}$

63. Let $f(x) = x^4 - 4x^3 + 4x^2 + c$, $c \in \mathbb{R}$. Then

- (A) $f(x)$ has infinitely many zeros in $(1, 2)$ for all c (B) $f(x)$ has exactly one zero in $(1, 2)$ if $-1 < c < 0$
 (C) $f(x)$ has double zeros in $(1, 2)$ if $-1 < c < 0$ (D) Whatever be the value of c , $f(x)$ has no zero in $(1, 2)$

Ans : (B)

Hint : $f(x) = x^4 - 4x^3 + 4x^2 + c$, $c \in \mathbb{R}$ then $F'(x) = 4x^3 - 12x^2 + 8x = 4x(x^2 - 3x + 2) = 4x(x-1)(x-2)$



if $-1 < c < 0$

$$f(1) = 1 - 4 + 4 + c \\ = 1 + c > 0$$

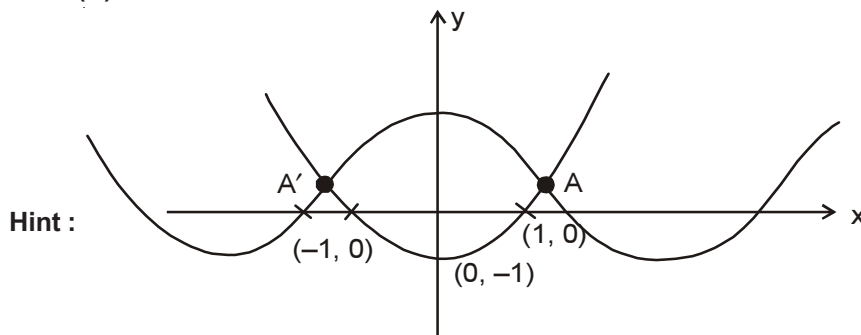
$$f(2) = 16 - 32 + 16 + c \\ = c < 0$$

$f(x)$ has exactly one zero in $(1, 2)$ if $-1 < c < 0$

64. The graphs of the polynomial $x^2 - 1$ and $\cos x$ intersect

- (A) at exactly two points (B) at exactly 3 points
 (C) at least 4 but at finitely many points (D) at infinitely many points

Ans : (A)



$$y = x^2 - 1, y = \cos x$$

$\cos x$ and $x^2 - 1$ intersect exactly at two points at A and B.

65. A point is in motion along a hyperbola $y = \frac{10}{x}$ so that its abscissa x increases uniformly at a rate of 1 unit per second. Then, the rate of change of its ordinate, when the point passes through (5, 2)

- (A) increases at the rate of $\frac{1}{2}$ unit per second
(B) decreases at the rate of $\frac{1}{2}$ unit per second
(C) decreases at the rate of $\frac{2}{5}$ unit per second
(D) increases at the rate of $\frac{2}{5}$ unit per second

Ans : (C)

Hint : $y = \frac{10}{x}$, $\frac{dx}{dt} = 1$ unit/second

$$\frac{dy}{dt} = -\frac{10}{x^2} \cdot \frac{dx}{dt}$$

$$= -\frac{10}{25} \times 1 = -\frac{2}{5} \text{ unit/sec}$$

decreases at the rate of $\frac{2}{5}$ unit per second.

CATEGORY - III (Q66 to Q75)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will be considered wrong, but there is no negative marking for the same and zero marks will be awarded.

66. Let $I_n = \int_0^1 x^n \tan^{-1} x \, dx$. If $a_n I_{n+2} + b_n I_n = c_n$ for all $n \geq 1$, then

- (A) a_1, a_2, a_3 are in G.P. (B) b_1, b_2, b_3 are in A.P. (C) c_1, c_2, c_3 are in H.P. (D) a_1, a_2, a_3 are in A.P.

Ans : (B,D)

Hint : $(n+1)I_n + (n+3)I_{n+2} = \int_0^1 \{(n+1)x^n + (n+3)x^{n+2}\} \tan^{-1} x \, dx$

$$= \left[\left\{ (n+1) \frac{x^{n+1}}{n+1} + (n+3) \frac{x^{n+3}}{n+3} \right\} \tan^{-1} x \right]_0^1 - \int_0^1 x^{n+1} (1+x^2) \cdot \frac{1}{1+x^2} dx = \left[(x^{n+1} + x^{n+3}) \tan^{-1} x \right]_0^1 - \left[\frac{x^{n+2}}{n+2} \right]_0^1$$

$$= \frac{\pi}{2} - \frac{1}{n+2}$$

$$c_n = \frac{\pi}{2} - \frac{1}{n+2}$$

$$\therefore a_n = n+3; b_n = n+1; c_n = \frac{\pi}{2} - \frac{1}{n+2}$$

67. Two particles A and B move from rest along a straight line with constant accelerations f and h respectively. If A takes m seconds more than B and describes n units more than that of B acquiring the same speed, then

- (A) $(f+h)m^2 = fhn$ (B) $(f-fh)m^2 = fhn$ (C) $(h-f)n = \frac{1}{2}fhn^2$ (D) $\frac{1}{2}(f+h)n = fhn^2$

Ans : (C)

Hint : $S + n = \frac{1}{2}f(t+m)^2$ and $S = \frac{1}{2}ht^2, V = ht$

$$\therefore \frac{1}{2}ht^2 + n = \frac{1}{2}f(t+m)^2 \text{ ----- (1)}$$

$$\text{Also } V = 0 + ht = 0 + f(t+m) \Rightarrow t+m = \frac{ht}{f}$$

From equation (1),

$$\frac{1}{2}ht^2 + n = \frac{1}{2}f\left(\frac{ht}{f}\right)^2 \Rightarrow t^2 = \frac{2nf}{h(h-f)}$$

Also,

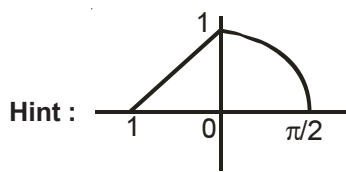
$$ht = f(t+m) \Rightarrow t^2 = \frac{m^2f^2}{(h-f)^2}$$

$$\therefore \frac{2nf}{h(h-f)} = \frac{m^2f^2}{(h-f)^2} \Rightarrow 2n = \frac{m^2fh}{h-f} \Rightarrow n(h-f) = \frac{1}{2}fhm^2$$

68. The area bounded by $y = x + 1$ and $y = \cos x$ and the x-axis, is

- (A) 1 sq. unit (B) $\frac{3}{2}$ sq. unit (C) $\frac{1}{4}$ sq. unit (D) $\frac{1}{8}$ sq. unit

Ans : (B)



Hint : $\text{Area} = \frac{1}{2} \times 1 \times 1 + \int_0^{\pi/2} \cos x dx = \frac{3}{2} \text{ sq. unit}$

69. Let x_1, x_2 be the roots of $x^2 - 3x + a = 0$ and x_3, x_4 be the roots of $x^2 - 12x + b = 0$. If $x_1 < x_2 < x_3 < x_4$ and x_1, x_2, x_3, x_4 are in G.P. then ab equals

- (A) $\frac{24}{5}$ (B) 64 (C) 16 (D) 8

Ans : (B)

Hint : $x_1 + x_2 = 3; x_1 \cdot x_2 = a$

$$x_3 + x_4 = 12; x_3 \cdot x_4 = b$$

Let r be the common ratio of G.P., then

$$\frac{x_1(1+r)}{x_1r^2(1+r)} = \frac{3}{12} \Rightarrow r = \pm 2$$

Take $r = 2$ ($\because$ G.P. is increasing)

$$\therefore x_1 + x_2 = 3 \Rightarrow x_1(1+r) = 3 \Rightarrow x_1 = 1$$

$$\therefore ab = x_1 x_2 x_3 x_4 = 1 \cdot 2 \cdot 4 \cdot 8 = 64$$

70. If $\theta \in \mathbb{R}$ and $\frac{1 - i \cos \theta}{1 + 2i \cos \theta}$ is real number, then θ will be (when I : Set of integers)

- (A) $(2n+1)\frac{\pi}{2}, n \in I$ (B) $\frac{3n\pi}{2}, n \in I$ (C) $n\pi, n \in I$ (D) $2n\pi, n \in I$

Ans : (A)

Hint : $\therefore \frac{1 - i \cos \theta}{1 + 2i \cos \theta}$ is real $\Rightarrow \frac{1 - i \cos \theta}{1 + 2i \cos \theta} = \frac{1 + i \cos \theta}{1 - 2i \cos \theta}$

$$\Rightarrow 1 - 3i \cos \theta - 2 \cos^2 \theta = 1 + 3i \cos \theta - 2 \cos^2 \theta \Rightarrow \cos \theta = 0$$

$$\Rightarrow \theta = n\pi + \pi/2 \quad (n \in I)$$

71. Let $A = \begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix}$. Then the roots of the equation $\det(A - \lambda I_3) = 0$ (where I_3 is the identity matrix of order 3) are

- (A) 3, 0, 3 (B) 0, 3, 6
(C) 1, 0, -6 (D) 3, 3, 6

Ans : (B)

Hint : Let $(A - \lambda I_3) = 0 \Rightarrow \begin{vmatrix} 3-\lambda & 0 & 3 \\ 0 & 3-\lambda & 0 \\ 3 & 0 & 3-\lambda \end{vmatrix} = 0$

$$\Rightarrow (3-\lambda)^3 - 9(3-\lambda) = 0 \Rightarrow (3-\lambda)[(3-\lambda)^2 - 3^2] = 0$$

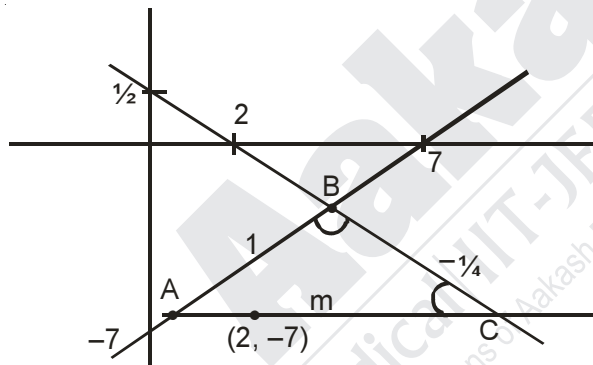
$$\Rightarrow 3-\lambda = 0 \text{ or } 3-\lambda-3 = 0 \text{ or } 3-\lambda+3 = 0 \Rightarrow \lambda = 0, 3 \text{ or } 6$$

72. Straight lines $x - y = 7$ and $x + 4y = 2$ intersect at B. Points A and C are so chosen on these two lines such that $AB = AC$. The equation of line AC passing through $(2, -7)$ is

- (A) $x - y - 9 = 0$ (B) $23x + 7y + 3 = 0$
(C) $2x - y - 11 = 0$ (D) $7x - 6y - 56 = 0$

Ans : (A, B)

Hint :



Let the slope of line AC be m ,

then, $AB = AC \Rightarrow \angle ABC = \angle BCA$

$$\therefore \left| \frac{m + \frac{1}{4}}{1 - \frac{m}{4}} \right| = \left| \frac{-\frac{1}{4} - 1}{1 - \frac{1}{4}} \right| \Rightarrow m = \frac{-23}{7}, 1$$

$\therefore$ Equation of line is: $23x + 7y + 3 = 0$, $x - y = 9$

73. Equation of a tangent to the hyperbola $5x^2 - y^2 = 5$ and which passes through an external point $(2, 8)$ is

- (A) $3x - y + 2 = 0$ (B) $3x + y - 14 = 0$
(C) $23x - 3y - 22 = 0$ (D) $3x - 23y + 178 = 0$

Ans : (A, C)

Hint : Let the tangent be $y = mx \pm \sqrt{m^2 - 5}$

Since it passes through $(2, 8) \Rightarrow (8 - 2m)^2 = m^2 - 5$

$$\Rightarrow 3m^2 - 32m + 69 = 0 \Rightarrow 3m^2 - 9m - 23m + 69 = 0 \Rightarrow (3m - 23)(m - 3) = 0 \Rightarrow m = 3 \text{ or } \frac{23}{3}$$

74. Let f and g be differentiable on the interval I and let $a, b \in I, a < b$. Then
- (A) If $f(a) = 0 = f(b)$, the equation $f'(x) + f(x)g'(x) = 0$ is solvable in (a, b) .
- (B) If $f(a) = 0 = f(b)$, the equation $f'(x) + f(x)g'(x) = 0$ may not be solvable in (a, b) .
- (C) If $g(a) = 0 = g(b)$, the equation $g'(x) + kg(x) = 0$ is solvable in $(a, b), k \in \mathbb{R}$
- (D) If $g(a) = 0 = g(b)$, the equation $g'(x) + kg(x) = 0$ may not be solvable in $(a, b), k \in \mathbb{R}$

Ans : (A, C)

Hint : $f(a) = 0 = f(b) \Rightarrow f'(a).f'(b) < 0$

let $h(x) = f'(x) + f(x)g'(x)$ $h(a) = f'(a), h(b) = f'(b)$

$\therefore h(a).h(b) < 0 \Rightarrow h(x) = 0$ has root(s) between (a, b)

Similarly, $g'(x) + kg(x) = 0$ has root(s) between (a, b) as $g(a) = 0 = g(b)$

75. Consider the function $f(x) = \frac{x^3}{4} - \sin \pi x + 3$
- (A) $f(x)$ does not attain value within the interval $[-2, 2]$
- (B) $f(x)$ takes on the value $2\frac{1}{3}$ in the interval $[-2, 2]$
- (C) $f(x)$ takes on the value $3\frac{1}{4}$ in the interval $[-2, 2]$
- (D) $f(x)$ takes no value $p, 1 < p < 5$ in the interval $[-2, 2]$

Ans : (B, C)

Hint : $f(-2) = 1$ and $f(2) = 5$ Also f is continuous.

Therefore by Intermediate value theorem, function f takes all values between 1 to 5.

□ □ □